⚠ DC-Analysis → neglect all small signals, AC analysis → neglect all large (bzw constant) signals

1 Basics

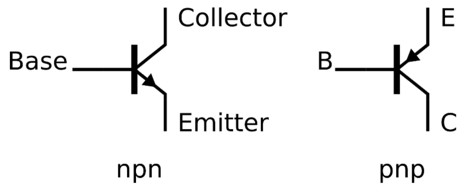
1.1 Constants

Elementary Charge	q	$1.602 \cdot 10^{-19}$	C
Thermal Voltage	$\boxed{V_T = \frac{kT}{q}}$	gleich für alle 26	mV
Early Voltage	V_A	$30 \ldots 100$	V
Saturation Current	I_C	$10^{-15} \ldots 10^{-12}$	A
Current Gain	β	$100 \ldots 500$	

1.2 Fundamentals

- **Resistor:** $u_R = Ri_R$
- **Capacitor:** $i_C = C\frac{du_c}{dt}$, $\quad E = \frac{1}{2}CU^2$
- **Inductor:** $u_L = L\frac{di_L}{dt}$, $\quad E = \frac{1}{2}LI^2$
- **Kirchhoff:** $\sum_{Mesh} U_k = 0$, $\quad \sum_{Node} I_k = 0$
- **Thevenin:** $U_0 = U_{Open}$
- **Norton:** $I_0 = I_{Short}$

1.3 BJT



1.3.1 Large Signal ≙ Operating point analysis

reverse saturation current of BJT. ~10^{-15} to 10^{-12} A

$$NPN : I_C = I_S e^{\frac{V_{BE}}{V_T}} \left(1 + \frac{V_{CE}}{V_A}\right)$$

=0 when neglecting early voltage ($V_A \to \infty$)

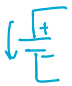
$$PNP : I_C = I_S e^{-\frac{V_{BE}}{V_T}} \left(1 - \frac{V_{CE}}{V_A}\right)$$

$$I_B = \frac{I_C}{\beta}$$

($I_B \to 0$ for $\beta \to \infty$
⇒ $I_E = I_C$ when $\beta \to 0$)

$$I_E = (1+\beta) I_B = \frac{1}{\alpha} I_C$$

$$\beta = \frac{\alpha}{1-\alpha} \Leftrightarrow \alpha = \frac{\beta}{1+\beta}$$



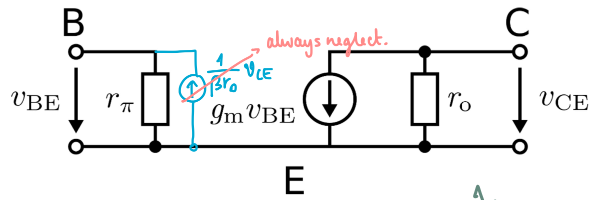
Voltage drop always + to - positive.

⚠ if all inputs are constant → every Signal in Circuit is also constant!

⚠ Both MOSFET and BJT have SSE & have to find operating point.

⚠ KVL: always same φ to same φ! (Best GND to GND).

1.3.2 Small Signal



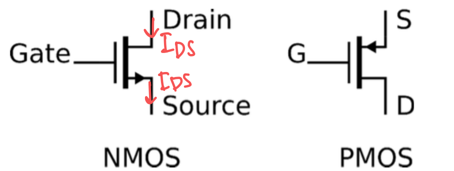
$$g_m = \frac{I_C}{V_T}, \frac{1}{g_\pi} = r_\pi = \frac{\beta}{g_m}, \frac{1}{g_o} = r_O = \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C} = \frac{1}{g_o}$$

1.3.3 Modes

Here for NPN, for PNP all operators flipped.

Mode	Voltage	Current
Active (means functioning for BJT)	$V_{BE} > 0,\ V_{CB} > 0$	$I_C = I_S e^{\frac{V_{BE}}{V_T}}$
Saturation	$V_{BE} > 0,\ V_{CB} \leq 0$	$I_C = -I_S$
Cutoff	$V_{BE} \leq 0,\ V_{CB} > 0$	$I_C = 0$

1.4 MOSFET



1.4.1 Large Signal → neglect all small signals!

$$NMOS : I_D^{>0} = \frac{K'}{2}\frac{W}{L}(V_{GS} - V_t)^2 (1 + \cancel{\lambda V_{DS}})$$

0 for $\lambda \approx 0$

$$PMOS : I_D^{<0} = -\frac{K'}{2}\frac{W}{L}(V_{GS} - V_t)^2 (1 - \cancel{\lambda V_{DS}})$$

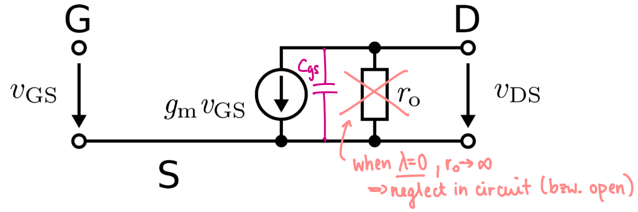
0 for $\lambda \approx 0$

⚠ $I_G = 0$ @DC (in saturation mode)

MOSFET Parameters:

K': Intrinsic transconductance coefficient
V_t: Threshold voltage
W/L: Gate width / Channel length (physical properties of the component!)
λ: Characteristic length (describes the channel length modulation. $\lambda \approx 0$ ⇒ we neglect the channel length modulation.

1.4.2 Small Signal → neglect all large signals!



$$\frac{1}{g_o} = r_O \approx \frac{1}{\lambda I_D}, \quad g_m = \sqrt{\frac{2K'W}{L} I_D}$$

→ Flip horizontally for PMOS

1.4.3 Modes

Here for NMOS, for PMOS all operators flipped.
($V_{DS} > 0, V_{GS} > 0$) ($V_{DS} < 0, V_{GS} < 0$)

Mode	Voltage	Current
Cutoff (not functioning)	$V_{GS} < V_t$	$I_D = 0$
Linear	$V_{DS} \leq V_{GS} - V_t$ ($V_{DS} < 0$)	$I_D = \frac{K'}{2}\frac{W}{L}(2(V_{GS} - V_t)V_{DS} - V_{DS}^2)$
Saturation (means functioning for MOSFET)	$V_{DS} > V_{GS} - V_t$ (≥ 0; ⇒ $V_{GS} < V_t$ ⇒ $V_{GS} > V_t$)	$I_D = \frac{K'}{2}\frac{W}{L}(V_{GS} - V_t)^2$

1.5 Operating Point Analysis

Use large signal equations and Kirchhoff's laws.

- $L \to$ short circuit
- $C \to$ open circuit
- Replace small signal sources

1.6 Small Signal Equivalent

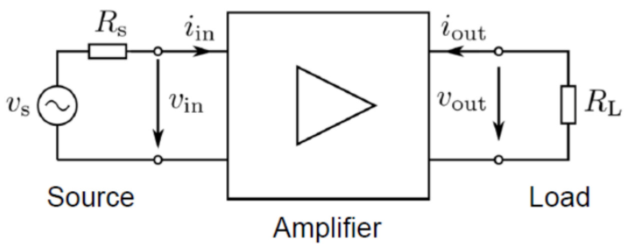
For small signals the transistors behave almost linearly and small signal models can be used.

1. Replace all BJTs and MOSFETs with their SSE
2. Keep all the passive elements
3. Replace constant voltage and current sources
   $U_{DC} \to$ short circuit — also $V_{DD} \to$ GND!
   $I_{DC} \to$ open circuit
4. Label elements, voltages and currents
5. Connect nodes with equal potential

Both BJT & MOSFET:
→ have SSE
→ have to find operating point

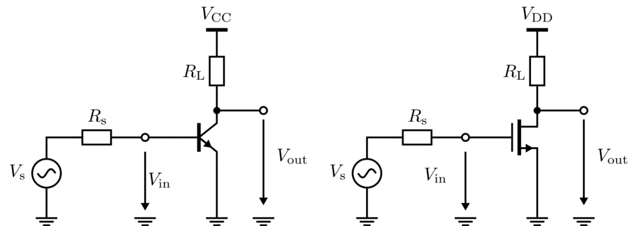
2 Single Transistor Amplifiers

2.1 Two-Port Network



Impedance	$Z_{in} = \frac{v_{in}}{i_{in}}\Big\vert_{i_{out}=0}$	$Z_{out} = \frac{v_{out}}{i_{out}}\Big\vert_{v_s=0}$
Gain	$A_V = \frac{v_{out}}{v_s}\Big\vert_{i_{out}=0}$	$A_i = \frac{i_{out}}{i_s}\Big\vert_{v_{out}=0}$
Transconductance	$G_m = \frac{i_{out}}{v_{in}}\Big\vert_{v_{out}=0}$	
Resistance	$R = \frac{v_{out}}{i_{in}}\Big\vert_{i_{out}=0}$	

2.2 Common-Emitter/Source Amplifier



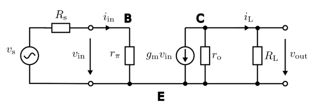
BJT (C.Emitter)

$$R_{in} = \frac{v_{in}}{i_{in}} = r_\pi$$

$$R_{out} = \frac{v_{out}}{i_{out}} = r_o$$

$$A_V = \frac{v_{out}}{v_s} = -g_m(r_o||R_L)\frac{r_\pi}{r_\pi+R_S} \approx -g_m R_L \frac{r_\pi}{r_\pi+R_S}$$

$$A_I = \frac{R_S}{R_S+r_\pi} g_m r_\pi = \frac{R_S}{R_S+r_\pi}\beta$$



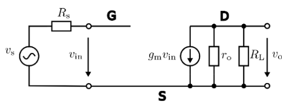
MOSFET (C.Source)

$$R_{in} = \infty$$

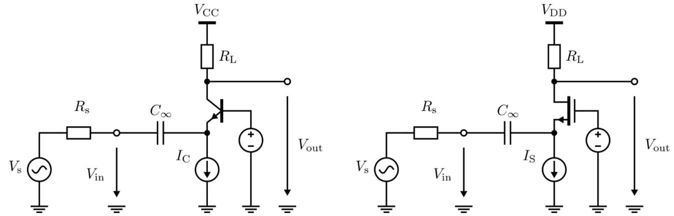
$$R_{out} = r_o$$

$$A_V = \frac{v_{out}}{v_s} = \frac{v_{out}}{v_{in}} = -g_m(r_o||R_L) \approx -g_m R_L$$

$$A_I = \infty$$



2.3 Common-Base/Gate Amplifier



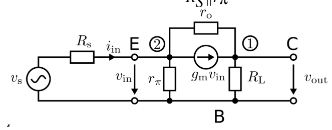
BJT (C.Base)

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{1}{\frac{1}{r_\pi}+\frac{1}{r_o}+g_m} \approx \frac{1}{g_m}$$

$$R_{out} \approx ((r_\pi \parallel R_S)g_m+1)r_o \approx \beta r_o$$

$$A_V = \frac{v_{out}}{v_s} \approx \frac{R_{in}}{R_s+R_{in}} g_m R_L \approx \frac{g_m R_L}{1+g_m R_S}$$

$$A_I = \frac{i_{out}}{i_s} = -\frac{g_m}{\frac{1}{R_S\|r_\pi}+g_m}$$



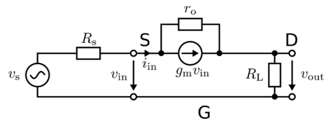
MOSFET (C.Gate)

$$R_{in} \approx \frac{1}{g_m}$$

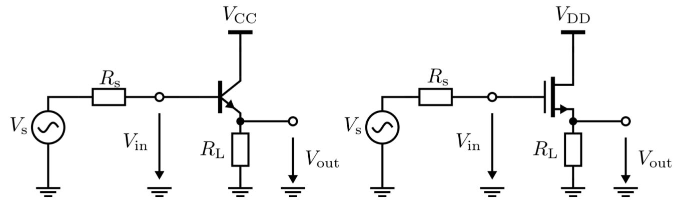
$$R_{out} \approx (1+g_m R_s)r_o$$

$$A_V = \frac{v_{out}}{v_s} \approx \frac{g_m R_L}{1+g_m R_S}$$

$$A_I = \frac{i_{out}}{i_s} = -1$$



2.4 Common-Collector/Drain Amplifier



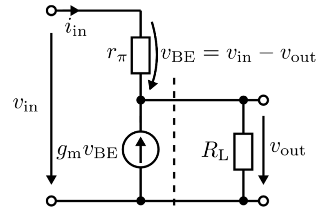
BJT (C.Collector)

$$R_{in} = r_\pi + (1+\beta)R_L$$

$$R_{out} = \frac{1}{g_m+\frac{1}{r_\pi}} \approx \frac{1}{g_m} \approx \beta r_o$$

$$A_V \approx \frac{1}{1+\frac{1}{g_m R_L}}$$

$$A_I = 1+\beta$$



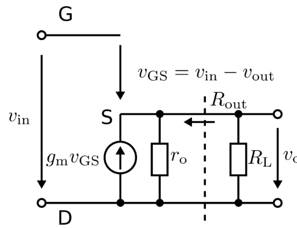
MOSFET (C.Drain)

$$R_{in} = \infty$$

$$R_{out} = \frac{1}{g_m}$$

$$A_V \approx \frac{1}{1+\frac{1}{g_m R_L}}$$

$$A_I = \infty$$



2.5 Comparison

	CE/CS	CC/CD	CB/CG
Voltage Gain	$\ll -1$	≈ 1	$\gg 1$
Current Gain	$\ll -1$	Moderate	≈ -1
R_{in}	High	High	Low
R_{out}	High	Low	High

This makes CC/CD good voltage sources, while CB/CG are good current sources.

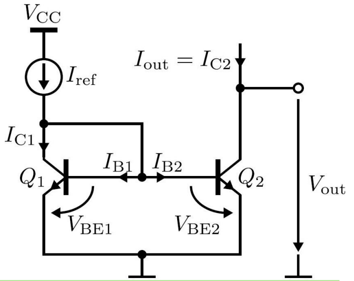
3 Current Mirrors

3.1 BJT

$$I_{out} = I_{C2} = \frac{I_{S2}}{I_{S1}} I_{C1} = \frac{I_{ref}}{1+\frac{2}{\beta}}$$

$$I_{ref} = I_{C1} + I_{B1} + I_{B2}$$

$$v_\tau = \ln\left(\frac{I_{C1}}{I_{S1}}\right) V_T \cdot \ln\left(\frac{I_{C2}}{I_{S2}}\right)$$



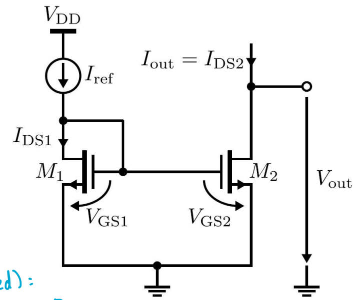
3.2 MOSFET

Gain $B = \frac{W_2}{W_1} = \frac{I_{out}}{I_{ref}}$

$$I_{out} = I_{DS2} = I_0 = \frac{W_2}{L_2}\frac{L_1}{W_1} \cdot I_{ref}$$

$$I_{ref} = I_{DS1}$$

$$R_{out} = r_{O2}$$

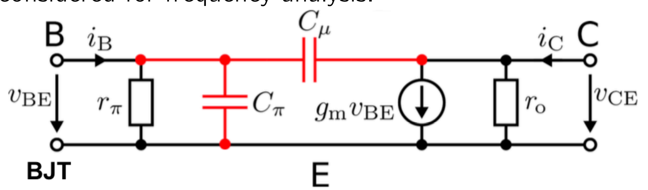


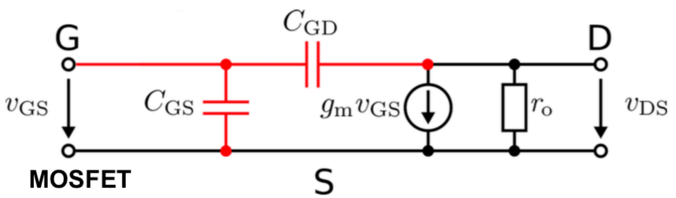
Wenn mehrere nebeneinander (Cascaded): multiply individual gains: $B = B_1 \cdot B_2 \cdot \ldots \cdot B_n$

4 Frequency Response of Amplifiers

4.1 Parasitic Capacitancies

Capacitive effects between the terminals may have to be considered for frequency analysis.

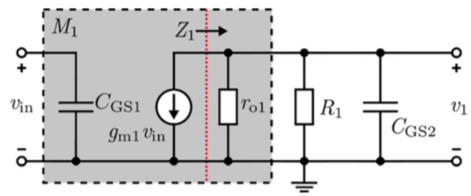




4.2 Multistage Amplifier

Single transistor amplifiers have limited gain range. Miltistage aplifiers help to spread the gain per stage.
$|A_v(j\omega)|$ = Amplification $\quad \phi_V(j\omega)$ = Phase shift

4.2.1 Calculation of 1st Stage



$$v_1 \approx -\frac{g_{m1}R_1}{1+sC_{GS2}R_1} v_{in}$$

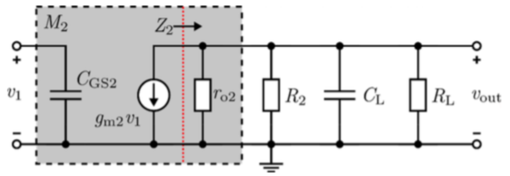
$$v_{out} = -\frac{g_{m2}R_2}{1+sC_LR_2} v_1 = \frac{g_{m2}R_2}{1+sC_LR_2}\frac{g_{m1}R_1}{1+sC_{GS2}R_1} v_{in}$$

For a sinusoidal signal with angular frequency ω:

$$|A_V(j\omega)| = \frac{g_{m2}R_2 \cdot g_{m1}R_1}{\sqrt{1+(j\omega C_LR_2)^2}\sqrt{1+(j\omega C_{GS2}R_1)^2}}$$

$$\phi_V(j\omega) = -\tan^{-1}(\omega C_{GS2}R_1) - \tan^{-1}(\omega C_LR_2)$$

4.2.2 Calculation of 2nd Stage



$$v_{out} = -\frac{g_{m2}R_2}{1+sC_LR_2} v_1$$

$$A_{v2}(s) = \frac{v_{out}(s)}{v_1(s)} = -\frac{g_{m2}R_2}{1+sC_LR_2}$$

For a sinusoidal signal with angular frequency ω

$$|A_{V2}(j\omega)| = \frac{g_{m2}R_2}{\sqrt{1+(j\omega C_LR_2)^2}}$$

$$\phi_{V2}(j\omega) = -180° - \tan^{-1}(\omega C_LR_2)$$

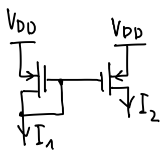
4.3 Bode Plots

	Amplitude	Phase
Constant	20dB $\log\lvert K\rvert$	0° if K≥0, 180° else
Negative Pole	-20dB/dec	-90° over 2 dec
Positive Pole	-20dB/dec	+90° over 2 dec
Negative Zero	+20dB/dec	+90° over 2 dec
Positive Zero	+20dB/dec	-90° over 2 dec
Zero at Origin	+20dB/dec	+90°
Pole at Origin	-20dB/dec	-90°

For voltage and current use: $20\log_{10}(|A(j\omega)|)$
For power use: $10\log_{10}(|A(j\omega)|)$

Resistor: A=const, $\phi = 0°$
Capacitor: A=-20dB/dec, $\phi = -90°$
Inductor: A=+20dB/dec, $\phi = +90°$

Unity Current Mirror:



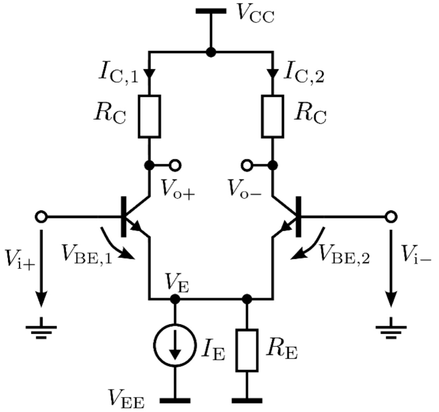
ideally, $\boxed{I_2 = I_1}$ (input current is copied to the output)

if $V_{GS1} = V_{GS2}$ and $V_{DS1} = V_{DS2}$, $I_1 = I_2$

5 Differential Amplifiers

5.1 Differential Amplifiers

5.1.1 BJT



General:

$$v_{icm} = \frac{V_{i+} + V_{i-}}{2}$$

$$v_{id} = v_{i+} - v_{i-}$$

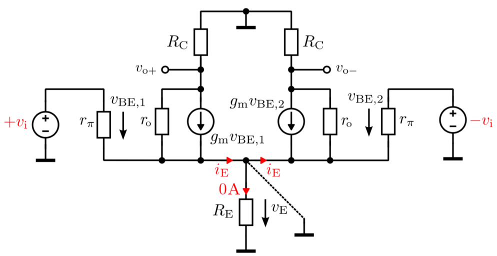
$$v_{od} = v_{o+} - v_{o-}$$

$$V_{i\pm} = v_{icm} \pm \frac{1}{2} v_{id}$$

Differential Mode: ($v_{i+} = -v_{i-} = v_{id}$)

$v_{o+} = -v_{o-} = v_{od}$

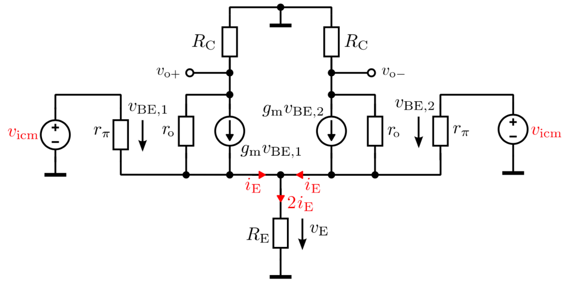
$$A_{vd} = \frac{v_{od}}{v_{id}} = -g_m R_C$$



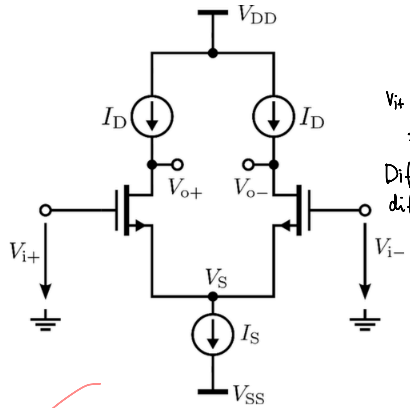
Common Mode: ($v_{i+} = v_{i-} = v_{icm}$)

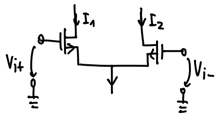
$v_{o+} = v_{o-} = v_{ocm}$

$$A_{vcm} = \frac{v_{ocm}}{v_{icm}} = -\frac{R_C}{2R_E}$$



5.1.2 MOSFET





Differential pair. The difference of their output currents is a function of the differential input voltage:

$$\boxed{\Delta I = I_1 - I_2 = f(V_{i+} - V_{i-})}$$

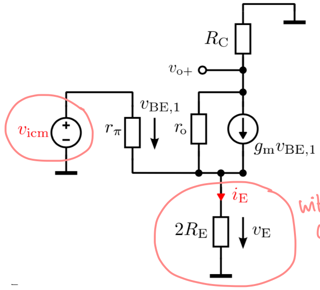
$I_1 = I_2$ if $V_{GS,1} = V_{GS,2}$ and $V_{DS,1} = V_{DS,2}$.

$$A_{vd} = -g_m r_o \quad (\gg -g_m R_L)$$

5.2 Half Circuit

Idea: Circuit symmetrical: only analyse half of circuit! Other half behaves identically!

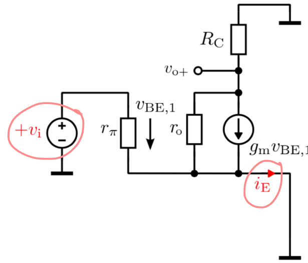
5.2.1 Common Mode



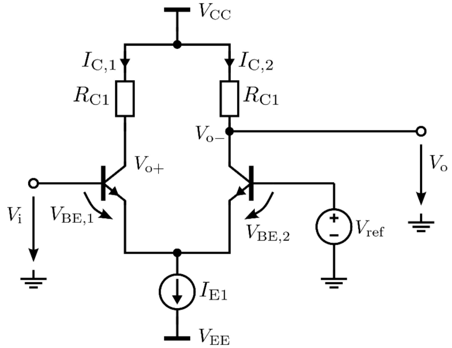
with this current here

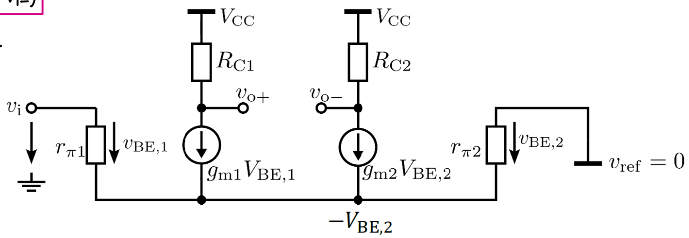
5.2.2 Differential Mode

Only possible if virtual ground at the emitter.



5.3 Single ended Differential Amplifier





$$v_{o+} = -\frac{g_{m1} R_{C1}}{2}, \quad v_{o-} = \frac{g_{m2} R_{C1}}{2}, \quad v_{id} = \frac{v_{i+}}{2}$$

$$g_{m1} = g_{m2} = g_m, \quad R_{C1} = R_{C2} = R_C$$

$$A_{vd} = \frac{v_{od}}{v_i} = -g_m R_C$$

5.4 Common-Mode Rejection Rate

How strong a common mode signal is attenuated compared to a differential signal.

$$G = \frac{A_{vd}}{A_{vcm}} = \frac{-g_m R_C}{-\frac{R_C}{2R_E}} = 2 g_m R_E$$

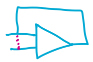
In dB: $CMRR = G_{dB} = 20 \cdot \log_{10} G$

5.5 Operational Amplifiers

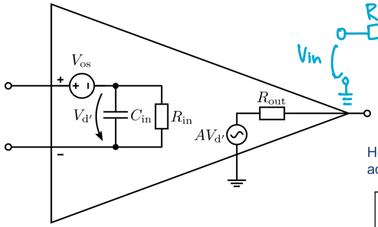
5.5.1 Ideal OpAmp

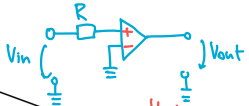
- No common mode gain
- Infinite input impedance
- Zero output impedance
- Infinite differential open loop gain
- Infinite bandwidth
- Infinite CMRR
- $V_{out} = A_V(V_{i+} - V_{i-}) = A_V \cdot V_D$

anykind of mit Rückkopplung:



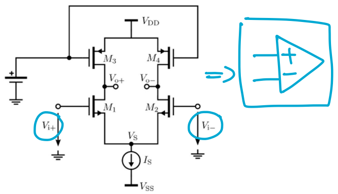
5.5.2 Non-ideal OpAmp:





$\hookrightarrow \frac{V_{out}}{V_{in}} = +\infty$

How an Op Amp actually looks like

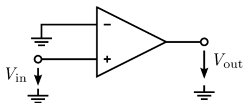


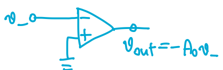
- Lowpass characteristic in TF
- Offset at input and output
- Load has impact on the TF
- $A_V(s) = \frac{V_{out}}{V_d} \approx \frac{A_0}{\left(1+\frac{s}{\omega_{po}}\right)}$

5.6 OpAmp Basic Circuits

Voltage Comparator

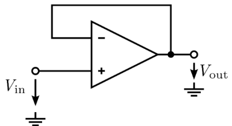
$V_{out} = sign(V_{in}) \cdot V_{cc}$





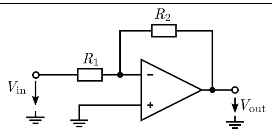
Voltage Follower

$V_{out} = V_{in}$



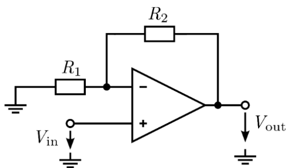
Inverting

$V_{out} = -\frac{R_2}{R_1} \cdot V_{in}$



Non-Inverting

$V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{in}$



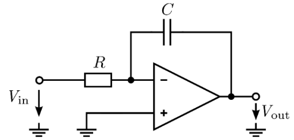
non inverting

Integrator

$V_{out} = + \frac{V_{in}(s)}{sRC}$

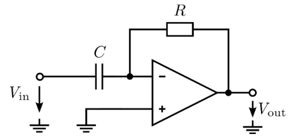
$= V_{out}(0) - \frac{1}{RC}\int_0^t V_{in}(\tau)d\tau$

Phase −90° for all frequencies



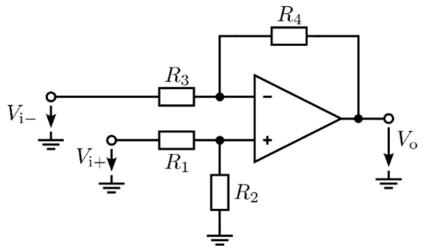
Diffentiator

$V_{out} = V_{in}(s)sRC = -RC\frac{dV_{in}(t)}{dt}$



6 Instrumentation Amplifiers

6.1 Basic Instrumental Amp

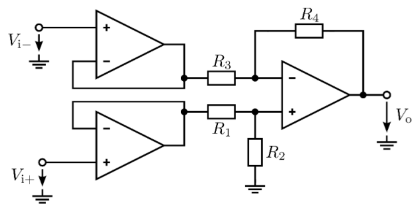


$$V_0 = \frac{R_2}{R_1}\frac{1+\frac{R_4}{R_3}}{1+\frac{R_2}{R_1}}V_{i+} - \frac{R_4}{R_3}V_{i-}$$

$$R_1 = R_3, \; R_2 = R_4$$

$$V_0 = G \cdot V_{i+} - G \cdot V_{i-}, \quad G = \frac{R_4}{R_3} = \frac{R_2}{R_1}$$

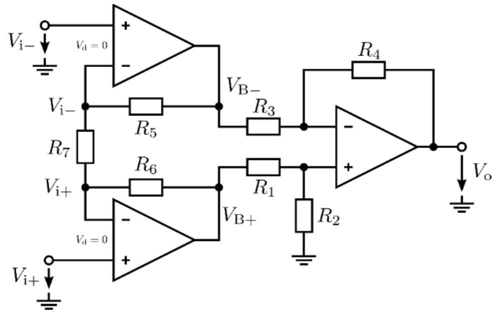
6.2 Buffered Instrumental Amp



$$V_0 = V_{icm}\left(\frac{R_2(R_3+R_4)}{R_3(R_1+R_2)} - \frac{R_4}{R_3}\right) + \frac{V_{id}}{2}\left(\frac{R_2(R_3+R_4)}{R_3(R_1+R_2)} + \frac{R_4}{R_3}\right)$$

$$CMRR = \frac{A_d}{A_{cm}} = \frac{V_0/V_{id}}{V_0/V_{icm}} = \frac{R_2(R_3+R_4)+R_4(R_1+R_2)}{2(R_2R_3 - R_4R_1)}$$

6.3 Input Stage Gain



Input Stage: (Differential and common mode gain)

$$A_B = \frac{V_{BD}}{V_{id}} = \frac{V_{B_+} - V_{B_-}}{V_{i_+} - V_{i_-}} = \frac{R_5 + R_6}{R_7}$$

$$A_{cm,B} = \frac{V_{B_+} + V_{B_-}}{V_{i_+} + V_{i_-}} = 1$$

(No current through R_5, R_6, R_7)

Total: (Differential and common mode gain)

$$A_{d'} = \frac{V_O}{V_{id}} = \frac{A_B}{2}\left(\frac{R_2(R_3+R_4)}{R_3(R_1+R_2)} + \frac{R_4}{R_3}\right)$$

$$A_{cm} = A_{cm,B}\left(\frac{R_2(R_3+R_4)}{R_3(R_1+R_2)} - \frac{R_4}{R_3}\right)$$

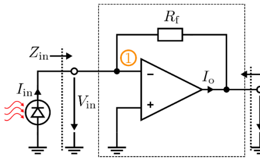
$$CMMR = \frac{A'_d}{A_{cm}} = A_B\frac{A_d}{A_{cm}}$$

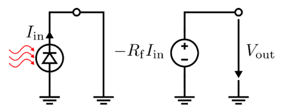
→ CMRR increased by factor A_B due to an input stage!

7 Various Amplifiers

7.1 Transimpedance Amplifier

$$V_{in} \approx 0, \quad Z_{in} \approx 0, \quad Z_{out} \approx 0$$





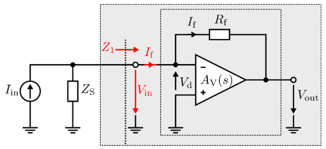
Frequency Response I

$Z_S \quad = (R_S || (C_S + C_{in}))$

$Z_1(s) = \frac{R_f}{1+A_V(s)}$

$I_f \quad = \frac{V_{in}+A_V V_{in}}{R_f}$

$Z_{T'} \quad = -A_V(s) Z_1 \approx \frac{-R_f}{1+\frac{s}{A_0 \omega_{p0}}}$



Frequency Response II

$Z_{in} \quad = (Z_S || Z_1)$

$Z_T \quad = \frac{V_{out}}{I_{in}}$

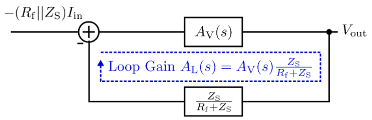
$I_f \quad = I_{in} - \frac{V_{in}}{Z_S} I_{in} - \frac{I_f Z_1}{Z_S}$

$V_{out} \quad = -A_V(s) Z_1 I_f = -\frac{Z_1 Z_S}{Z_1+Z_S} A_V(s) I_{in}$

$Z_T(s) = -(R_f || Z_S) \frac{A_V(s)}{1+A_V(s)\frac{Z_S}{R_f+Z_S}}$

Loop Gain $\quad A_L(s) \quad = A_V(s) \frac{Z_S}{R_f+Z_S}$

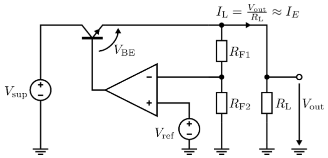
Feedback factor $\quad \beta(s) \quad = \frac{Z_S}{R_f+Z_S}$



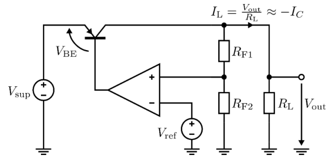
$\rightarrow$ Higher transimpedance gain results in lower bandwidth

7.2 Linear Voltage Regulator

Non-inverting topology



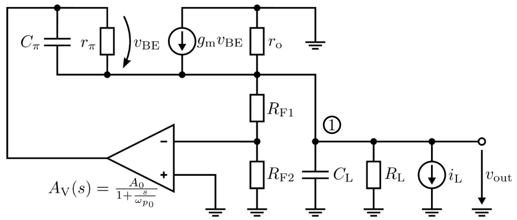
Inverting topology



$$R_{F1}, R_{F2} \gg R_L \Rightarrow I_L \approx \frac{I_E}{-I_C}$$

$$V_{out} = \left(1 + \frac{R_{F1}}{R_{F2}}\right) V_{ref}, \quad A_0' = A_0 \frac{R_{F2}}{R_{F1}+R_{F2}}$$

7.2.1 Small Signal Equivalent



$$Z_{out} = \frac{v_{out}}{i_o} = \frac{1}{g_m A_0'} \frac{1+\frac{s}{\omega_{p0}}}{\left(1+\frac{s}{\omega_{p0} A_0'}\right) \cdot \left(1+\frac{sC_L}{g_m}\right)}$$

7.3 Logarithmic Amplifiers

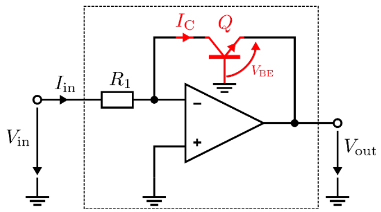
Non-linear circuit with output voltage proportional to the logarithm/exponential of the input voltage.

7.3.1 Logarithmic Amplifier

$$V_{in} > 0$$

$$I_{in} = \frac{V_{in}}{R_1} = I_C = I_S e^{\frac{V_{BE}}{V_T}} = I_S e^{\cdot}$$

$$V_{out} = -V_T \cdot \ln\left(\frac{I_{in}}{I_S}\right) = -V_T$$

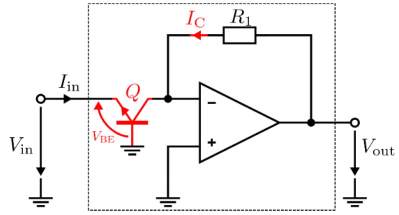


7.3.2 Anti-logarithmic Amplifier

$$V_{in} < 0$$

$$I_C = I_S e^{\frac{V_{BE}}{V_T}} = I_S e^{\frac{-V_{in}}{V_T}}$$

$$V_{out} = I_C R_1 = I_S R_1 e^{-\frac{V_{in}}{V_T}}$$



8 Filters

TF: Transfer Function

AR: Amplitude Response $\quad |T(j\omega)| = \frac{|V_0(j\omega)|}{|V_i(j\omega)|}$

PR: Phase Response $\quad \angle T(j\omega) = \angle \frac{V_0(j\omega)}{V_i(j\omega)}$

8.1 Damping factor

$$\zeta = \frac{\omega_{p0} + \omega_{pS}}{2\sqrt{A_0 \omega_{p0} \omega_{pS}}}$$

- **Overdamped:** $\zeta > 1, Q < \frac{1}{2}$
  $\rightarrow$ solutions of characteristic equation: $p_{1,2} \in \mathbb{R}, < 0$
- **Critically damped:** $\zeta = 1, Q = \frac{1}{2}$
  $\rightarrow$ identical solutions: $p_1 = p_2 = -\omega_n$
- **Underdamped:** $0 < \zeta < 1, Q > \frac{1}{2}$
  $\rightarrow$ complex conjugates: $p_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

8.2 Standard Transfer Function

$$H(s) = \frac{N(s)}{\frac{s^2}{\omega_n^2} + \frac{s}{Q\omega_n} + 1}$$

$N(s) = k$: lowpass filter with DC-Gain: k

$N(s) = k\frac{s^2}{\omega_n^2}$: highpass filter with high frequency gain k

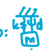
$N(s) = k\frac{s}{Q\omega_n}$: bandpass filter with max gain k

$N(s) = k\left(1 - \frac{s^2}{\omega_n^2}\right)$: notch filter with gain k

Resonance: (may appear in Systems with two or more poles).

$$\boxed{G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$

ω_n.. Natural frequency
ζ.. damping ratio
K.. gain

Bsp: 

$\omega_n = \sqrt{\frac{k}{m}}$, $\zeta = \frac{d}{2\sqrt{km}}$
$K = -\frac{1}{k}$

↳ frequency response: $G(j\omega) = \frac{K\omega_n^2}{(\omega_n^2 - \omega^2) + j(2\zeta\omega_n\omega)}$

$$\Rightarrow \boxed{|G(j\omega)| = \frac{K\omega_n^2}{\sqrt{(\omega_n^2-\omega^2)^2 + (2\zeta\omega_n\omega)^2}}}, \quad \boxed{\angle G(j\omega) = -\arctan\left(\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2}\right)}$$

1. For Stability, need $\zeta \geq 0$
2. For $\zeta \geq 1$ poles real (overdamped system)
3. For $\zeta = 1$ poles real & equal (critical damping)
4. For $0 < \zeta < 1$ poles complex (under-damped system)
5. For $\zeta = 0$ poles imaginary (undamped system)
6. For $\zeta \geq 1/\sqrt{2}$ magnitude bode plot decreasing in ω
7. For $0 \leq \zeta < 1/\sqrt{2}$ magnitude bode plot has a max. at $\omega = \omega_n\sqrt{1-2\zeta^2}$ and $|G(j\omega)| = \frac{K}{2\zeta\sqrt{1-\zeta^2}}$

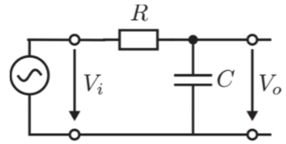
8.3 First Order Passive Filters

8.3.1 Low-Pass Filter

TF: $T(s) = \frac{V_0}{V_i} = \frac{1}{1+sRC}$

AR: $\frac{|V_0(j\omega)|}{|V_i(j\omega)|} = \frac{1}{\sqrt{1+(\omega RC)^2}}$

PR: $\angle\frac{V_0(j\omega)}{V_i(j\omega)} = -\arctan(\omega RC)$

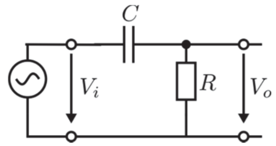


8.3.2 High-Pass Filter

TF: $T(s) = \frac{V_0}{V_i} = \frac{sRC}{1+sRC}$

AR: $\frac{|V_0(j\omega)|}{|V_i(j\omega)|} = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}}$

PR: $\angle\frac{V_0(j\omega)}{V_i(j\omega)} = 90 - \tan^{-1}(\omega RC)$

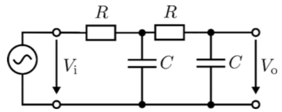


8.4 Second Order Passive Filters

8.4.1 General

TF: $T(s) = \frac{\frac{1}{R^2C^2}}{s^2+\frac{3s}{RC}+\frac{1}{R^2C^2}}$

$$D(s) = s^2 + \frac{\omega_0}{Q_0}s + \omega_0^2$$

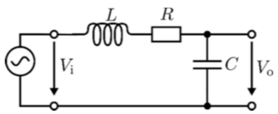


2nd-Order Low-pass Passive RC Filter

8.4.2 Low-Pass Filter

TF: $T(s) = \frac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$

$$= \frac{K\omega_0^2}{s^2+\frac{\omega_0}{Q_0}s+\omega_0^2}$$



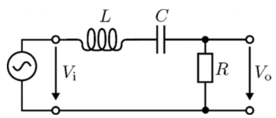
8.4.3 High-Pass Filter

TF: $T(s) = \frac{s^2K}{s^2+\frac{\omega_0}{Q_0}s+\omega_0^2}$

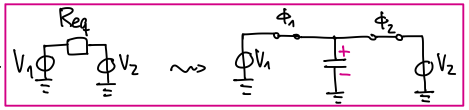
8.4.4 Band-Pass Filter

TF: $T(s) = \frac{s\frac{R}{L}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$

$$= \frac{sK\frac{\omega_0}{Q_0}}{s^2+\frac{\omega_0}{Q_0}s+\omega_0^2}$$



Switched Capacitors:



↳ Vorgehen: ①

②

③

④

⚠ Wenn es steht "... $\Phi_1 \to \Phi_2$ ..." : analyse circuit in stage Φ_2 !

8.4.5 Comparison with First Order Passive Filter

Passive Filters	Active Filters
2 passive elements that determine pole	3 passive elements, pole determined by elements in feedback
Fixed gain of 1	Variable pass band gain
No power consumption	OmAmp consumes power
Real filter transfer function close to ideal	Real filter transfer function dependent on gain

8.5 First Order Active Filters

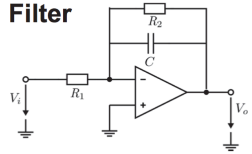
8.5.1 Low-Pass Filter

TF: $T(s) = -\frac{R_2}{R_1}\frac{1}{1+sR_2C}$

AR: $|T(j\omega)| = \frac{R_2}{R_1}\frac{1}{\sqrt{1+(\omega R_2C)^2}}$

PR: $\angle T(j\omega) = 180 - \tan^{-1}(\omega RC)$

Filter



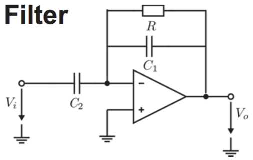
8.5.2 High-Pass Filter

TF: $T(s) = \frac{V_0}{V_i} = -\frac{sRC_2}{1+sR_2C}$

AR: $|T(j\omega)| = \frac{R_2}{R_1}\frac{\omega RC_2}{\sqrt{1+(\omega RC_1)^2}}$

PR: $\angle T(j\omega) = -90 - \tan^{-1}\omega RC_1$

Filter



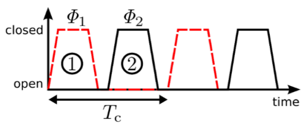
9 Switched Capacitor Filters

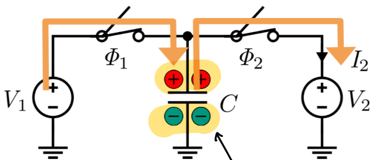
Bed: $f_{V_{in}} \ll f_c$

Resistors take up too much space, so replace them by switched capacitors.

9.1 Equivalent Resistor

We transfer charge ΔQ from potential V_1 to V_2 at a fixed rate $f_c = \frac{1}{T_C}$



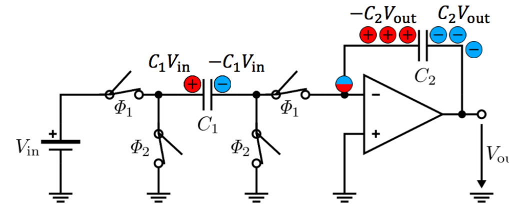


define yourself @ beginning which plate is positive and negative & keep it throughout the whole task!

$$\Delta Q = C(V_1 - V_2) \qquad Q = C \cdot V$$

$$I_{2,avg} = \frac{\Delta Q}{T_C}, \quad R_{eq} = \frac{T_c}{C} = \frac{1}{f_cC}$$

9.2 Inverting Integrator



Initial condition:
$V_{out} = 0$

Phase 1: ϕ_1 on — charge accumulates on C_1 and C_2

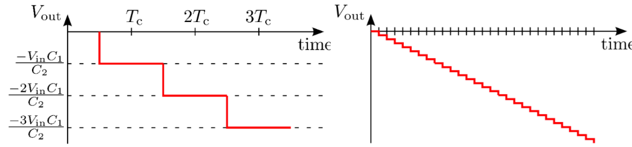
$$Q_1 = C_1 \cdot V_{in}(n-1)$$
$$Q_2 = -C_2 \cdot V_{out}(n-1)$$

Phase 2: ϕ_2 on — C_1 is discharged

$$Q_1 = 0$$
$$Q_2 = -C_2 \cdot V_{out}(n-0.5)$$

$$C_2V_{out}(n) = C_2V_{out}(n-0.5)$$
$$C_2V_{out}(n) = C_2V_{out}(n-1) - C_1V_{in}(n)$$

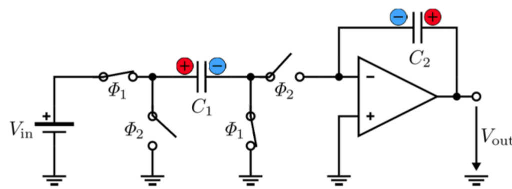
$$\frac{V_{out}}{V_{in}} = -\frac{C_1}{C_2}\frac{1}{1-z^{-1}}$$



→ Seems continuous for small T_c

$$V_{out}(nT_c) = -\frac{C_2}{T_cC_2}\int_0^{nT_c} V_{in}(t)\, dt$$

9.3 Non-Inverting Integrator



Same circuit as before, but change of switching circuit

→ charge on C_2 is inverted

Phase 1: ϕ_1 on C_1 is charged to V_{in}

$$Q_1 = C_1 \cdot V_{in}(n-1)$$
$$Q_2 = C_2 \cdot V_{out}(n-1)$$

Phase 2: ϕ_2 on Charge is transferred to C_2

$$Q_1 = 0$$
$$Q_2 = C_2 \cdot V_{out}(n-0.5)$$

$$C_2 V_{out}(n) = C_2 V_{out}(n-0.5)$$
$$C_2 V_{out}(n) = C_2 V_{out}(n-1) + C_1 V_{in}(n-1)$$

$$\frac{V_{out}}{V_{in}} = \frac{C_1}{C_2} \frac{z^{-1}}{1-z^{-1}}$$

9.4 Procedure

1. Select a polarity for each capacitor
2. Draw circuit diagrams for each phase
3. Determine charge of each capacitor during each phase ($Q_k = C_k V_k$)
4. Identify nodes where charge conservation can be applied
5. Derive charge equations at phase transitions using polarity of the phase to come
6. Z-Transform for transfer function

9.5 Z-Transform

- **Definition:** $Z\{x[nT_c]\} = X(z) = \sum_{k=-\infty}^{\infty} x[kT_c] z^{-k}$
- **Time Delay:** $Z\{x[(n-k)T_c]\} = z^{-k} X(z)$
- **Integration:** $\frac{T_c}{1-z^{-1}}$
- **Differentiation:** $\frac{1-z^{-1}}{T_c}$
- **Mapping to** $j\omega$**-axis:** $z = e^{j\omega T_C} = e^{j\frac{2\pi f}{f_C}}$
- **Mapping to** s**-axis:** $s = \frac{z-1}{T_C}$, or $1 - z^{-1} = sT_C$ (equivalent)

↳ forward euler transformation

9.6 Transient Response

Capacitor:

$$u_C = u_C(t \to \infty) - [u_C(t \to \infty) - u_C(t = t_0)] e^{-\frac{t-t_0}{RC}}$$
$$i_C = \frac{1}{R} [u_C(t \to \infty) - u_C(t = t_0)] e^{-\frac{t-t_0}{RC}}$$

Inductor:

$$u_L = R[i_L(t \to \infty) - i_L(t = t_0)] e^{-\frac{R}{L}(t-t_0)}$$
$$i_L = i_L(t \to \infty) - [i_L(t \to \infty) - i_L(t = t_0)] e^{-\frac{R}{L}(t-t_0)}$$

10 Appendix

10.1 Passive Elements

[R] Seriell: $R_{ges} = \sum_{k=1}^{n} R_k$

[R] Parallel: $\frac{1}{R_{ges}} = \sum_{k=1}^{n} \frac{1}{R_k}$ $\quad G_{ges} = \sum_{k=1}^{n} G_k$

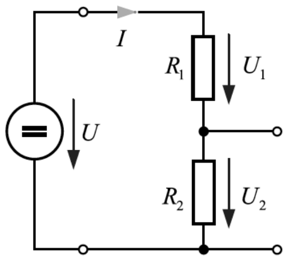
[C] Seriell: $\frac{1}{C_{ges}} = \sum_{k=1}^{n} \frac{1}{C_k}$

[C] Parallel: $C_{ges} = \sum_{k=1}^{n} C_k$

[L] Seriell: $L_{ges} = \sum_{k=1}^{n} L_k$

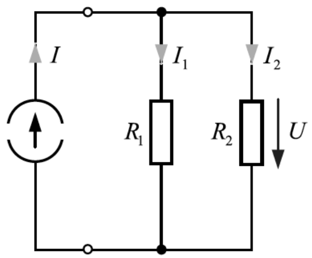
[L] Parallel: $\frac{1}{L_{ges}} = \sum_{k=1}^{n} \frac{1}{L_k}$

10.2 Dividers



$\frac{U_2}{U} = \frac{R_2}{R_1+R_2}$

$\frac{U_1}{U_2} = \frac{R_1}{R_2}$



$\frac{I_2}{I} = \frac{R_1}{R_1+R_2} = \frac{G_2}{G_1+G_2}$

$\frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{G_1}{G_2}$

10.3 Superposition Principle

- Only look at one voltage/current source at a time
- Set all other sources to 0
  - Current source → open circuit
  - Voltage source → short circuit
- Add the contributions

10.4 Miscellaneaous

- **Gain:** Voltage: $A_{dB} = 20\log(A)$, $\quad A = 10^{A_{dB}/20}$
  Power: Factor 10 instead of 20
- **Gain Bandwidth Product:** $GBP = \omega_p \cdot A(j\omega = 0)$
  → Trade-off between DC-gain and cutoff frequency
- **Simplification:** Often $g_m \gg \frac{1}{r_o}$, $g_m \gg \frac{1}{r_\pi}$
- **Error:** $E = |\frac{A_{approx} - A_{ideal}}{A_{ideal}}| \cdot 100\%$

low-pass filter: some Ausdruck like: $A(s) = \frac{R_1}{R_2} \cdot \frac{1}{1+sCR}$

pass-band gain

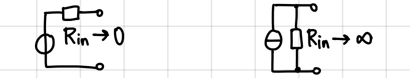
$\frac{1}{w_c}$

w_c : cutoff frequency

= pole frequency

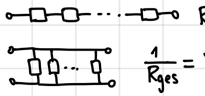
Zusatz

Ideal Voltage Source / Ideal Current Source



Resistances:

⚠ $R \to \infty$: LL, $R \to 0$ KS



$R_{ges} = \sum_i R_i$

$\frac{1}{R_{ges}} = \sum_i \frac{1}{R_i}$ Wenn 2: $R_{ges} = \frac{R_1 R_2}{R_1 + R_2}$

in EC: "neglect R" means $R \to \infty$

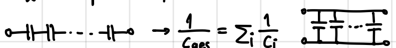
if $R_1 \ll R_2$: $R_1 \| R_2 \approx R_1$

Capacitances:

@ DC → kein Strom fliesst durch! $i_c = C \cdot \frac{du_c}{dt}$

@ AC → $\underline{Z}_c = \frac{1}{j\omega C} = \frac{1}{sC}$

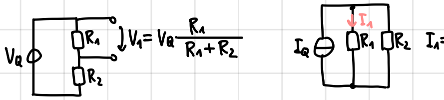
C_∞ → open @ DC, short @ AC



→ $\frac{1}{C_{ges}} = \sum_i \frac{1}{C_i}$ → $C_{ges} = \sum_i C_i$

Impedances: $\underline{Z}_L = j\omega L$, $u_L = L \cdot \frac{di_L}{dt}$

Voltage divider: / Current divider:



$V_1 = V_Q \cdot \frac{R_1}{R_1 + R_2}$ $I_1 = I_Q \cdot \frac{R_2}{R_1 + R_2}$

Transconductance of the Circuit: $g_s = \frac{i_{out}}{v_{in}}$

Transconductance of the transistor: g_m

Transimpedance of the circuit: $\frac{v_{out}}{i_{in}}$

Output Resistance: $\frac{v_{out}}{i_{out}} = r_{out}$ Voltage gain: $\frac{v_{out}}{v_{in}}$

Input Resistance:

"Operating point" means: bestimme voltages / currents s.t. component is in operating (≙ active) mode!
→ "saturation" for MOSFET, "active" for BJT!

↳ meistens eine Grösse gegeben (z.B. Vout) die erfüllt sein muss.

Voltage gain: $A_v = \frac{v_{out}}{v_{in}}$

Resonance: (may appear in Systems with two or more poles).

$$\boxed{G(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}}$$

ω_n.. Natural frequency
ζ.. damping ratio
K.. gain

Bsp: $\omega_n = \sqrt{\frac{k}{m}}$, $\zeta = \frac{d}{2\sqrt{km}}$, $K = -\frac{1}{k}$

↳ frequency response: $G(j\omega) = \frac{K\omega_n^2}{(\omega_n^2 - \omega^2) + j(2\zeta\omega_n\omega)}$

$$\Rightarrow \boxed{|G(j\omega)| = \frac{K\omega_n^2}{\sqrt{(\omega_n^2-\omega^2)^2 + (2\zeta\omega_n\omega)^2}}}, \boxed{\angle G(j\omega) = -\arctan\left(\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2}\right)}$$

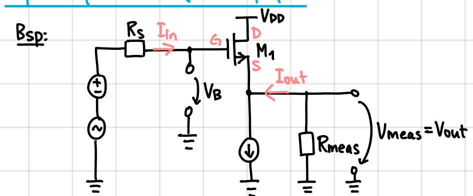
1. For Stability, need $\zeta \geq 0$
2. For $\zeta \geq 1$ poles real (overdamped system)
3. For $\zeta = 1$ poles real & equal (critical damping)
4. For $0 < \zeta < 1$ poles complex (under-damped system)
5. For $\zeta = 0$ poles imaginary (undamped system)
6. For $\zeta \geq 1/\sqrt{2}$ magnitude bode plot decreasing in ω
7. For $0 \leq \zeta < 1/\sqrt{2}$ magnitude bode plot has a max. at $\omega = \omega_n\sqrt{1-2\zeta^2}$ and $|G(j\omega)| = \frac{K}{2\zeta\sqrt{1-\zeta^2}}$

Resolution: The smallest increment of measurement, movement or other output that a mashine, instrument, or component is capable of making.

Bsp: $V_{res,s} = 50 \cdot V_{res,meas}$ means the output (≙ measuring device) can only "see" jumps of $n \cdot 50$ mV, $n \in \mathbb{N}$ at the sensor! D.h. if V_s changes by z.B. ± 25 mV, we won't notice it. → Bad resolution!
→ sensor "feels" precise changes, but measurement device (bzw. we) won't be able to make use of it :/ → Sol.: smash in MOSFETs :)

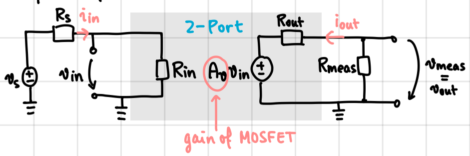
2-port replacement of an amplifier:

Bsp:



1) Calculate $R_{in} = \frac{v_{in}}{i_{in}}$, $R_{out} = \frac{v_{out}}{i_{out}}\Big|_{v_{in}=0}$ in SSE.

2) 2-port replacement:



Frequent assumptions:

$r_o \gg R_L$
$g_m r_o \gg 1$
$R_o \gg r_\pi$
$\frac{1}{r_\pi} \ll g_m$

Zehnerpotenzen:

m	10^{-3}	k	10^{3}
µ	10^{-6}	M	10^{6}
n	10^{-9}	G	10^{9}
p	10^{-12}	T	10^{12}
f	10^{-15}	P	10^{15}
		E	10^{18}

Poles of a transfer function:

Dort wo Im = Re im Nenner. (oder einfach dort wo Nenner 0 wird, und s einfach mit w ersetzen.)

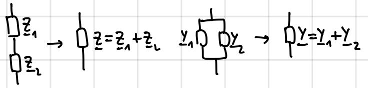
Poles angeben als ω, "frequencies" angeben als f.

Grenzfrequenz ≙ Eckfrequenz = Cutoff frequency ⚠ $[\omega] = $ rad/s

Impedanz / Admittanz:

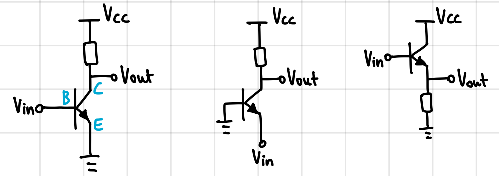
$\underline{Z}_C = \frac{1}{j\omega C} = \frac{1}{sC}$ $\underline{Y}_C = \frac{1}{\underline{Z}_C} = sC$ $\boxed{s = j\omega}$

$\underline{Z}_L = j\omega L = sL$ $\underline{Y}_L = \frac{1}{\underline{Z}} = \frac{1}{sL}$



Common α → no input / output connected to α.

BJT: Common Emitter / Common Base / Common Collector



MOSFETS: Common Gate / Common Source / Common Drain

⚠ Transfer function: $A(s) = \ldots$ → highest order s^n w.o. coeff.!

Filter 2. ordnung: LC-Hochpass / Tiefpass

Resonanzfrequenz $\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$ (hier $Re(\underline{Z}_1 + \underline{Z}_2) = 0$, $\varphi = 90°$) Phasenverschiebung

↳ denominator of the transfer fct. is in the form $\boxed{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$ Resonanzf. Güte (quality)

Strom fliesst nur, wenn NW geschlossen ist!

Why is it favourable to have a high CMRR for a differential circuit:
In differential circuits, the signal of interest that one wants to have amplified is symmetric, i.e. $v_{i+} = -v_{i-}$. Unwanted / parasitic signals usually affect both v_{i+} and v_{i-} equally and with the same polarity, consequently they appear as common mode signals at the input of the amplifier. Therefore, common mode signals must be amplified with the smallest gain or the highest CMRR possible. Supply or ground variations as interference coupling from the mains are examples of unwanted common mode signals.

Capacitor:
The voltage when charging a capacitor with a voltage source and a resistor (step response of a 1.order System) is:

$$v_c(t) = V_0(1 - e^{-t/\tau}), \quad \tau = RC$$

genauer: $v_c(t) = v_c(t\to\infty) - [v_c(t\to\infty) - v_c(t=t_0)]e^{-\frac{t-t_0}{RC}}$

and $i_c(t) = \frac{1}{R}[v_c(t\to\infty) - v_c(t=t_0)]e^{-\frac{t-t_0}{RC}}$

effektive C, die geladen wird.

Reff, R an den Klemmen von C. ($\phi \to$ ¦ $\phi \to |$)

Induktivität: $v_L(t) = R[i_L(t\to\infty) - i_L(t=t_0)]e^{-\frac{R}{L}(t-t_0)}$

$i_L(t) = i_L(t\to\infty) - [i_L(t\to\infty) - i_L(t=t_0)]e^{-\frac{R}{L}(t-t_0)}$

Laplace

Laplace Transform, properties

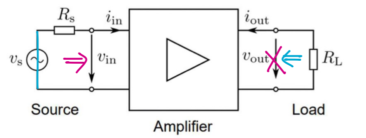
1) Linearity: $\mathcal{L}\{\alpha f(t) + \beta g(t)\} = \alpha\mathcal{L}\{f(t)\} + \beta\mathcal{L}\{g(t)\} = \alpha F(s) + \beta G(s)$
2) s-shift: $\mathcal{L}\{e^{-at} f(t)\} = F(s+a)$
3) Time derivative: $\mathcal{L}\{\frac{d}{dt}f(t)\} = sF(s) - f(0)$, $\mathcal{L}\{\frac{d^2}{dt^2}f(t)\} = s^2F(s) - sf(0) - f'(0)$
4) Convolution: $\mathcal{L}\{(f*g)(t)\} = F(s)\cdot G(s)$ and $\mathcal{L}\{f(t)\cdot g(t)\} = (F*G)(s)$

Some important Laplace Transforms:

$\delta(t) \circ\!-\!\bullet\ 1$ $\sin(\omega t) \circ\!-\!\bullet\ \frac{\omega}{s^2+\omega^2}$
$H(s) \circ\!-\!\bullet\ 1/s$ $\cos(\omega t) \circ\!-\!\bullet\ \frac{s}{s^2+\omega^2}$
$e^{-at} \circ\!-\!\bullet\ \frac{1}{s+a}$

When to neglect what:

↳ When calculating $r_{out} = \frac{v_{out}}{i_{out}}$, set $v_{in} = 0$



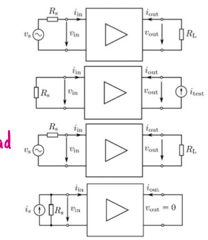
Input Impedance: $\boxed{Z_{in} = \frac{v_{in}}{i_{in}}}$ with $v_{out} = 0$

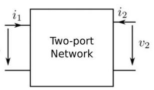
Output Impedance: $Z_{out} = \frac{v_{out}}{i_{out}}$, with $v_s = 0$

Voltage Gain: $A_V = \frac{v_{out}}{v_s}$, → assume no load

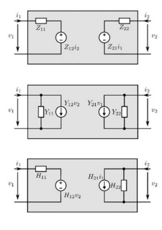
Current Gain: $A_I = \frac{i_{out}}{i_s}$, with $R_L = 0$

$A_s = \frac{v_{out}}{v_{in}}$





- General linear two-port network
- The (linear) relationship among i_1, v_1, i_2, v_2 can always be expressed in terms of 4 parameters but they can take different forms



$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$ Impedance

$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ Admittance

$\begin{bmatrix} v_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ v_2 \end{bmatrix}$ Hybrid

Einheiten:

[U] = V [L] = H
[I] = A [ρ] = Ωm
[R] = Ω [σ] = Sm⁻¹
[C] = F

Spannungsquelle off ⇒ KS

Stromquelle off ⇒ LL

"Keine Spannung" ≠ KS i.A.! (d.h. heisst nicht dass Strom einfach so durchfliessen kann. ⚡ gleich heisst nicht Stromfluss!

(Virtueller KS → nur wenn kein Stromfluss!)

Bode-Diagramm

Gezeichnet wird von kleinen zu grossen Frequenzen, d.h. links nach rechts / Darstellung in dB-Skala $\rightarrow F(\omega)[dB] = 20\log_{10}(F(\omega))$

1. Faktorisieren der Funktion: $\underline{F}_{ges}(j\omega) = K_0\,(j\omega)^r\,\underbrace{\underline{F}_1(j\omega)\cdot\underline{F}_2(j\omega)\cdot\ldots\cdot\underline{F}_n(j\omega)}_{\underline{F}^*_{ges}(j\omega)}$

   ⚠ Grenzfrequenz ≙ Eckfrequenz ≙ Cutoff-frequency (all same!)

   ⚠ Amplitudenplot: $20\text{dB}\cdot\log_{10}(|G(j\omega)|)$

   Teilsysteme $F_i(j\omega)$ in Standardform

$F_i(j\omega) = 1 + j\omega T_{n,i}$	Steigung +20dB/Dekade	Phase +90°
$F_i(j\omega) = \frac{1}{1+j\omega T_{p,i}}$	Steigung -20dB/Dekade	Phase -90°
$F_i(j\omega) = 1 + 2d_i T_{n,i}(j\omega) + (j\omega)^2 T^2_{n,i}$	Steigung +40dB/Dekade Bedingung: $d_i \leq 1$, sonst Polynom mit 2 reellen Nullstellen	Phase +180°
$F_i(j\omega) = \frac{1}{1+2d_i T_{p,i}(j\omega)+(j\omega)^2 T^2_{p,i}}$	Steigung -40dB/Dekade Bedingung: $d_i \leq 1$, sonst Polynom mit 2 reellen Polstellen	Phase -180°

2. Teilsysteme nach aufsteigenden Eckfrequenzen $\omega_i = 1/T_{ni}$ bzw. $\omega_i = 1/T_{pi}$ sortieren (ω_1 = kleinste Eckfrequenz)

   Amplitudengang (doppellogarithmische Darstellung)

3. Startpunkt: $\omega_1\ /\ F_{dB}(\omega_1) = 20\log_{10}(|K_0 F^*_{ges}(0)|\cdot\omega_1^r)$
4. Startpunkt nach links: Gerade mit Steigung $r\cdot 20\text{dB}/\text{Dekade}$ (Für $r = 0$ waagerechte Gerade)
5. Startpunkt nach rechts: Geradensegmente von einer Eckfrequenz bis zur nächst höheren Eckfrequenz. Bei jeder Eckfrequenz ω_i ändert Amplitudengang Steigung je nach Teilsystem, das zur Eckfrequenz gehört (s.o.).
   Mehrfache Pol-/Nullstellen: Steigungsänderung mehrfach nehmen. → immer zur jetzigen Steigung dazuaddieren!
6. Annäherung: Ecken bei Eckfrequenz noch um ± 3dB bzw. Vielfachen davon bei mehrfachen Pol-/Nullstellen abrunden (+n·3dB bei konvexem / -n·3dB bei konkavem Verlauf). Dies gilt nur für konjugiert komplexe Pole mit Dämpfung $d_i > 1/2$.

   Falls $d_i < 1/2$:
   - Resonanzüberhöhung bei ω_i um $-20\log_{10}(2d_i)$dB oberhalb Geradennäherung
   - Amplitude: $|F(j\omega_i)| = \frac{1}{2d_i}$
   - Resonanzkreisfrequenz $\omega_r = \omega_i\sqrt{1-2d_i^2} \Rightarrow$ Punkt um $-20\log_{10}(2d_i\sqrt{1-d_i^2})$dB oberhalb Geradennäherung

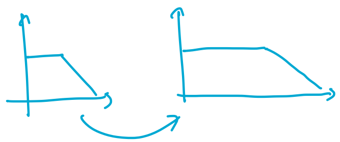
   Phasengang (logarithmische x-Achse)

7. Startfrequenz ω_1 nach links:

$$\varphi(0) = \begin{cases} r\cdot 90^\circ & \text{falls}\quad K_0 F^*_{ges}(0) > 0 \\ -180^\circ + r\cdot 90^\circ & \text{falls}\quad K_0 F^*_{ges}(0) < 0 \end{cases}$$

8. Startpunkt nach rechts: Phase ändert sich bei jeder Eckfrequenz ω_i je nach Teilsystem (s.o.).
9. Annäherung: Glieder 1. Ordnung – Phasenverlauf mit ±45°/Dekade zwischen $0.1\omega_i$ und $10\omega_i$
   Konjugiert komplexe Pole: Phasenänderung bei Eckfrequenz um so steiler, je kleiner d_i
   Phasengang für Teilsystem $F_i(j\omega)$ ist punktsymmetrisch zu dazugehörigen Eckfrequenz ω_i.
10. $\omega \rightarrow \infty$: Phase φ_{ges} strebt gegen $(m-n)\cdot 90^\circ$ (n Grad Nenner- & m Grad Zählerpolynom).

Tiefpass:



more broad band

Bei Filter (bzw. Glieder) 1. Ordung:

@ Grenzfrequenz:
↳ $|\text{Re}(\text{den. von } G(s))| = |\text{Im}(\text{den. von } G(s))|$
↳ Phasenverschiebung $\varphi = 45°$
↳ Ausgangsamplitude $= \frac{1}{\sqrt{2}}\cdot$ Eingangsamplitude
  ↳ da $20\text{dB}\cdot\log_{10}(1/\sqrt{2}) = -3\text{dB}$ → also called -3dB-Eckfrequenz!

Faster Way:

	$\phi(\omega=0)$	$\phi(\omega=\infty)$	$\Delta M(\omega=0)$	$\Delta M(\omega=\infty)$
Integrator $\frac{1}{s}$	-90°	-90°	-20dB	-20dB
Differentiator s	90°	90°	20dB	20dB
Negative Pole $\frac{1}{(s+\alpha)}$	0	-90°	0	-20dB
Positive Pole $\frac{1}{(s-\alpha)}$	0	90°	0	-20dB
Negative Zero $(s+\alpha)$	0	90°	0	20dB
Positive Zero $(s-\alpha)$	0	-90°	0	20dB

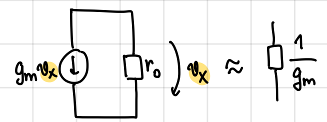
⚠ Poles of Transferfunction: Dort wo Im = Re im Denominator. (oder einfach dort wo $|den| = 0$, mit $s = j\omega$ (s mit ω "ersetzen")

First order high pass transfer function: $\beta\cdot\frac{s\alpha}{1+s\alpha}$ (β: Passband gain) $\quad \omega_0 = \frac{1}{\alpha}\ (=\frac{1}{RC})$

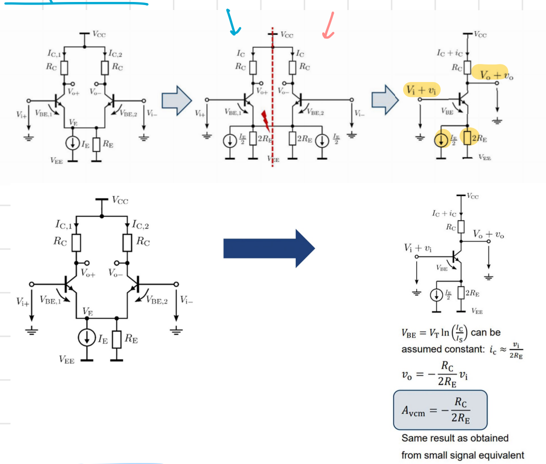
First order low pass transfer function: $\frac{1}{1+s/\omega_0}$

When asked nach Poles: angeben als ω (rad/s)

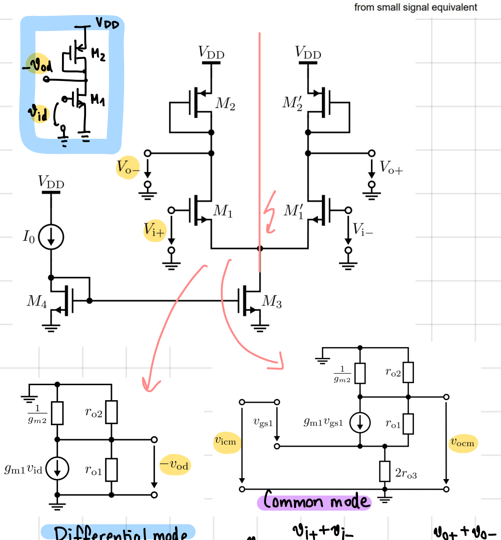
Simplifying SSE:



Half-Circuit:



Bsp.



with $v_{id} = v_{i+} - v_{i-}$, $v_{od} = v_{o+} - v_{o-}$ $\qquad v_{icm} = \frac{v_{i+}+v_{i-}}{2}$ $\quad v_{ocm} = \frac{v_{o+}+v_{o-}}{2}$