Superconductivity[1]

Prof. Manfred Sigrist, HS 2020
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Summary Room at HIT, me writing on the provided Ipad which was connected to the beamer. Examiner and co-examiner are sitting in the back row of the room (Corona). Language: English, even though German would have also been possible - Swiss German however not.

Description of the content: 2.1, 2.2, 3.1, 3.2, 7.1, 7.2[2]

Ablauf Entered the room, the professor showed me how to use an Ipad. Mathematical formulary was interestingly provided, but this was probably an exception as the preceding exams were about mechanics of continua and not about superconductivity.

Prof: So we looked at a phenomenological approach to describe a superconductor. Can you tell me something about it.

Me: The approach is known as the Ginzburg-Landau theory, which is an application of the Landau theory to a superconductor. The Landau theory is based on the assumption that close to the transition temperature, the free energy functional can be expanded in a power series in the order parameter.

$$F = \sum_i a_i \Phi^i \tag{1}$$

We further note that the free energy is a scalar and shows the underlying symmetry of the general problem. So the order parameter used in this approach was a complex wave function Ψ, which is connected to the superconducting electron density by $\Psi = \gamma n_s$ (*Beware, this statement is wrong and was corrected later during the exam*). This is in correspondence to the London theory where the first postulate was: $n_0 = n_s + n_n$. As Ψ is a wave function we need gauge symmetry where the wave function transforms as follows

$$\hat{\Phi}_\chi \Psi = \Psi e^{i\chi(\mathbf{r})}. \tag{2}$$

Therefore, only even terms can appear in the power series expansion. Following, the free energy functional is given by

$$F[\Psi, \mathbf{A}; T] = F_N(T) + \int d^3r \Big[a|\Psi(\mathbf{r})|^2 + \frac{b}{2}|\Psi(\mathbf{r})|^4 + \frac{1}{2m^*}|\,\mathbf{\Pi}\Psi(\mathbf{r})|^2 + \frac{\mathbf{B}(\mathbf{r})^2}{8\pi} \Big]. \tag{3}$$

[1] MSc Physics, Elective Subjects
[2] The chapter number refer to the script .

The last term corresponds to the magnetic field contribution. The second last is known as the Wilson term in Landau theory leading to a rigidity in the problem. It includes the covariant gradient

$$\mathbf{\Pi} = \frac{\hbar}{i}\mathbf{\nabla} - \frac{e}{c}\mathbf{A} \tag{4}$$

which is invariant under gauge transformations. The remaining terms in the integral correspond to the well known Φ^4-Theory. The first term simply includes all the contributes to the free energy of a normal conductor. As we have found an expression for the free energy, we know from thermodynamics that it has to be minimal with respect to its parameters. Hence,

$$\frac{1}{2m^*}\mathbf{\Pi}^2\Psi + a\Psi + b|\Psi|^2\Psi = 0 \tag{5}$$

as well as

$$\frac{e^*}{2m^*c}[\Psi^*\mathbf{\Pi}\Psi + \Psi(\mathbf{\Pi}\Psi)^*] - \frac{1}{4\pi}\mathrm{rot}\mathbf{B} = 0 \tag{6}$$

follow from the functional derivatives $\frac{\delta F}{\delta \Psi^*} = 0$ and $\frac{\delta F}{\delta A} = 0$. The equations derived above are known as the Ginzburg-Landau equations.

We can now use them to derive some properties in a superconductor with the assumptions that there is no magnetic field and the covariant gradient of the wave function vanishes. From Φ^4 theory we find that

$$|\Psi(\mathbf{r},T)|^2 = \begin{cases} 0 & T > T_c \\ -\frac{a}{b} & T < T_c \end{cases} \tag{7}$$

from which we deduce that $n_s \sim -\tau$ and essentially the critical exponent is $\beta = 1/2$ corresponding to the mean field universality class. On the other hand we know from Casimir-Gorter that

$$n_s = n_0\Big[1 - (\frac{T}{T_c})^4\Big] \tag{8}$$

which seems to contradict our solution. However, we note that this is a result of Landau theory which bases on the assumption that the temperature is close to T_c. Using a Taylor expansion, one finds that the derived solution is in agreement with Casimir-Gorter as shown in Figure 1. *I messed things up here as I denoted the vertical axis by n_s instead of Ψ and therefore got the wrong graph.*

Prof: You said that we find a linear behaviour. There is something wrong in your sketch.

Me: Well, seems like I screwed things up.

Prof: By which assumption was the superconducting electron density coming into the problem.

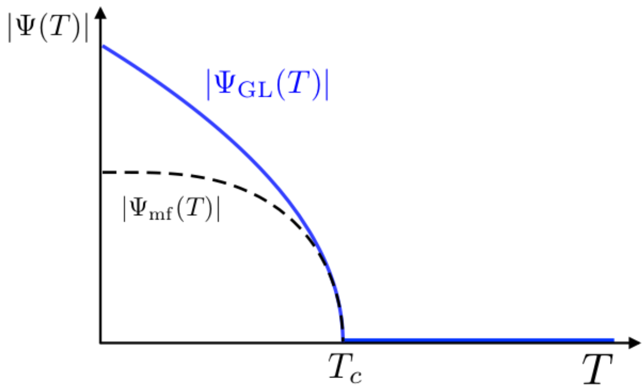

Figure 1: From the script.

Me: Ahh, I forgot the square! The correct assumption is

$$|\Psi|^2 = \gamma n_s. \tag{9}$$

I redrew the sketch with n_s which is linear at $T = T_c$ and coincides with the behaviour of Casimir-Gorter. In the next step we looked at the thermodynamics of a superconductor where we found

$$F = F_n - \frac{H_c(T)^2}{8\pi} = F_n - \frac{a^2}{2b} \tag{10}$$

leading to $a \sim -\tau$. On the other hand, measurements have shown that $H_c(T) \propto 1 - (T/T_c)^2$ where we get the same problem as above. Close to T_c the phenomenological approach coincides with the measured results. As the Ginzburg-Landau theory gives us a term for the free energy, we can derive an expression for the specific heat by taking twice the derivative with respect to temperature. It follows that

$$\frac{C}{T} = \begin{cases} \gamma & T > T_c \\ \gamma + \frac{a'2}{bT_c^2} & T < T_c \end{cases} \tag{11}$$

where γ is the electron contribution to the specific heat of a metal. We can now draw a very beautiful connection to the microscopic derivation shown in Chapter 7 of the script that

$$\Delta C = \frac{8\pi^2}{7\zeta(3)} N(\varepsilon_F) k_B^2 T_c \tag{12}$$

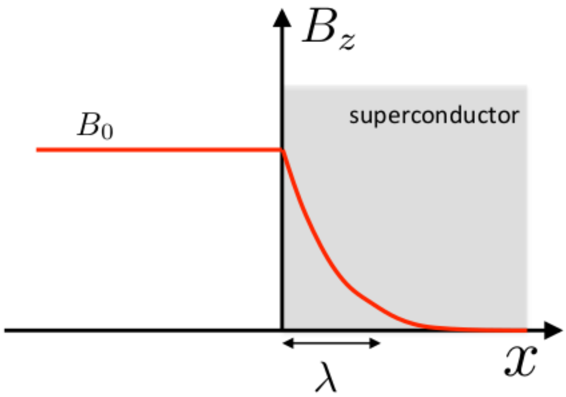

Figure 2: From the script.

and by realising that $\gamma \propto N(\varepsilon_F)$ we find the equation

$$\frac{\Delta C}{C_n} = \frac{1}{\gamma T_c} \frac{8\pi^2}{7\zeta(3)} N(\varepsilon_F) k_B^2 T_c \approx 1.43 \tag{13}$$

which is a universal constant. This result was due to Gorkov and shows a very nice example where the phenomenological constants in Landau theory were actually derived from a microscopic consideration.

Using the same assumptions as above, we then looked at the current density in a superconductor which is given by

$$\mathbf{j} = \frac{e^*\hbar}{2m^*i}\left[\Psi^*\boldsymbol{\nabla}\Psi - \Psi\boldsymbol{\nabla}\Psi^*\right] - \frac{(e^*)^2}{m^*c}|\Psi|^2\mathbf{A} \tag{14}$$

and where the first term vanishes by the assumption that we have no rigidity in the problem. Using Maxwell's equation

$$\mathrm{rot}\mathbf{B} = \frac{4\pi}{c}\mathbf{j} \tag{15}$$

we find the London equation

$$\boldsymbol{\nabla}^2\mathbf{B} = \frac{4\pi(e^*)^2}{m^*c^2}|\Psi|^2\mathbf{B} = \lambda^{-2}\mathbf{B} \tag{16}$$

which shows an exponential decay in the magnetic field as shown in Figure 2. We can now once again use Casimir-Gorter to show that $\lambda = \infty$ for $T > T_c$. In the other limit we find that...

Prof: We need to stop here, as the exam is only 20 minutes and we are already over the limit.

Final Remarks The style of the exam was exactly as announced in the lecture that Professor Sigrist wants you to tell him a story. It is probably best to prepare small presentations of 20 minutes about a chapter. Maybe including some connections to different chapters is also helpful so you cover the essentials of the script. No guarantee, but I think that Professor Sigrist will not interrupt as long as you are not making horrible mistakes or getting stuck somewhere. I mostly used the books: Landau Lifshitz Vol. 9, Abrikosov - Fundamentals of the Theory of Metals and Tinkham - Introduction to Superconductivity. For those who took the Statistical Physics and the Quantum Mechanics 2 course, I can really recommend this course. Please do not take the derived results for granted, as I might have done some mistakes. Good luck!

Expected mark: 5.75
Received mark: 6