Criticality and scaling

1

Sclaing relations

2

Critical dynamics

3

Hohenberg-Mermin-Wagner

4

Quantum critical phase diagram

5

Quantum vs. thermal fluctuations

6

Phase boundary shape

7

Dynamic critical properties at quantum critical point

8

$$
\begin{aligned}
C_v &\sim |T_c - T|^{-\alpha} \\
l &\sim (T_c - T)^{\beta} \\
\chi_l &\sim |T_c - T|^{-\gamma} \\
l &\sim |H_c|^{1/\delta} \\
\langle l(0) l(\mathbf{r}) \rangle &\sim r^{d-2+\eta}
\end{aligned}
$$

1

$$
\text{Homogeneity} \Rightarrow \begin{cases} \alpha + 2\beta + \gamma = 2 & \text{(Rushbrooke)} \\ \gamma = \beta(\delta - 1) & \text{(Widom)} \end{cases}
$$

$$
\text{Hyperscaling} \;\; \Rightarrow \begin{cases} d\nu = 2 - \alpha & \text{(Josephson)} \\ \gamma = \nu(2 - \eta) & (\mathit{Fisher}) \end{cases}
$$

Hyperscaling is not fulfilled in mean field treatment.
Magnetic systems are short ranged $\rightarrow$ Hyperscaling.

2

The characteristic time at which the correlated cluster of size $\xi$ disappears is given by

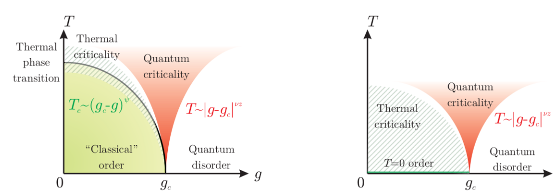
$$
\begin{aligned}
\tau &\sim \xi^{z} \\
\omega &\sim q^{z}
\end{aligned}
$$

$\rightarrow$ critical slowing down

3

Continuous symmetry can not be spontaneously broken at finite temperature in 2D and 1D in a system with the interaction strength falling off fast (faster than a certain power law) with the distance. Beware that the theorem only deals with thermal fluctuations and no statement regarding $T = 0$ can be made. Realistic materials have discreet symmetry but ordering is suppressed.

4




Absence of thermal fluctuations not sufficient for order. Critical behaviour is at $T \leqslant 0$.

5

$$
\begin{aligned}
F &\propto \xi^{-d}(T + A\xi^{-z}) \\
F &\propto \xi^{-d-z}, \quad d_{\text{eff}} = d + z \\
\hbar\omega_0 &\propto \xi^{-z} \propto |g - g_c|^{\varphi}
\end{aligned}
$$

Within the cone, critical thermal fluctuations are dominant but there is no phase transition. For $g = g_c$, unable to find any other characteristic energy apart from temperature $T$. Unlike thermal, in QPT thermodynamics can not be separated from dynamics.

6

The semiclassical phase boundary is determined by the shift exponent $\Psi$

$$
T_c \propto ((g_c - g)^{\Psi}
$$

where in Landau mean field theory

$$
\Psi = \frac{z}{d + z - 2} \neq \varphi = z\nu
$$

If different, then hyperscaling is violated.

7

Hyperscaling is often violated at QPT as $d_{\text{eff}} \leq 4 \Rightarrow$ mean field. If $d_{\text{eff}} < 4$, then along $g = g_c$ we get

$$
S(\mathbf{q}, \omega, T) \propto \sigma\left(\frac{\mathbf{q}^{z}}{T}, \frac{\omega}{T}\right)
$$

Absence of intrinsic energy scales.

8

Goldstone bosons

9

Higgs boson

10

Weakly anisotropic case
Goldstone mode

11

Spin wave theory
General

12

Spin wave theory
Derivation

13

Magnon decay
Collinearity

14

Two-magnon decays

15

Decays of acoustic mode

16

Continuous broken symmetry. Infinitesimal in-plane oscillation correspond to a gapless Nambu-Goldstone boson (Goldstone's Theorem). Typically, $\omega \propto k$ and #gapless modes = #broken symmetry
Beware Ferromagnet, 2 symmetries broken but only one Goldstone mode with $\omega \propto k^2$. Modes are not independent.

9

Quasiparticle associated with longitudinal fluctuations are gapped where the mass scales with the order parameter. At $g = g_c$ the Higgs and Goldstone modes are both gapless, degenerate and indistinguishable.

10

$\Rightarrow$ discreet symmetry $\Rightarrow$ Goldstone breaks down and excitations become massive. For weak anisotropy, the Goldstone bosons have a small gap leading to pseudogoldstone modes.

11

Assumptions: dilute approximation, weakly deviate from equilibrium and fully ordered. Linear spin wave theory can handle the spectrum of excitations in a structure described by a single propagation vector $\mathbf{Q}$.

12

Introduce pseudospin operators to map onto a ferromagnetic structure. Define the operators which create and destroy the minimal possible on-site deviation $\rightarrow$ Holstein-Primakoff transformation. Diagonalize Hamiltonian using a Bogoliubov transformation and find the dispersion. Full account of all the correlation functions.

13

In collinear structures magnons can not spontaneously decay into two as this does not conserve the wave function parity under $\pi$-rotation around the collinearity axis. Therefore the number of magnons is conserved and no magnon-magnon interaction in leading order.

14

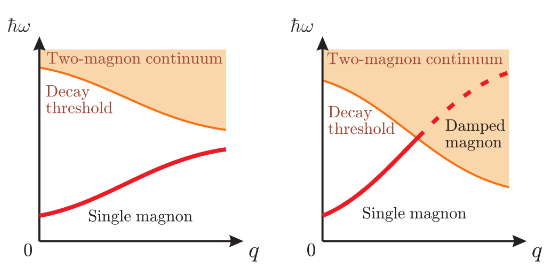
Kinematic condition

$$\hbar\omega(\mathbf{q}) = \hbar\omega(\mathbf{k}) + \hbar\omega(\mathbf{q}-\mathbf{k})$$

$$E_2^{\min}(\mathbf{q}) \leq \hbar\omega(\mathbf{k}) + \hbar\omega(\mathbf{q}-\mathbf{k}) \leq E_2^{\max}(\mathbf{q})$$

If $\hbar\omega(\mathbf{q}) < E_2^{\min}(\mathbf{q}) \Rightarrow$ magnon is safe in whole Brillouin zone and higher-order processes also forbidden.

15




For $\hbar\omega(\mathbf{q}) = cq + \alpha q^3$, then decays allowed for $\alpha = \frac{c\varphi^2}{6(q-k)^2} > 0$ (convex).

16

Field induced decays

17

Decays at zero field

18

Batyev-Braginskii approach

19

Bose gas analogy

20

Nonlinear sigma model
General

21

Nonlinear sigma model
Formula

22

Beresinskii-Kosterlitz-Thouless transition
Concept

23

Beresinksii-Kosterlitz-Thouless transition
Mathematical description

24

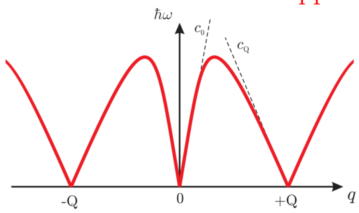
Antiferromagnet:

$$\begin{cases} \text{Magnons stable} \quad H \geq H_{\text{sat}} \\ \qquad\qquad \text{(collinear but convex)} \\ \text{Decay possible} \quad H < H_{\text{sat}} \\ \qquad\qquad \text{(not collinear and convex)} \\ \text{Magnons stable} \quad H = 0 \\ \qquad\qquad \text{(collinear and concave)} \end{cases}$$

The threshold field at $H^* \sim 0.76 H_{\text{sat}}$ changes the sign of $\alpha$.



For multiple branches the condition $\alpha = \frac{c\varphi^2}{6(q-k)^2}$ is not valid. There are 3 Goldstone modes in a spin structure with propagation vector $\mathbf{Q}$. Fast magnons decay into slow even within linear approximation.



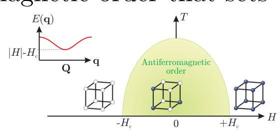



Antiferromagnet, use Matsubara-Masuda transformation to ascribe a particle state to each spin state. Hard-core constraint is required to enforce single particle occupation. The particles have bosonic statistics. One then finds

$$\hbar\omega(\mathbf{q}) = J \sum_{\mathbf{R}} (1 + \cos \mathbf{qR})$$



In the dilute limit ($-H_c \leq H$), $E(\mathbf{q})$ is minimized by $\mathbf{Q} = (\pi/2, \pi/2, \pi/2)$ and the gap is "negative" for values above $-H_c$. This is in analogy to the Bose-Einstein condensate of $a$-particles and corresponds to the antiferromagnetic order that sets in perpendicular to the field.






Assume short-range ordering with $\infty \gg \xi \gg a \Rightarrow$ avoid dealing with lattice and focus on long wave length properties (hydrodynamic approach). The actual spins on the lattice are approximated by a continuous field. Low energy behaviour without the assumption of long range order.



$$\frac{\hbar^2}{2} \frac{\partial \mathbf{n}(\mathbf{r}, t)}{\partial t^2} = 4(SJda)^2 \boldsymbol{\nabla}^2 \mathbf{n}(\mathbf{r}, t)$$

$$\mathcal{S} = \frac{\hbar S}{2} \int dt \int d^3 r \Big[ \frac{1}{JS^2} \big( \frac{\partial \mathbf{n}}{\partial t} \big)^2 - 8J(da)^2 (\boldsymbol{\nabla} \mathbf{n})^2 \Big]$$

Dynamics reduce to minimization of a classical action corresponding to a fixed length vector field.



Vortex and antivortex are robust topological charges which cannot be turned into one another by a uniform rotation of the spins. There is charge conservation. The interaction between them is attractive. A strongly bound pair resembles the uniform ferromagnetic state which will be broken apart by the fluctuations at high temperatures.



$$F_{\text{vortex}} = (\pi |J| S^2 - 2T) \log \left( \frac{L}{a} \right)$$

$$\Rightarrow \quad T_{\text{BKT}} = \frac{\pi |J| S^2}{2}$$

Corresponds to a spontaneous dissociation of vortex-antivortex pairs. The resulting state is still lacking LRO but QLRO below $T_{\text{BKT}}$. Form of hidden order (order by nonexisting) and no symmetry broken at $T_{\text{BKT}}$.

Correlation length and susceptibility of BKT-transition

25

Easy-plane hamiltonian

26

Antiferromagnets
Square lattice 2D Heisenberg

27

Antiferromagnets in a field

28

Nonmagnetic magnets

29

Quantum paramagnets

30

Magnetic Bose-Einstein condensation

31

Pressure induced quantum phase transition
Difference to mBEC

32

Remember: No LRO due to HMW. $\xi$ and $\chi$ diverge at $T_{\text{BKT}}$ and grow faster than any power law. Below $T_{\text{BKT}}$ both the correlation length and the uniform magnetic susceptibility remain infinite. $\Rightarrow$ Field induced ferromagnet.

25

Any easy-plane anisotropy leads to BKT transition.

$$\mathcal{H} = J \sum_{\mathbf{r},d\mathbf{r}} \left[ (\mathbf{S}_{\mathbf{r}} \cdot \mathbf{S}_{\mathbf{r}+d\mathbf{r}}) - \eta S^z_{\mathbf{R}} S^z_{\mathbf{r}+d\mathbf{r}} \right]$$

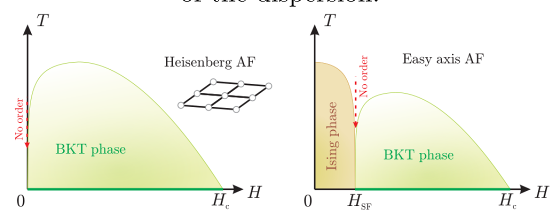
$\eta = 0$ Heisenberg, $\eta = 1$ XY, $\eta < 0$ Ising. For almost Heisenberg ($\eta \ll 1$)

$$T_{\text{BKT}} = \frac{4\pi J S^2}{\log(\pi^2/\eta)}$$

with fast but modified diverging correlation length and susceptibility.

26

Has long range Néel order at $T = 0$ with $\langle \hat{S}^z \rangle \sim 0.6S$. LSWT applicable, but reduced magnetic moment must be taken into account leading to renormalization of the dispersion.




27

The Heisenberg system may display a BKT transition in the presence of a magnetic field. The reason is the staggered magnetization preferring being perpendicular to the infinitesimally weak external field $\rightarrow$ BKT behaviour.

28

At very low temperatures it may happen that the ground state of a particular magnetic ion with even number of electrons is a singlet and the external magnetic field has nothing to couple to. At high temperature all states are present with equal probability. In $Ni^{2+}$ singlet ground state separated by energy gap. In ($S = 1/2$) A.F. dimer spins decouple from external field as singlet is ground sate. At high temperature, $S = 1/2$ paramagnet. Odd number of $e^-$ $\Rightarrow$ at least doubly degenerate.

29

The states that can couple to the magnetic field are at the same time energetically expensive and con not be reached at low temperatures. The energy of the excited state is lowered by Zeeman and at $H_c$ magnetization is restored abruptly. Assume no interaction between the magnetic molecules. Discreetness of magnetization is a 100% quantum effect.

30

Assume interaction between magnetic molecules and a magnetic field. The excited states are then no longer localized and form a band with dispersion $\hbar\omega = \sqrt{\Delta^2 + 2\Delta J'(\mathbf{q})}$ $\Rightarrow$ level-crossing becomes extended to field interval, no jump-like behaviour. Quantum disorder $\Rightarrow$ mBEC $\Rightarrow$ A.F. $\Rightarrow$ mBEC $\Rightarrow$ fully polarized. Shift exponent $\Psi = 2/3$ and $z = 2$. Example of a field induced quantum phase transition.

31

Strong interactions $\Rightarrow$ ordinary magnet, weak interaction $\Rightarrow$ quantum disorder. Interaction can be tuned by changing the pressure and altering the superexchange geometries. Difference is found in dynamical exponent. $z = 1$ for pressure induced as wells as dispersion in A.F. But before mBEC has happened, the dispersion is quadratic $\Rightarrow$ $z = 2$.

32

General spin wave theory
Introduction

33

General spin wave theory
Mean field approximation

34

Magnetization and staggered moment in easy place $S = 1$ A.F.

35

XXZ spin chain
Jordan-Wigner transformation

36

XY chain
Free fermions

37

XY mode response function

38

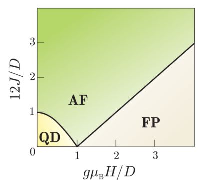
XY chain
Ground state

39

Ising chain
Excitation

40

Assume most of the bosonic particles are in the condensate (ground state) and little residue. The Hamiltonian is $\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_1 + \mathcal{H}_2$, where $\mathcal{H}_1$ only depends on the parameters of the unitary transformation and by minimization gives the ground state.






Interaction within magnetic molecules are much stronger than between them.

$$|\Psi\rangle = \prod_{\mathbf{r}} |\Psi\rangle_{\mathbf{r}}, \quad |\Psi\rangle_{\mathbf{r}} = \sum_{\lambda} m_{\lambda} |\lambda\rangle_{\mathbf{r}}$$

Flavours correspond to the state in the Hilbert state of a single magnetic molecule. Due to the interdimer interaction, an excitation would propagate between them. Find a basis by a unitary transformation in which the ground state can be created by a single operator $|\Psi\rangle_{\mathbf{r}} = \hat{b}^{\dagger}_{\mathbf{r},\Psi} |0\rangle$



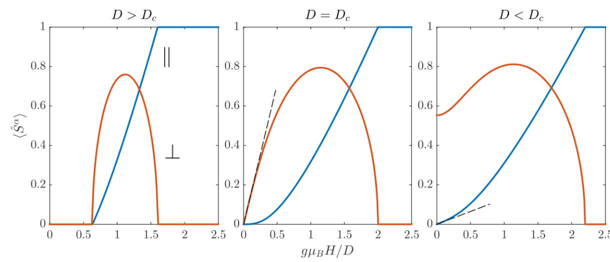
$$\hat{\mathcal{H}} = \sum_n J_{xy}(\hat{S}^x_n \hat{S}^x_{n+1} + \hat{S}^y_n \hat{S}^y_{n+1}) + J_z \hat{S}^z_n \hat{S}^z_{n+1} - g\mu_B H \hat{S}^z_n$$

The modification of the transformation has to be non-local for proper fermionic statistics. Introduce new operators by attaching an infinite string of $\hat{S}^z$.

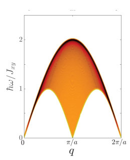
$$\hat{c}_n = \hat{S}^-_n \prod_{m<n} (-2\hat{S}^z_m)$$

Particles are topological, know state of all other sites.







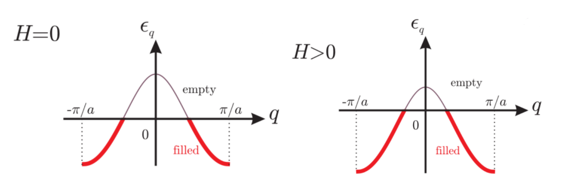
Use ideal electron gas model results. Maximal intensity at upper boundary. Soft mode at $\pi/a$ due to nesting (A.F). Linear dispersion $q \to 0$, very narrow, well defined quasi particle. Gapless.



$J_z = 0$, Fourier transform $\Rightarrow \hat{\mathcal{H}} = \sum_q (\varepsilon_q + 1/2)\hat{c}^{\dagger}_q \hat{c}_q$

$$\varepsilon_q = J_{xy}\cos(qa) - g\mu_B H$$

HMW $\Rightarrow$ no magnetization in zero field $\Rightarrow$ # particles = # holes






Ground state: Néel antiferromagnet. The lowest energy excitation is the domain wall which have no mean of propagating along the chain, localized, $\varepsilon_q = J_z/2$.

$$\xi(T) \propto e^{\frac{J_z}{2T}}$$

The gapped transition to the ordered state at $T = 0$ is reminiscent of the BKT universality class.



At $T = 0$ no LRO but correlations falling off very slowly, QLRO, on the verge to ordering (quantum critical state).

Non-ideal Ising chain
Derivation of existence of mobile domains

41

Non-ideal Ising chain
spin of quasi particles

42

Lieb-Schultz-Mattis theorem

43

Heisenberg chain

44

Fermi liquid

45

Tomonaga-Luttinger spin liquid
Introduction

46

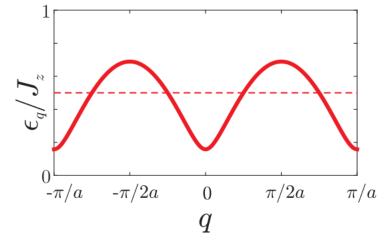
Low energy physics in Tomonaga-Luttinger liquid

47

Tomonaga-Luttinger liquid
Bosonization idea

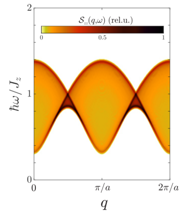
48

For $0 \neq J_{xy} \ll J_z$, domain walls become mobile even at $T = 0$. The deviation from the Néel state is small and we can treat them as weakly interacting particles.






Single spin flip gives $\Delta S = \pm 1$ and improper domain can grow at no energy cost. Single domain wall is $S = 1/2$ and the gapped excitations are fermions and in experiment can only be excited in pairs. Energy conservation $\Rightarrow$ open parameter $k$ $\Rightarrow$ continuum.



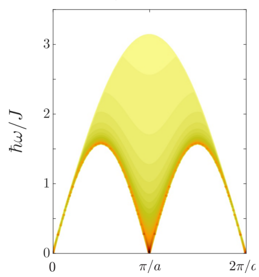



$S = 1/2$ dimer, the singlet state is separated from the excited states by a gap $\Delta$. For half-odd integer spins there exists an excited state with energy that vanishes as $N \to \infty$. $\Rightarrow$ $S = 1/2$ chain would have a gapless spin.



Heisenberg chain is gapless and quantum critical. Spinons have $S = 1/2$ and can only be excited in pairs

$\Rightarrow$ continuum.






2D and 3D, abrupt cut-off at $\varepsilon_F$ with reduced jump $\mathcal{Z} < 1$. Landau quasi particles characterized by linearized dispersion but lifetime is now finite. Particles are well defined at $\varepsilon_F$ and also present for $T = 0$. Excited electrons are dressed by weak density fluctuations.



Spatial restriction leads to infinitely large density fluctuations $\Rightarrow$ smooth boundary at $\varepsilon_F$. The Tomonaga-Luttinger liquid is the analogue of the Fermi liquid in 1D. The singular behaviour is approximated by

$$n(k) \propto |k - k_F|^{K/2 + 1/(2K-1)}$$

$K = 1$, non interacting, $K < 1$ repulsive and $K > 1$ attractive.



Low energy excitations $k \sim k_F$ do not have well defined $\varepsilon_F$ dispersion but rather form a continuum

$$E_F + u|k - k_F|$$

and fully describe the low energy physics by $u$ and $K$. Beware that $d_{\text{eff}} = 2$ and far from mean field.



Fermionic particle operator $\hat{c}_i^\dagger$ and $\hat{c}_i$ as continuous quantum bosonic fields. Introduce field $\varphi(x)$ as the polar canting angle w.r.t. z-axis and field $\theta(x)$ as orientation in xy-plane.They obey bosonic statistics and resemble quantum oscillators. In one dimension: Switch between bosonic and fermionic description but interaction between quasi particles are affected. Trade statistics for interaction. Non commuting as $\hat{S}^\alpha$ do not commute.

TLL Hamiltonian

49

TLL
Correlations and response formula

50

TLL
Transverse and longitudinal spectrum

51

XXZ chain: TLL graphs

52

XXZ chain: Phase diagram

53

Ordering temperature in chains

54

Spin ladder basics

55

strong rung case

56

$$\langle s_n^z s_{n+l}^z \rangle \sim A_z \left(\frac{1}{l}\right)^{2K}$$

$$\langle s_n^+ s_{n+l}^- \rangle \sim A_x \left(\frac{1}{l}\right)^{1/2K} - B_x \left(\frac{1}{l}\right)^{2K+1/2K}$$

$$K \begin{cases} < 1/2 & \text{longitudinal dominant} \\ = 1 & \text{isotropic} \\ > 1/2 & \text{transverse dominant} \end{cases}$$

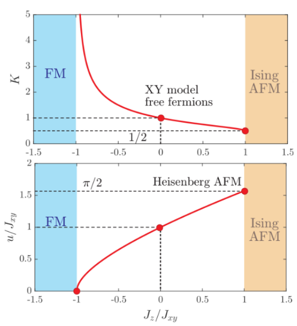
50

The most primitive TLL Hamiltonian is given by

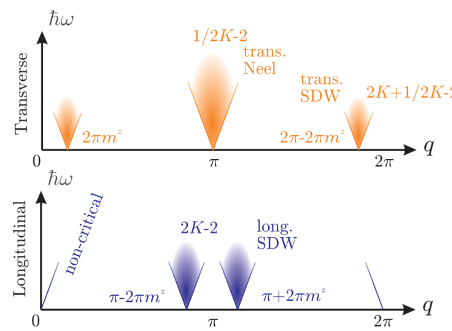
$$\hat{\mathcal{H}} = \frac{U}{2\pi} \int \Big[ K(\boldsymbol{\nabla}\theta(x))^2 + \frac{1}{K}(\boldsymbol{\nabla}\varphi(x))^2 \Big] dx$$

and is in the majority case the only part responsible for low-energy behaviour.

49




52




$\hbar\omega = uq$

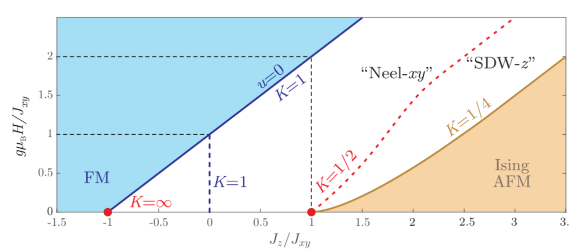
51

Interchain coupling $J'$ is always present in realistic materials. It follows that $\chi$ will diverge not at $T = 0$ but earlier thanks to the interchain coupling.

$$\chi^{MF}(T) = \frac{\chi}{1 - J'\chi}$$

and $T_N \propto (J')^{\lambda}, \lambda = 2K/(4K-1)$. Heisenberg: $\lambda = 1$, XY-model: $\lambda = 2/3$

54



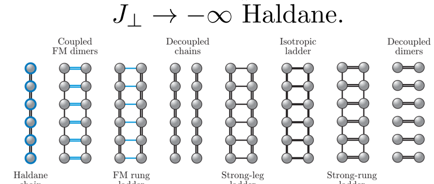

53

$J_\perp \ll J_\parallel$, for the dimerized there is a triplet and a singlet state. For small magnetic field, the ground state remains the direct product of rung singlets. For strong magnetic field, the triplet state is split and gap closes within a QPT. The resulting state has no LRO (long-wavelength fluctuations) and is a TLL state.

56

$$\hat{\mathcal{H}} = \sum_n J_\parallel(\hat{\mathbf{S}}_{n,1}\hat{\mathbf{S}}_{n+1,1} + \hat{\mathbf{S}}_{n,2}\hat{\mathbf{S}}_{n+1,2}) + J_\perp \hat{\mathbf{S}}_{n,1}\hat{\mathbf{S}}_{n,2}$$

$J_\perp = 0$ uncoupled chains, $J_\parallel = 0$ non interacting and $J_\perp \to -\infty$ Haldane.




55

Strong rung case: XXZ chain mapping

57

Strong leg case

58

Symmetric and anti-symmetric excitations in spin ladder

59

Haldane chains

60

AKLT model

61

AKLT ground state

62

Topological order

63

Haldane and AKLT model

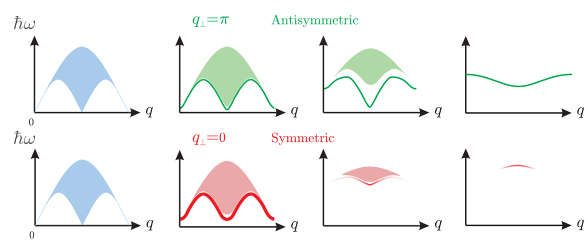
64

In magnetic field only consider low energy state $|s\rangle$ and $|t_+\rangle$ which can be mapped to some effective spin-1/2 chain system in a fictitious magnetic field ⇒ each rung of the ladder corresponds to a single pseudospin object. Using pseudospin operators one finds the XXZ Hamiltonian with $J_z/J_{xy} = 1/2$. The low energy states are described by the TLL. Strong rung assumption needed to neglect the remaining two triplet states.



Spinon excitations are bound in the strong leg case. Symmetric excitation: Singlet, $S = 0$, 2 spinon excitation (1/2 spin excitation forbidden), bound state between the legs $q_{\|} = 0$. Antisymmetric excitation: Triplet, $S = 1$, bound state along leg, $q_\perp = \pi$, transverse momentum. Excitations will be found at different position in Brillouin zone.

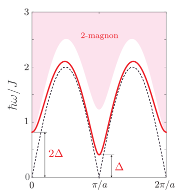







$S = 1$, short-range correlations and gapped . $\xi \propto \exp\{\pi S\} a$ holds for integer spins (half odd ⇒ Lieb-Schultz-Mattis Theorem. From NL$\sigma$M we get

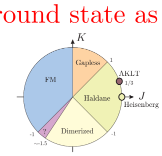
$\Delta \propto \xi^{-1}$.






$$\hat{\mathcal{H}} = \sum_n J(\hat{\mathbf{S}}_n \cdot \hat{\mathbf{S}}_{n+1}) + K(\hat{\mathbf{S}}_n \cdot \hat{\mathbf{S}}_{n+1})^2, \quad \mathbf{S} = 1$$

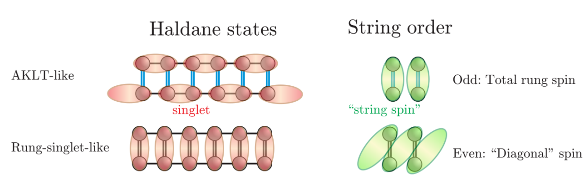
The AKLT model corresponds to $K = 1/3J$ which has an exact solution. The Heisenberg model has the same thermodynamic ground state as the AKLT model.






$S = 1$ ion physically consists of 2 spin 1/2 electrons and is formed by ferromagnetic coupling of rung exchange. The singlets are formed between the spins of site $n$ and $n + 1$. The resulting ground state as a crystal of singlets. Translational invariance "broken" by forming dimerized pairs, but the periodicity remains the same.








The Haldane state at $K = 0$ may be seen as the AKLT state with a finite number of excited states (topological order) present. All the other key properties, such as gapped ground state and presence of pseudospin- 1/2 degrees of freedom at the open chain ends can also be found in the Heisenberg limit. Therefore, despite being a somewhat artificial construction, AKLT model perfectly captures the essential physics of the S = 1 Haldane chain and all of its exotic properties.

Ladders in a field

65

Universal phase diagram (Heisenberg)

66

Harris' criterion

67

Rare regions: depleted magnets

68

Destroying order by impurities

69

Depletion of dimerized antiferromagnet

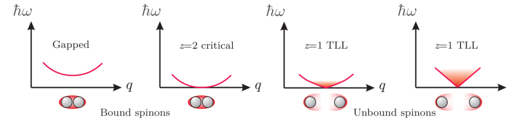
70

Magnetic frustration
Introduction

71

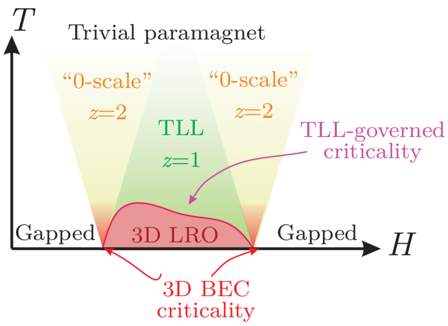
Frustration
Magnetic susceptibility of antiferromagnet

72




Triplet $S = 1$ state will be split by applied magnetic field and lowered. No mBEC to LRO due to HMW but resulting TLSL state $\Rightarrow$ continuum of states.








$d\nu > 2$, $\alpha < 0$ for the irrelevance of disorder effects at the phase transition $\Rightarrow$ correlated volume averages out the randomness, transition remains sharp, tolerates finite amount of disorder. Otherwise, it becomes washed out, no prediction what happens. This criterion is necessary but not sufficient. In QPT, $d$ is not replaced by $d + z$.



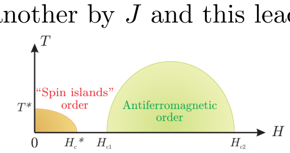
In the case where the Harris' criterion is not fulfilled, local effects have to be taken into account. Site depletion by substituting an atom with a chemical similar one but different spin. Rare regions can be the driving force behind a phase transition.



$x \sim 1$: By removing spins one will slowly get a depleted paramagnet. At some critical concentration $x_c$ the answer is given by percolation theory. $T = 0$ square lattice $x_c \sim 60\%$. For the case when $x \sim 0$, one gets antiferromagnetic ordering.



Assume many body dimer, if one spin is removed, then one spin remains as a degree of freedom, which is not localized $\Rightarrow$ spin islands (correlated droplet). They interact with one another by $J$ and this leads to ordering at $T^* \sim J$.



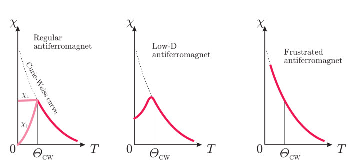



Marshall's theorem $\Rightarrow$ for A.F. interaction, the collinear state hast he lowest possible energy. But even in bipartite lattices frustration can occur (villain lattice). Toulouse criterion

$$\prod_{\text{contour}} \text{sign}(-J) = -1 \Rightarrow \text{frustrated}$$

Macroscopic degeneracy in the ground state which does not occur due to the symmetry in Hamiltonian but from geometric frustration $\Rightarrow$ accidental degeneracy.





Order from disorder



Order from disorder
Classic vs quantum



Triangular lattice in a magnetic field



Frustrated spin chains



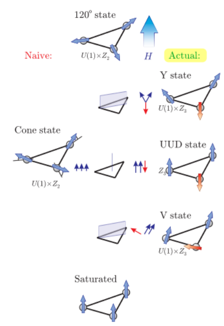
Spin nematics
Introduction

Consider $F = U - TS$, search for the point in phase space with the largest entropy contribution (thickest coating). This is connected to the excitation spectrum. Width inverse proportional to rigidity $\Rightarrow$ To maximize the entropy, one needs to find the ground state with the softest excitation spectrum.

73

Classic: The selection of the true ground state is performed by thermal fluctuations $\Rightarrow \sim \sum \log w$
Quantum: $T = 0$ but zero point fluctuations present $\Rightarrow \sim \sum w$. Therefore, quantum fluctuations can drive the order from disorder mechanism too.

74




75

$$\hat{\mathcal{H}} = \sum J_1(\hat{S}_n \hat{S}_{n+1}) + J_2(\hat{S}_n \hat{S}_{n+2})$$

Exact solution at the Majumdar-Ghosh point: $J_1 = 2J_2$.

76

If $\langle \mathbf{S_r} \rangle \neq 0 \Rightarrow$ local magnetic field detectable and referred as dipolar. Break time reversal symmetry and rotational symmetry of spin space. Frustration can provoke spontaneous breaking of spin rotational symmetry without formation of dipolar magnetic moments. The quadrupolar components go ordered which is a time reversal invariant rank 2 tensor $\Rightarrow$ tensorial order parameter. The spin rotational symmetry remains broken and leads to different outcomes for measuring the spins along different directions.

77