Franck-Hertz Experiment

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Abstract

The Franck-Hertz experiment was conducted on a mercury tube and the data analysed using the theory of non equal spacing in the Franck-Hertz curve. The acquired data revealed an increase in distance to the successive maxima and minima. Using a linear fit the excitation energy of the mercury atom was determined to be (4.88 ± 0.05) eV for the maxima and (4.52 ± 0.05) eV for the minima. This was then used to calculate Plancks constant h to be $(6.62 \pm 0.07) \cdot 10^{-34}$ Js which deviates by 0.1% from the defined value.

Contents

1 Introduction

In the beginning of the 19th century several experiments indicated a quantisation of physical quantities, which at that time could not be explained by classical mechanics nor by electrodynamics. The discreetness of energy levels was first shown by James Franck and Gustav Hertz in the years of 1911 till 1914 [1]. The paper presenting the experiment mostly known as the Franck-Hertz experiment was published on April 24, 1914. James Franck and Gustav Hertz were awarded the Nobel Prize in Physics in 1925 ≪for their discovery of the laws governing the impact of an electron upon an atom ≫[2].

2 Theory

2.1 Energy Levels in Atoms

Considering an electron orbiting around a nucleus at constant radius, the electron is accelerated by centripetal force and therefore must emit radiation. By energy conservation, the effective potential energy of the electron must decrease which contradicts the possibility of a stable orbit. In 1913, Niels Bohr found a solution of this apparent contradiction by introducing two postulates:

- The energy levels of an orbit can only take well-defined discrete energies E_n.
- At ground state, electrons do not emit any radiation. However, they emit radiation (photons) while passing from a state with lower binding energy to a state with higher binding energy. Contrariwise the electron can be excited into a state with lower binding energy by absorption of light. This relation is described using the formula

$$E_n - E_m = h\nu, \tag{1}$$

where ν is the frequency of the photon and h the Planck's constant.

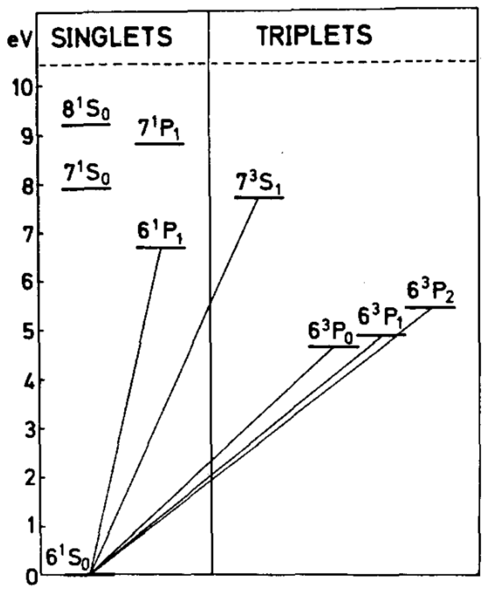

Figure 1: A simplified energy level scheme of the first excited states of the mercury atom. The first excited state is the 6^3P_0 state at 4.67 eV. Illustration from [9].

A higher state is referred to a state with lower binding energy but higher potential energy. The lowest state corresponds to the ground state and the remaining bond states are called excited states. By absorption of a discrete amount of energy ΔE, an electron of the atom can be excited to a higher state. Such a transition can be achieved by inelastic collision between a free particle and an atom [4]. Usually, the excited states of an atom are unstable and will decay to the stable ground state by a spontaneous emission of a photon. The energy of the emitted photon is given by

$$E = h\nu, \tag{2}$$

where h is the Planck's constant and ν is the frequency. Using the fact that photons travel at the speed of light, it follows that

$$E = \frac{hc}{\lambda}, \tag{3}$$

where λ is the wavelength of the photon and the relation $c = \lambda\nu$ was used. In order to convert the energy E in eV, the value of E has to be divided by the elementary charge e. For a photon with wavelength 2537 Å the energy is given by

$$E = \frac{6.63 \cdot 10^{-34}[\mathrm{Js}] \cdot 2.998 \cdot 10^{8}[\frac{\mathrm{m}}{\mathrm{s}}]}{2537 \cdot 10^{-10}[\mathrm{m}] \cdot 1.602 \cdot 10^{-19}[\mathrm{C}]} = 4.89[\mathrm{eV}].$$

Franck and Hertz observed a line in the spectrum of the target gas at 2537 Å [4], which corresponds to a photon with an energy of 4.89 eV. The line in the spectrum is relevant as it corresponds to the first maximum in the Franck-Hertz curve and to the state 6^3P_1.

2.2 Scattering in Gases

In the following, a collision between an electron with small mass and a mercury atom with much larger mass is considered. If the kinetic energy of the electron is less than the lowest excitation energy of the mercury atom $E_a = 4.67$ eV [5], the collision is elastic [3]. This would correspond to the 6^3P_0 state, which has a small cross section. In contrast to the 6^3P_0 state, the 6^3P_1 state has a greater cross section [11] and has the energy of 4.9 eV as shown in Figure 1. Since the mass of the electron is negligible[1] in comparison to the mass of the mercury atom, the loss in kinetic energy of the electron is very small. If the energy of the electron exceeds E_a, inelastic collisions occur, where the electron transfers a significant amount of energy onto the atom. Consequently the electron looses kinetic energy and the mercury atom is in an excited state.

The mean free path λ is defined as the distance an electron travels after gaining kinetic energy of the amount of E_a. The energy E_a corresponds to the energy required in order to excite an atom from ground state into the first excited state. Figure 2 (a) shows the energy of the electron as a function of the position, where the electric field points from right (G_2) to left (G_1). The gradient of the curve is proportional to the magnitude of the electric field, where a steeper gradient corresponds to a stronger electric field and therefore higher acceleration. The additional energy gained over the distance λ is defined as δ_1 (see Figure 2 (a)). In contrast to Figure 2 (a), Figure 2 (b) shows the energy of the electron as a function of the distance for an electric field with increased magnitude. Similar to Figure 2 (a), the number of peaks in the distance L is an integer. Considering that the mean free path λ stays constant, the additional gained energy δ_2 is larger than δ_1. The additional energy gained in Figure 2 (b) is $E_2 = 2E_2 + 2\delta_2$, which can be generalised to [3]

$$E_n = n(E_a + \delta_n). \tag{4}$$

The energy E_n corresponds to the total energy an electron gains in the electric field which has a magnitude such that the electron scatters exactly n times within the distance L. The gradient of the curve is given by $nE_a/(L - \lambda)$ which can be used to define δ_n as

$$\delta_n = n\frac{\lambda}{L}E_a. \tag{5}$$

[1] The mass of the electron is $2.75 \cdot 10^{-6}$ times smaller than the mass of the mercury atom.

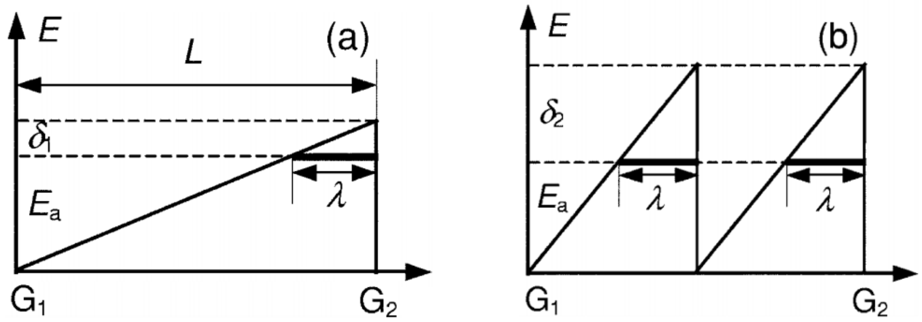

Figure 2: The energy of the electron as a function of the distance travelled in the Franck-Hertz tube. The gradient is proportional to the voltage applied on the Franck-Hertz tube, which induces an electric field from right to left. Illustration from [3].

Inserting Equation (4) into Equation (5), the following term can be derived [3]:

$$\Delta E(n) = E_n - E_{n-1} = [1 + \frac{\lambda}{L}(2n-1)]E_a. \tag{6}$$

The quantity $\Delta E(n)$ describes the additional energy an electron gains when an electric field with greater magnitude is applied. Equation (6) reveals that $\Delta E(n)$ increases linearly in n and has a temperature T dependence, since the free path λ is given by

$$\lambda = \frac{k_B T}{p\sigma}, \tag{7}$$

where k_B is Boltzmann's constant, σ the cross section for inelastic collisions and p the pressure [3].

2.3 Construction of a Franck-Hertz Tube

T	ρ_{Hg}
[°C]	[Torr]
170	6.2
180	8.8
190	12.4
200	17.3

Table 1: Vapor pressure of mercury as a function of the temperature [4].

In order to construct a Franck-Hertz tube the characteristics of the tube need to be defined initially. A good Franck-Hertz tube should

- recreate a Franck-Hertz curve,
- have realizable feasible temperature and voltage range and
- show increasing spacing in dips of the Franck-Hertz curve.

In [4] the temperature has a range of 150 °C to 220 °C. Mercury drops can cause a short-circuit between the anode and the cathode [4]. In order to measure an increase in spacing between the dips λ from Equation (7) needs to be maximised. Table 1 shows that the quotient T/p increases for decreasing temperature. Therefore the optimal temperature for the tube is 170 °C and the pressure 6.2 Torr. Assuming that σ is equal to $3.5 \cdot 10^{-19}\text{m}^2$ [9], the mean free path λ is 0.02 mm. This value for λ can be found in [3] for the temperature of 190 °C. Therefore the value for σ found in [9] does not coincide with the value assumed in which is equal to $(2.1 \pm 0.1) \cdot 10^{-19}$ m^2. Even though this deviation is about of the factor of two, the value for σ is taken as $3.5 \cdot 10^{-19}$m^2. If the voltmeter has an error of 0.05 V, this gives an upper bound for the length L. Setting n equal to 2 results in the smallest increase in spacing which should exceed the error of U_2. The value for n equal to 1 is not measurable since the Franck-Hertz curve has no distinct dip before the first peak. Therefore, the length L should not exceed 0.6 cm as the increase cannot be measured for larger L values due to the error of 0.05 V. This boundary can be found using Equation (5). Overall this can be summarized in a possible temperature range of 150 °C to 170 °C at a tube length of 0.6 cm.

3 Experimental Setup

3.1 Franck-Hertz Curve

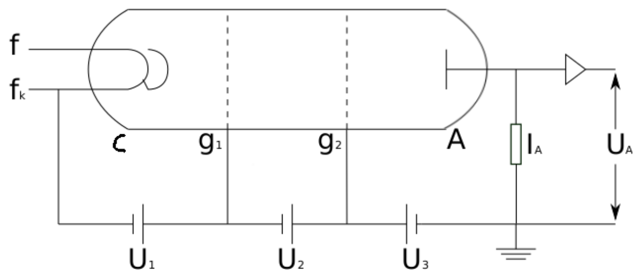

Figure 3: Schematic of the experimental setup for the Franck-Hertz experiment. Adapted from [4].

Figure 3 shows a schematic of the experimental setup the Franck-Hertz experiment. In the Franck-Hertz experiment, electrons are accelerated in an electric field of a Franck-Hertz tube which is filled with a dense gas consisting of mercury atoms[2]. The gas inside the tube can be heated from 150 °C to 220 °C [4]. In this experiment mercury is used due to the low electronegativity and easy availability in high purity [4]. There are two gratings g_1 and g_2 inside the tube, which are connected to a voltage source U_1 ranging from $0-5$ V and U_2 ranging from $0-30$ V as shown in Figure 3. The two gratings, the cathode and the anode generate three different electric fields within the tube:

- between C and g_1,
- between g_1 and g_2 and
- between g_2 and A.

At the cathode C electrons are ejected into the gas due to thermionic emission induced by a constant DC-current. After ejection the electrons are pre accelerated by the electric field. After gaining the energy eU_1, the electrons pass through the grating g_1. Here the electrons are accelerated by the electric field and scatter from the mercury atoms. Finally the electrons pass through g_2, where the repulsive electric field is applied on the electrons. At the anode A the electrons which surpassed the repulsive potential U_3 ranging from $0-5$ V are detected. In this experimental setup, the potential difference U_A is measured over a resistance R.

[2]The Franck-Hertz experiment is not limited to mercury atoms, but can also be done using neon atoms. See [3] for more information.

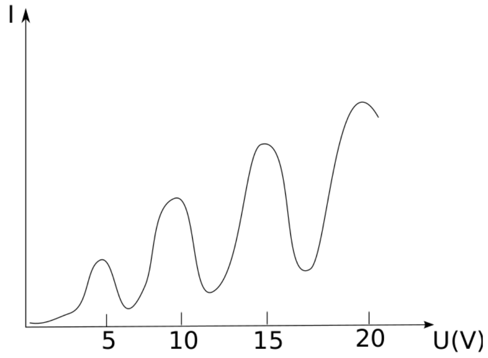

Figure 4: The expected Franck-Hertz curve. In the notation introduced in Section 3, I is proportional to U_A and U corresponds to U_2. Illustration from [4].

Figure 4 shows the expected signal measured as U_A at the anode (A) shown in Figure 3. The typical form arises from the fact that the energy levels in an atom are discreet and where the first maxima is expected at 4.89 V.

Rise of the curve: As the voltage U_2 increases, the gain in kinetic energy of the electron increases which leads to a higher probability for the electron to reach the anode. The fact that for each collision the electron looses a small amount of energy results in electrons which cannot surpass U_3 and eventually are absorbed at g_2. This explains the rising current I of the curve in dependence on the applied voltage U in Figure 4.

Peak and decline: An electron with higher energy than E_a has to possibility to scatter inelastically with a mercury atom. This leads to a decrease in kinetic energy of the electron as described in Section 2.2. Hence, the electron does not gain enough energy in the remaining distance to the grating g_2 and will be absorbed by the grating. This phenomenon can be seen in Figure 4 where the curve decreases for the first time. The minima then corresponds to the situation where most of the electrons are scattered inelastically.

Second rise of the curve: The following increase of the curve can then be explained by the same argument as initial rise. The electrons gain enough energy in the remaining distance to g_2 to surpass the electric field between g_2 and A even though they might have been scattered inelastically before.

Periodicity: The periodicity of the occurring minima was reported to constantly be E_a [6] even though the data contradicts equal spacing of the minima [3]. At the minima the amount of electrons detected at the anode is a local minimum as a function of the applied voltage U_2. This requires that the electrons have the least energy and, therefore, have scattered in the vicinity of the grating g_2. Figure 2 shows the situation where an electron is scattered inelastically close to the grating g_2. Therefore, Equation (6) can be used to calculate the total kinetic energy of the electron at the minima. Since the charge of the electron stays constant, the increase in energy can only result by an increase in applied voltage. Therefore, the distance between minima ΔU_2 increases as a function of the order of minima. Further the temperature T has an impact on the curve, where higher temperature leads to a faster increase in spacing between the minima according to Equation (7).

The intrinsic resolution of the anode is less than 2 mV which is prone to noise in the signal. The signal-to-noise ratio is:

$$\kappa = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}. \tag{8}$$

The goal is to find the parameters of the setup which result in the maximum value for κ.

4 Measurement and Data Analysis

In order to measure Franck-Hertz curves with maximum κ value, the setup needs a preliminary calibration. The goal of this part is to find the optimal values for U_1, U_3 and T which result in the largest κ value. This is done by initially taking a fixed value for U_1. For three different temperature T and three different braking voltage U_3 the Franck-Hertz curve is measured. The calibration measurements are shown in Figure 5, Figure 6 and Figure 7.

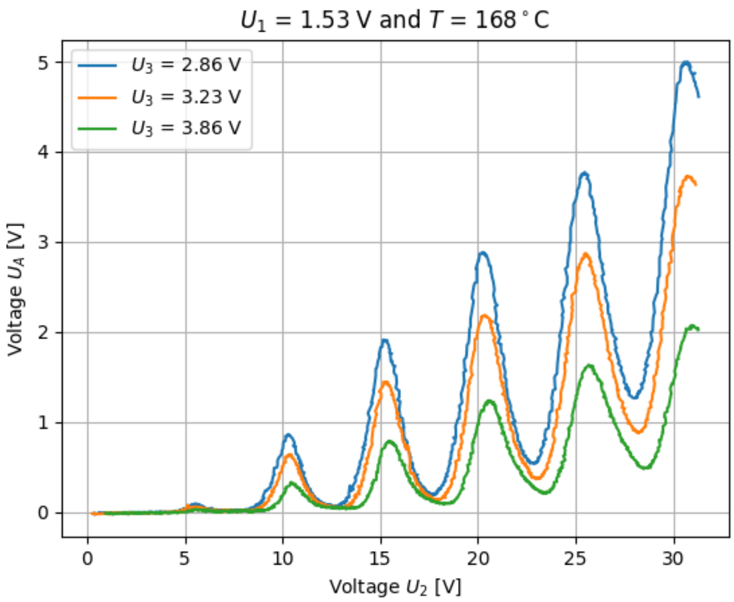

Figure 5: Data of the described measurement in Section 3 for the temperature T set to 168 °C

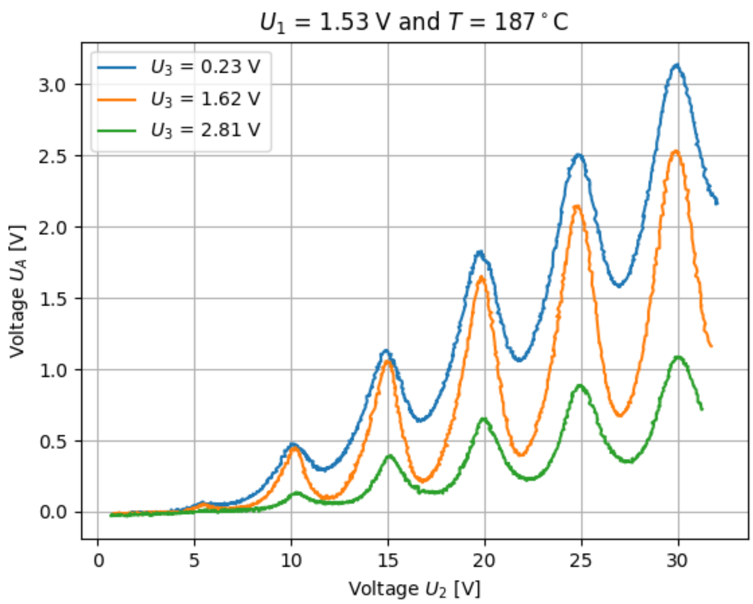

Figure 6: Data of the described measurement in Section 3 for the temperature T set to 187 °C

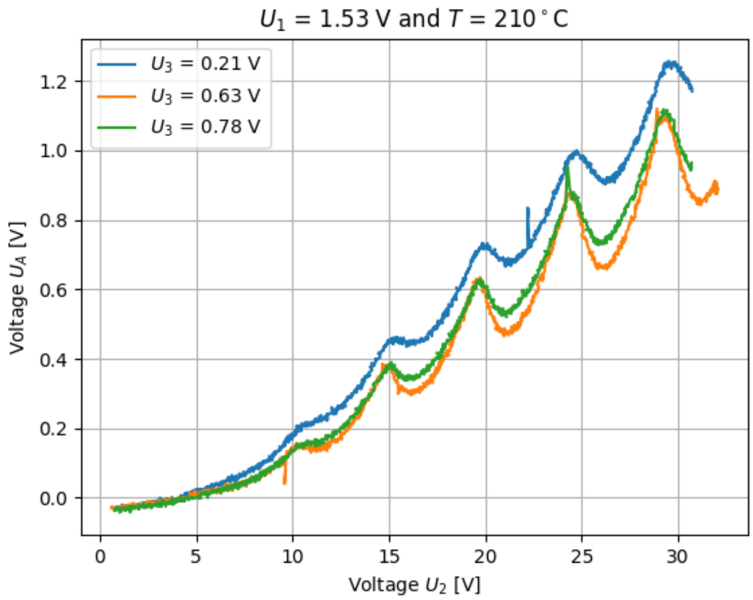

Figure 7: Data of the described measurement in Section 3 for the temperature T set to 210 °C.

After finding the optimal temperature T at 174 °C, it is left constant throughout the experiment. Further the pre-accelerating voltage U_1 is then changed and the Franck-Hertz curve is measured for different U_3 values. In the same way as the temperature T was found, the optimal values for U_1 and U_3 are determined and the measurement shown in Figure 8, Figure 9, Figure 10 and Figure 11. The optimal parameters were found to be 174 °C for the temperature T, 2.74 V for U_1 and 3.57V for U_3.

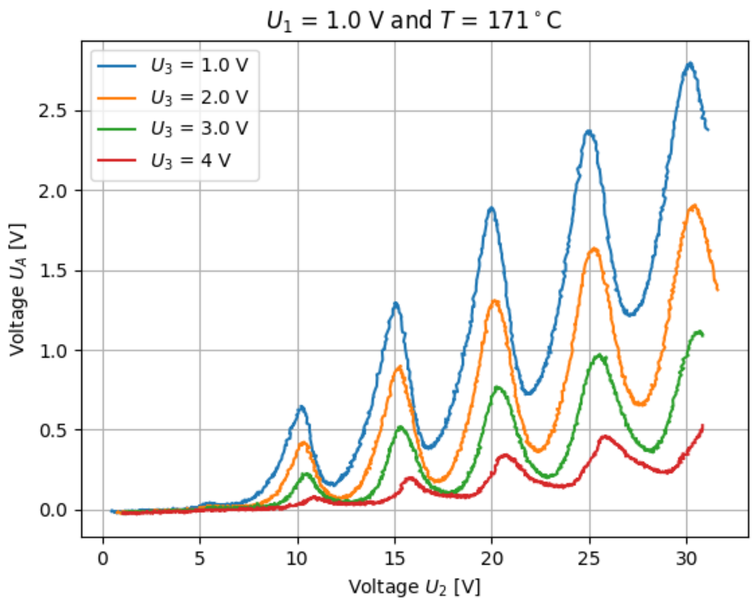

Figure 8: Data of the described measurement in Section 3 for the temperature T set to 171 °C and U_1 set to 1.0 V.

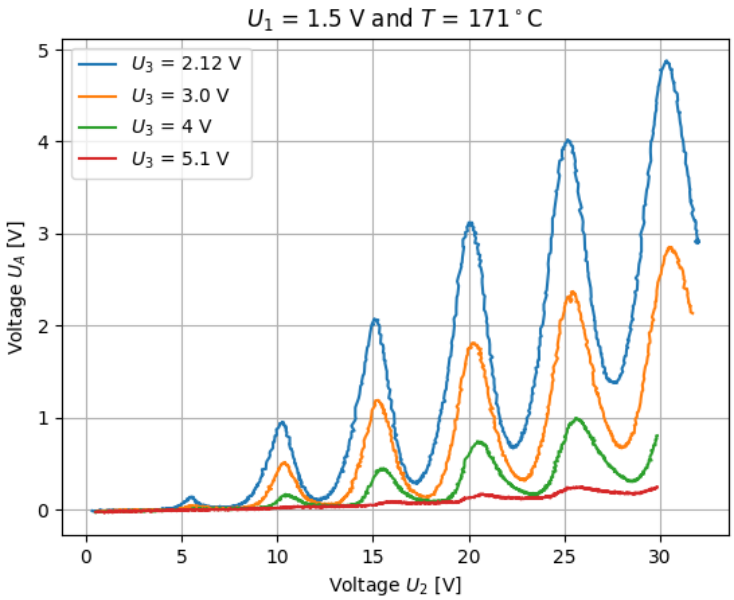

Figure 9: Data of the described measurement in Section 3 for the temperature T set to 171 °C and U_1 set to 1.5 V.

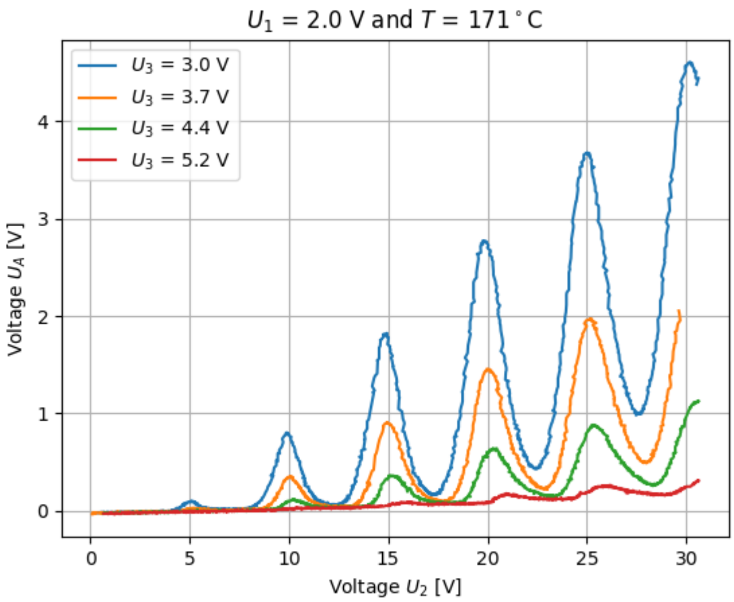

Figure 10: Data of the described measurement in Section 3 for the temperature T set to 171 °C and U_1 set to 2.0 V.

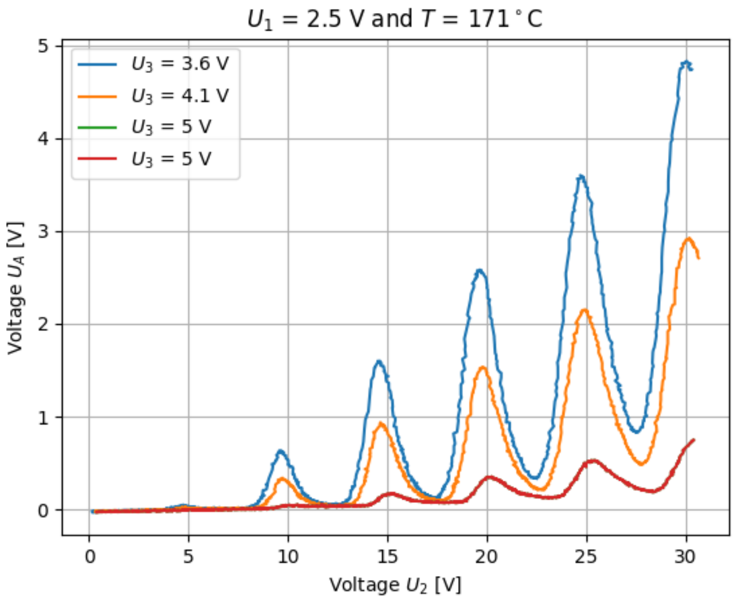

Figure 11: Data of the described measurement in Section 3 for the temperature T set to 171 °C and U_1 set to 2.5 V.

Using the parameters found, five measurements of the Franck-Hertz curve are conducted. Throughout the whole experiment the uncertainties were predicted by linear error propagation theory using the python package *uncertainties* [7]. Rounding was done using the rule recommended by PDG [8]. Figure 12 shows the five measured Franck-Hertz curves at optimal parameters

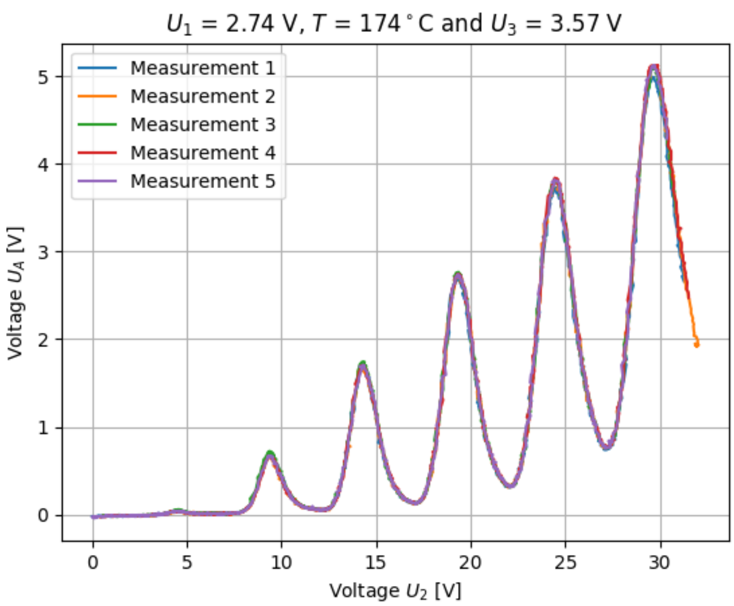

Figure 12: After the best condition for the measurement was found as described in Section 3, five measurements were done at U_1 equal to 2.74 V and T at 174 °C.

The analysis of the curve are shown in Table 2 for the maxima. In order to identify the minima and maxima of the measurements, the package NumPy was used for analysis. The peaks were found by using the numpy.max and numpy.min function in the specific range, where the peaks were expected.

The measured Franck-Hertz curve has the characteristic form which is proposed in the theory. The first maxima can not be fully determined which is fine as it can be calculated using Equation (6) for the value of $E(0.5)$. The uncertainty of U_2 was found to be 0.04 V which was propagated to 0.05 V for ΔU_2.

Order of maxima	1	2	3	4	5	Error
U_2 [V]	9.40	14.30	19.35	24.50	29.68	0.04
ΔU_2 [V]	4.90	5.05	5.15	5.18	-	0.05

Table 2: Data acquired from the 5 measurements at best condition which is further illustrated in Figure 12. U_2 describes the voltage of U_2 at which the peak was detected. In the second row, the spacing between the successive U_2 values corresponding to the peak are noted.

Figure 13 and Figure 14 illustrate the correspondent linear fits of the data acquired in the five measurements.

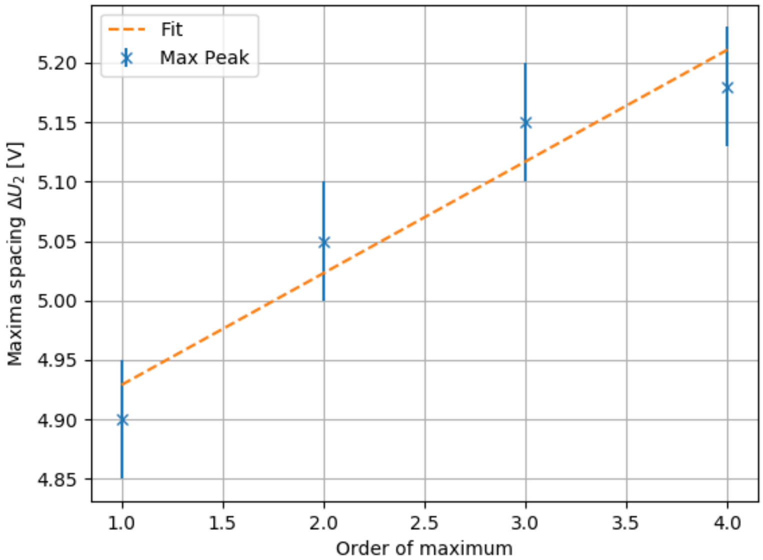

Figure 13: Linear fit of ΔU_2 as a function of order of maximum.

Figure 13 shows the spacing between the peaks as a function of the order, where the fit lies within the error bound for every measurement. This does not coincide with [3] since the model proposes equal distances between maxima.

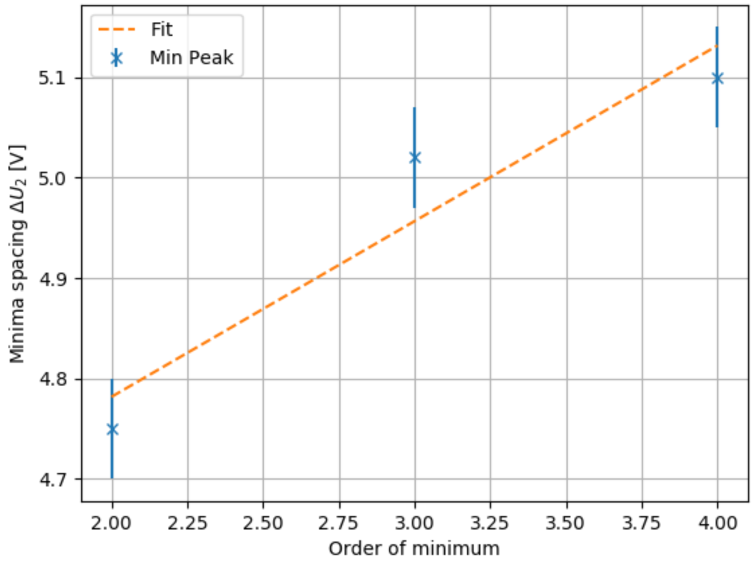

Figure 14: Linear fit of ΔU_2 as a function of order of minimum.

Figure 14 shows the spacing between the dips as a function of the order. The fit does not lie within the error bound which can be explained by the fact that the first dip could not be precisely identified.

	$\Delta E(0.5)$ [eV]	h [10^{-34} Js]
Maxima	4.88	6.62 ± 0.07
Minima	4.52	6.13 ± 0.07

Table 3: For the calculated value of E_a the Planck's constant h is calculated using Equation (3).

Table 3 shows the values found for E_a, which are in accordance with [9] where values of 4.89 V are given with an uncertainty of a few tenths of a volt. Equation (3) was then used to determine the Planck's constant h using the calculated values of E_a. The calculation revealed the values of $(6.62 \pm 0.07) \cdot 10^{-34}$ Js for the fit of the maxima spacing. This has a deviation from the literature value [10] of 0.1%. Likewise the h value was calculated using the fit for the minima spacing. By a deviation of 7.5% the value for h was calculated to be $(6.13 \pm 0.07) \cdot 10^{-34}$ Js. This deviation arises from the fact that the first dip could not be determined precisely.

5 Conclusion

The Franck-Hertz experiment shows that mercury atoms have discreet energy levels. This could be quantitatively observed in this experiment, after the optimal parameters were found by calibration. At the expense of the first dip, the κ value was maximised to gain best signal-to-noise ratio. The measured Franck-Hertz curve is in accordance to the theory of non equal distance. The fit of the minima spacing does not lie within the error bound for the acquired data which can be explained by the first dip which could not be identified precisely. Therefore, the determination of the h value was less accurate revealing the value of $6.13 \cdot 10^{-34}$ Js. Overall the Franck-Hertz experiment offers an opportunity to see discreetness in nature without doing optical spectroscopy and using a rather simple experimental setup.

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