# Interacting electrons

Stoner: Coulomb $\underbrace{\text{long range}}_{\substack{\text{mean field} \\ \text{changes energy of} \\ \text{Groundstate by} \\ \text{a constant}}}$, but also $\underbrace{\text{divergent}}_{}$

at short: Pauli ⇒ only $e^-$ of opposite spin are close

⇒ only affects $e^-$ with different spin

$$\mathcal{H}_H = \underbrace{U}_{>0 \text{ (repulsive)}} \sum_{\underbrace{j}_{\text{cells}}} n_{j\uparrow} n_{j\downarrow} \qquad \text{(Hubbard Hamiltonian)}$$

$$\mathcal{H}_H^{MF} = U \sum_j \left( n_{j\downarrow} \langle n_\uparrow \rangle + n_{j\uparrow} \langle n_\downarrow \rangle \right) - U \sum_j \langle n_\downarrow \rangle \langle n_\uparrow \rangle$$

def.

$$\Rightarrow \quad \mathcal{H}_H^{MF} = -U \sum_j n_{j\uparrow} \frac{MV_0}{g\mu_B} + U \sum_j n_{j\downarrow} \frac{MV_0}{g\mu_B} + UNn^2/4 + \underbrace{\frac{UV_0 M^2}{(g\mu_B)^2}}_{\frac{H_{eff}}{2} M} V$$

⇝ exactly like a Hamiltonian of non interacting electrons subject to an effective external field.

$$M = \chi_p (H + H_{eff}) = \chi H$$

important!

$$\Rightarrow \quad \chi = \frac{\overbrace{\chi_p}^{\chi_{\text{pauli}}}}{1 - \frac{2UV_0}{(g\mu_B)^2} \chi_p}$$

for large $U$ the denominator will go to zero and susceptibility will diverge. ⇝ Ferromagnet

diverge ⇔ $\underbrace{U \nu(\varepsilon_F) V_0 > 2}_{\text{important}}$ (Stoner Criterion for ferromagnetism)

# Magnetic Insulator

$$\mathcal{H} = \sum_{i,j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

Interaction MF $\Longrightarrow$

$$H^{eff} = \left\{ H^{ext} - \frac{\mu V_0}{(g\mu_B)^2} J(\vec{q}) \right\} e^{i\vec{q}\cdot\vec{r}} + c.c.$$

def. $\Longrightarrow$

non interacting susceptibility

$$J(0) = \sum_j J(r_j)$$

$$\chi(\vec{q}) = \frac{\chi_0}{1 + \frac{J(\vec{q}) V_0}{(g\mu_B)^2} \chi_0} \quad , \quad J(\vec{q}) = \sum_j J(\vec{r}_j) e^{i\vec{q}\cdot\vec{r}}$$

$\rightarrow 0$ if $T_c \sim -J(Q)$

$T_W \sim -J(0)$ (Weiss temperature)

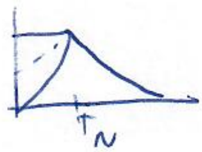
non interacting

Measure $\chi := \chi(0)$ uniform for high temperature $\overset{MF}{\Longrightarrow} \xi \to 0 \Rightarrow \chi = \chi_0$

$J(Q) = \text{minimum} \Rightarrow \chi(Q)$ large

**Ferromagnet:** $Q = 0$, $T_W = T_c$

**Antiferromagnet:** Example: $Q = (\frac{\pi}{a}, \frac{\pi}{b}, 0)$




bcc with different spin in center

**Helimagnets:** $Q$ incommensurate, spiral, geometric frustration

Now include DM interactions:

$\rightarrow$ helimagnet / $\rightarrow$ weak ferromagnet

$\rightarrow$ multiferroics

# Helimagnets

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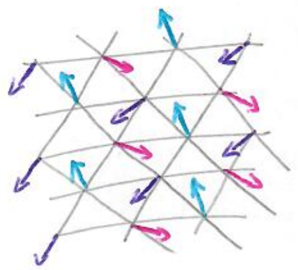
What if $Q$ is incommensurate?

Not a spin density wave as magnitude on each site is fixed.

→ helimagnet, a ~~result of geometric frustration~~ non-colinear structure
can be ~~comens~~ commensurate but
also incommensurate

Example

geometric frustration



nearest neighbourgh antiferromagnet

breaks inversion symmetry, no net magnetization

Measure something and find $Q$

if $Q = 0 \Rightarrow$ ferromagnet because simple cosine starting negative

as


if $Q = \frac{\pi}{a} \Rightarrow$ antiferromagnet


if $Q$ incommensurate $\Rightarrow$ helimagnet

simple ~~easy~~ model


it rotates as $\phi = Q \cdot a = \arccos\left(\frac{-J_1}{4J_2}\right)$

$Q$ is the wavevector with which magnetization is modulated

Bragg peak

# Dzyaloshinskii-Moriya

Definition:

Antisymmetric exchange is a contribution to the total magnetic exchange interaction between two neighboring magnetic spins, $H_{DM} = \vec{D}_{ij} \cdot (\vec{S}_i \times \vec{S}_j)$. In magnetically ordered systems, it favors spin canting of other-wise (anti) parallel aligned magnetic moments and thus, is a source of weak ferromagnetic behaviour in an antiferromagnet.

Derivation: Originates from spin-orbital coupling.

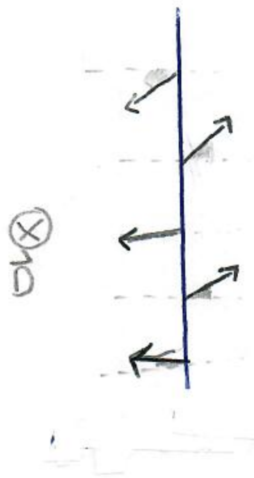
Experimental evidence:

$\vec{D}$ is typically a few % of J. Inversion symmetry $\Rightarrow$ $\vec{D} = 0$

Helimagnet

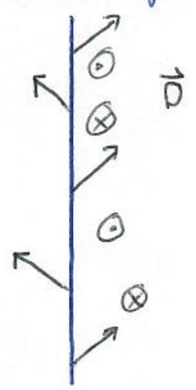
$\vec{D} = \text{const.}$

DM vs. exchange $\Rightarrow$ frustration




Weak ferromagnet

$\vec{D}$ = alternating $\Rightarrow$ canted antiferromagnetic structure has non-zero net magnetic moment



# Coulomb and spin

Spin-Independant Coulomb Interactions $(e^- - e^-)$ and $(e^- - \text{core})$ produce effective spin interactions through Pauli exclusion principle.

Interactions are short range. $\mathcal{H} = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$

## Susceptibility: interacting spins

$$\chi(\vec{q}) = \frac{\overset{\swarrow \frac{C}{k_B T}}{\chi_0}}{1 + \frac{J(\vec{q}) V_0}{(g\mu_B)^2} \chi_0} \quad , \quad \overset{\swarrow \sum \cos(\ldots)}{J(\vec{q})}, \text{ F.T. of coupling, not from fermi-surface}$$

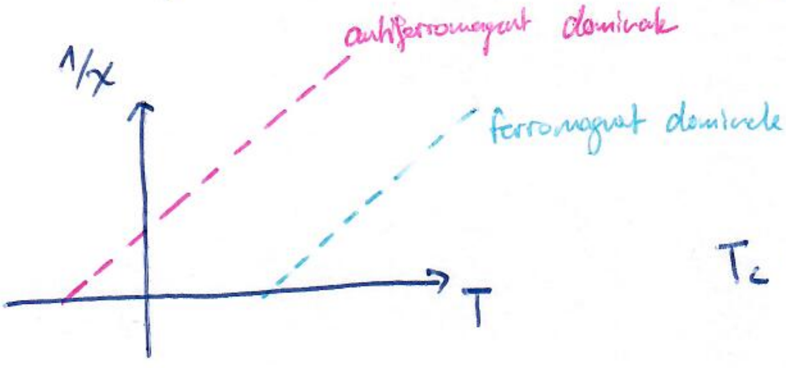
## Weiss temperature

p. 88 chaikin for structure

$\chi(0) = \chi$

$$\frac{1}{\chi} = \frac{k_B(T - \overset{\swarrow \text{weiss temperature}}{T_W})}{\underset{\nwarrow \text{curie constant}}{C}} \quad , \quad T_W \sim -J(0)$$

$T \to \infty$, MF theory exact, due to $\xi \to 0$




} does not guarantee the order!

$T_c \sim -J(Q)$

$Q$ is the wave vector, where $J(q)$ is a minimum.

# Metallic ferromagnetism

**Definition:**

The emergence of spontaneous magnetization in metals due to an electron-electron interaction arising from spin dependant coulomb interaction.

**Derivation:**

$$\text{Hubbard Hamiltonian} \overset{MF}{\Longrightarrow} \chi = \frac{\chi_P}{1 - \frac{2UV_0}{\mu_0 (g\mu_B)^2}\chi_P}$$

$$\overset{\text{Paulisusceptibility}}{\Longrightarrow} \quad U\nu(\varepsilon_F)V_0 > 2$$

(Stoner Criterion for ferromagnetism)

# Ferromagnet


Fig. 1,4 Zheludev

$Q=0$ $\Leftarrow$ all constants are negative

$\rightarrow$ is ferromagnetic, $T_W = T_C \sim -J(Q)$

# Antiferromagnet

p.350 Ziman

The magnetic bragg peak occurs at the global minimum of $J(q)$. Eg. $Q=\left(\frac{\pi}{a},\frac{\pi}{b},\frac{\pi}{c}\right)$ orthorombic

I: all spins are collinear

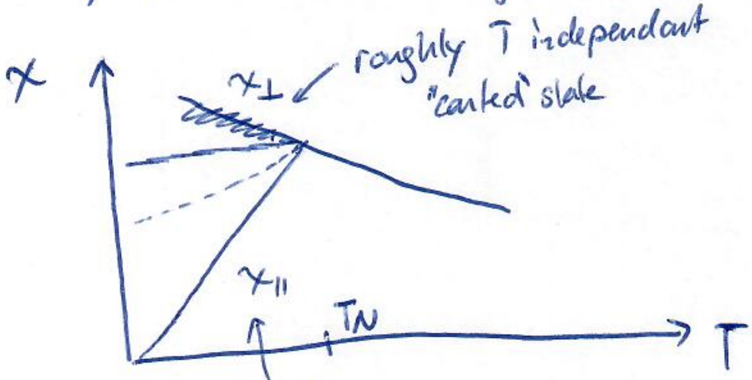
II: no net magnetic moment

III: commensurate with crystal lattice, doubling of unit cell

uniform susceptibility ~~[illegible]~~ never diverges.

$\gamma = 1$ (MF)

but doesn't hold as magnetic short range fall off exp.




no effect on antiferromagnet remain ↑ or ↓

antiferromagnet on BCC-lattice

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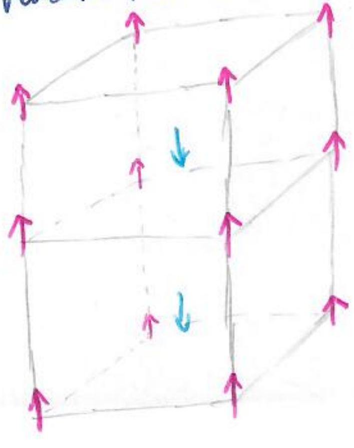
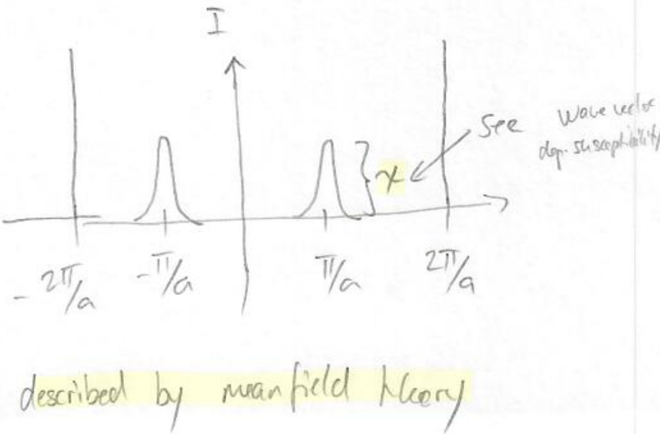


Measure using neutron diffraction




described by mean field theory

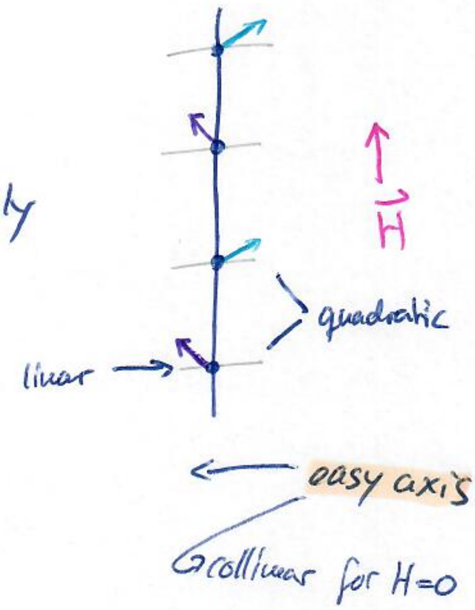
# Spin flop

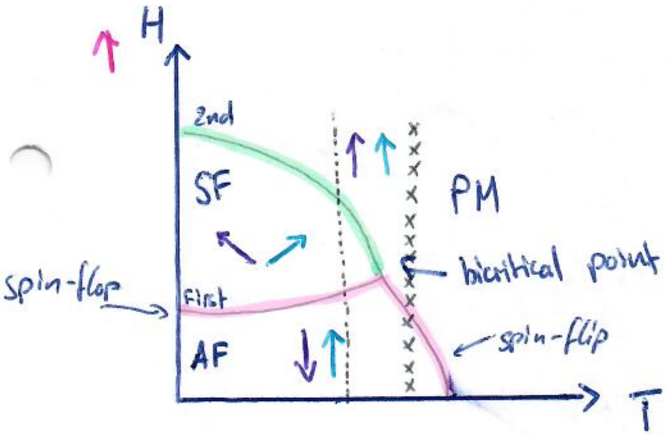
antiferromagnet & helimagnet no net magnetization

⇒ align perpendicular to the field

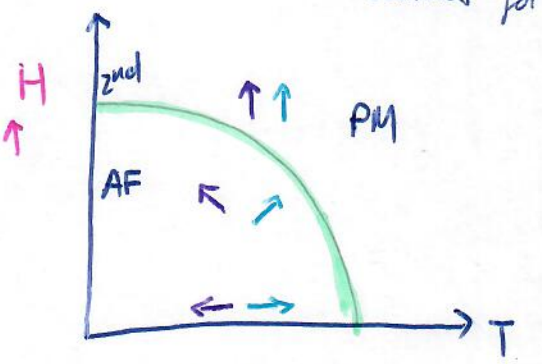
Spin flop is first order (along easy axis), magnetization discontinuously

but no phase coexistence




aligned with easy axis




misaligned with easy axis