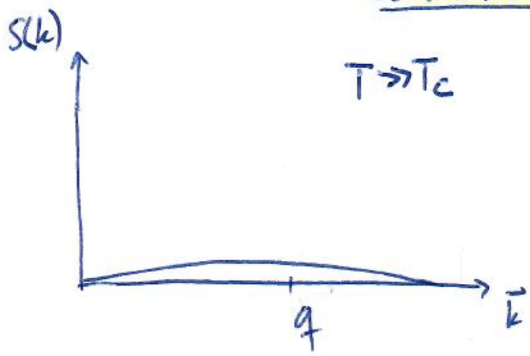
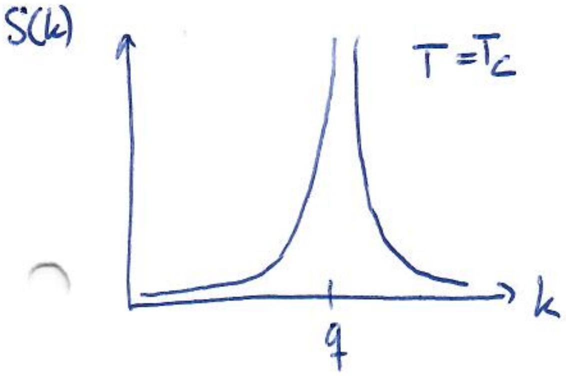
Correlations of order parameter

continuous transition



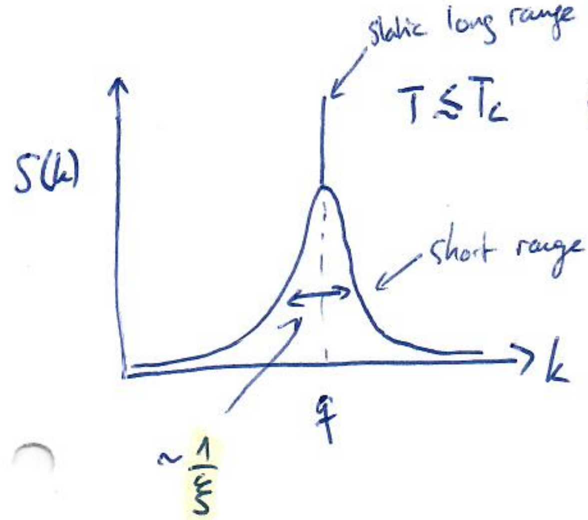
$T \gg T_c$

$\xi \to 0$, high symmetry phase



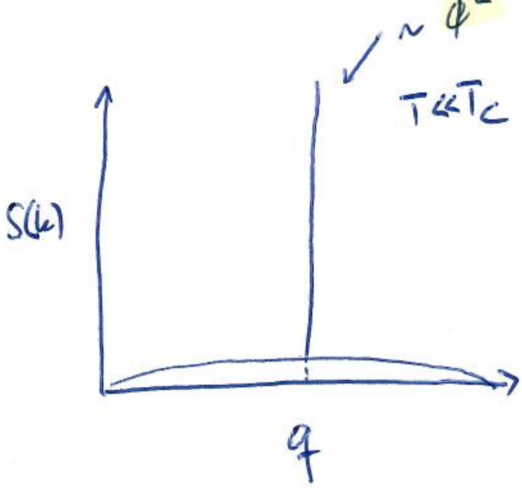
$T = T_c$

$\xi \to \infty$, long range order



$T \lesssim T_c$

two diffraction peaks



$T \ll T_c$

$\xi \to 0$

Wave vector dependant susceptibility

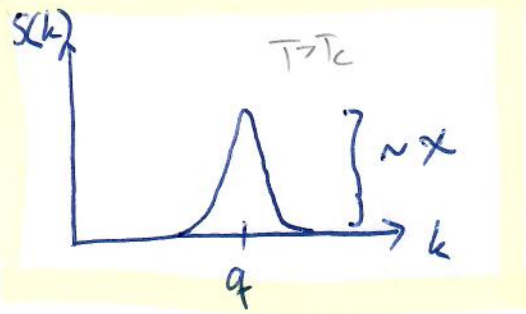
1.7.2. Zheludev

Apply $h(\vec{r}) = h_{\vec{k}}\, e^{i\vec{k}\cdot\vec{r}} + c.c.$ $\Rightarrow$ response in conjugate quantity $\phi(\vec{r})$

$\Rightarrow$ $\delta\phi(\vec{r}) = \phi_{\vec{k}}\, e^{i\vec{k}\cdot\vec{r}} + c.c.$ (same periodicity)

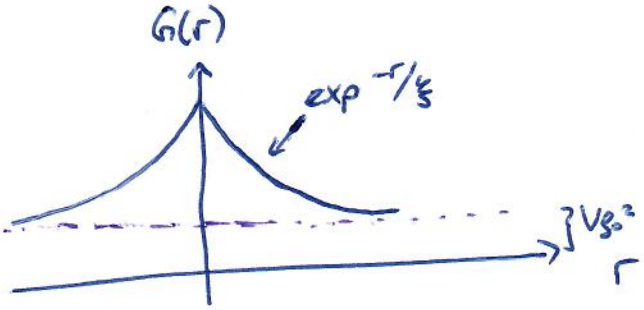
$$\Rightarrow \quad \chi(\vec{k}) := \left.\frac{d\phi_{\vec{k}}}{dh_{\vec{k}}}\right|_{h_{\vec{k}}=0} \overset{\text{Fluctuation dissipation Theorem}}{=} \frac{1}{V}\,\frac{S(k)}{k_B T}$$

If $q=0 \Rightarrow \chi$ can be measured directly. For $q \neq 0$



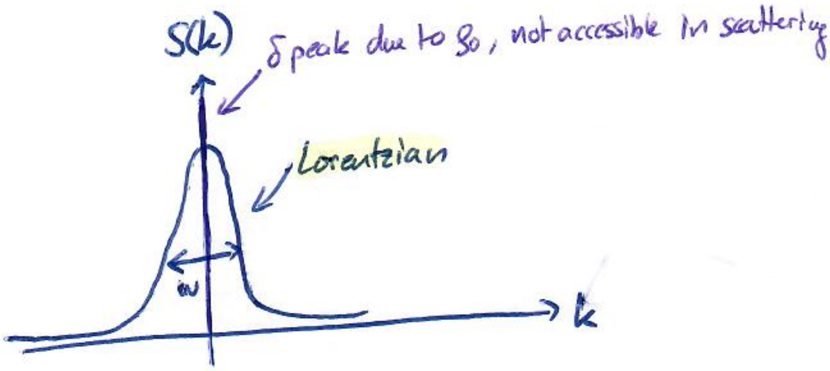
Short range correlations and diffraction

$G(r) \sim G_0 + e^{-r/\xi}$



Fourier-Transformation

$S(k) \propto \frac{1}{1+k^2\xi^2}$



The width is inversely proportional to the correlation length ξ.

Periodic structures

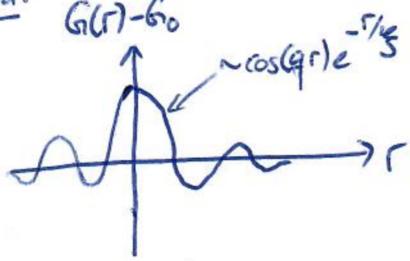
Eg. Density in crystal is spatially modulated, but there are also incommensurate periodic modulation.

Bragg peaks

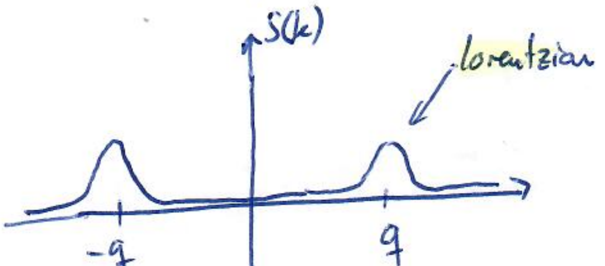
if $\langle \varrho(r) \rangle \sim \cos(qr) \Rightarrow G(r) \sim \cos(qr) \Rightarrow S(k) \sim [\delta(q+k) + \delta(q-k)]$

$\Rightarrow$ periodicity q can be determined using diffraction

Short range corr. with periodic mod.



Idea: apply static H-Field
cool down sample
measure $S(q)$
close to T_c Lorentzian
gets narrower until
finally δ-Bragg



Landau quasiparticles

Coulomb interactions, strong ($T \sim 10^4 K$) and long range

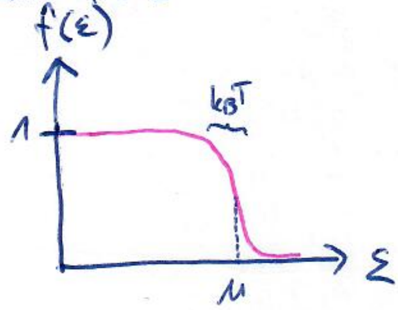
Landau: Low energy of strongly interacting Fermions behave exactly as those of some effective gas of non-interacting quasiparticles. ← $N_{qp} = N_{e^-}$

$$E = \sum_{\vec{k}} n_{\vec{k}} \, \varepsilon(\vec{k})$$

actually depends on $n_{\vec{k}}$ ⇝ interaction

← disp. relation chosen, that hides all interaction of $e^- - e^-$

← quasiparticle state



Fermions →

$$n_{\vec{k}} = \frac{1}{e^{\beta(\varepsilon - \varepsilon_F)} + 1}$$

Remember: $k_F^3 = 3\pi^2 n$ also holds for quasiparticles, $\varepsilon(k_F) = \varepsilon_F$

$T \to 0$ only those quasiparticles with energy $\varepsilon_F \pm k_B T$ are relevant.

$$\Rightarrow \quad \varepsilon(\vec{k}) = \varepsilon_F + \hbar v_F (k - k_F) \quad , \quad v_F \overset{Q.P.}{=} \frac{\hbar k_F}{m^*}$$

← effective mass

Large effective mass signify that the excitation includes many particles.

V_F related?

$$\Rightarrow \quad \nu(\varepsilon_F) = \frac{1}{V} \frac{dN}{d\varepsilon}\Big|_{\varepsilon = \varepsilon_F}$$

Only two numbers characterize all properties of isotropic metall.

Metal: Dielectric susceptibility p. 147 Ziman

Small external periodic potential: $\phi(\vec{r}) = \phi e^{i\vec{q}\cdot\vec{r}} + c.c.$, $\hat{\phi}$ the operator

Assume $\langle \vec{r} | \vec{k} \rangle = \frac{1}{\sqrt{V}} e^{i\vec{k}\cdot\vec{r}}$ (free particle) electrons

$$\Rightarrow |\psi_{\vec{k}}\rangle = |\vec{k}\rangle + \sum_{\vec{k}'} \frac{\langle \vec{k}' | e\hat{\phi} | \vec{k} \rangle}{\varepsilon(\vec{k}) - \varepsilon(\vec{k}')}$$

$\langle \vec{k}' | e\hat{\phi} | \vec{k} \rangle$: $\phi \delta_{\vec{k}+\vec{q}-\vec{k}'} + \phi^* \delta_{\vec{k}-\vec{q}-\vec{k}'}$

$$|\psi_{\vec{k}}\rangle = |k\rangle + \frac{e\phi}{\varepsilon(\vec{k}) - \varepsilon(\vec{k}+\vec{q})} |\vec{k}+\vec{q}\rangle + \frac{e\phi^*}{\varepsilon(\vec{k}) - \varepsilon(\vec{k}-\vec{q})} |\vec{k}-\vec{q}\rangle$$

$$\Rightarrow \delta\rho(\vec{r}) = e \sum_{\vec{k}} |\psi_{\vec{k}}(\vec{r})|^2 f(\vec{k}) - \frac{Ne}{V}$$

$\delta\rho(\vec{r})$: same periodicity as perturbing potential

$f(\vec{k})$: Fermi-dirac

$\frac{Ne}{V}$: Ground charge

By definition $\delta\rho = \alpha_{\vec{q}} \phi$

$$\Rightarrow \alpha_{\vec{q}} = \frac{2e^2}{V} \sum_{\vec{k}} \frac{f(\vec{k}) - f(\vec{k}+\vec{q})}{\varepsilon(\vec{k}) - \varepsilon(\vec{k}+\vec{q})} < 0 \quad \text{(Lindhard's formula)}$$

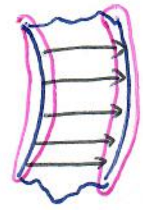
2: spin

Charge density is pushed out of regions with high applied electrical potential.

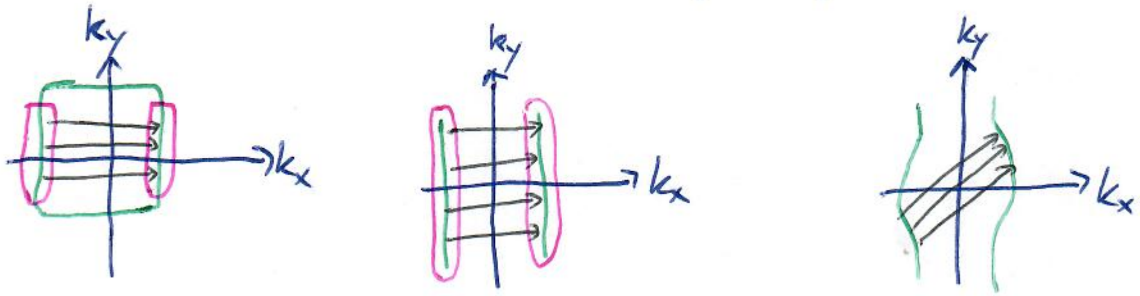
p.155 Ziman ?
p.1 Grüner

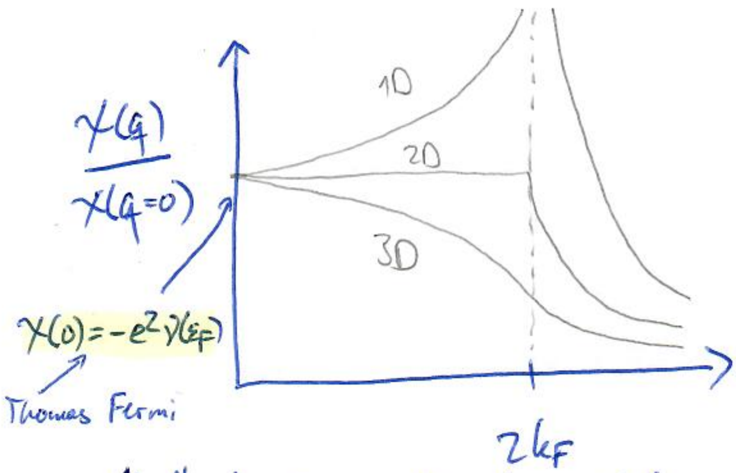
Nesting and dimensionality

$$\chi_q = \frac{2e^2}{(2\pi)^d} \int \frac{\overset{\text{occupied}}{f(\vec{k})} - \overset{\text{empty}}{f(\vec{k}+\vec{q})}}{\varepsilon(\vec{k}) - \varepsilon(\vec{k}+\vec{q})} \, d\vec{k}$$



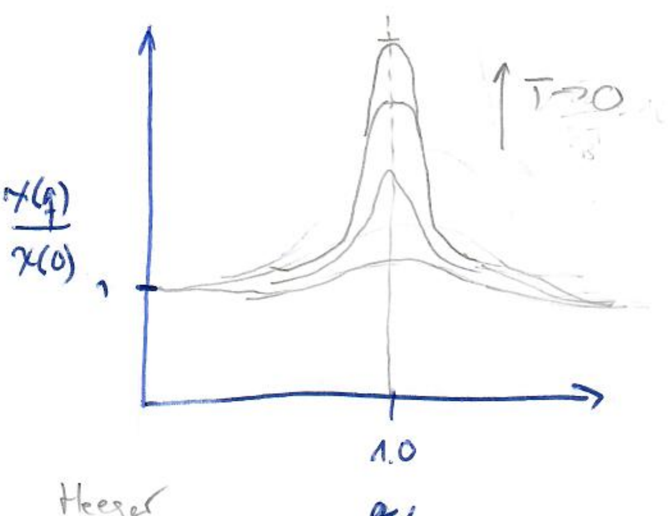
the closer the better $\Rightarrow$ q connects large part of Fermi surface $\Rightarrow$ nesting





Remember: The uniform susceptibility provides a direct measure of the density of states at the Fermi level. TF approx.

Lindhard response for free electron gas at T=0



response function of 1-dim free e gas

Spin susceptibility (ASSP)

p. 333 Ziman

$$H(\vec r) = H\exp(i\vec q\cdot\vec r) + c.c.$$

2 Fermi seas (spin ↑, spin ↓) $\Rightarrow$ $U_\sigma(\vec r) = -\overset{\pm 1}{\sigma} H \frac{g\mu_B}{2} e^{i\vec q\cdot\vec r} + c.c.$

$$\Rightarrow \rho_\sigma(\vec r) = -\sigma \frac{1}{2}\chi_q^{(el)} H \frac{g\mu_B}{2e} e^{i\vec q\cdot\vec r} + c.c.$$

spin

$M = \chi_q H$
$\Rightarrow$

$$\chi_q^{magn} = -\frac{(g\mu_B)^2}{2(2\pi)^d}\int d\vec k \, \frac{f(\vec k) - f(\vec k + \vec q)}{\varepsilon(\vec k) - \varepsilon(\vec k + \vec q)} > 0$$

$\Rightarrow$ paramagnetic

important $\rightarrow$ $$\boxed{\chi_P = \frac{(g\mu_B)^2}{4}\nu(\varepsilon_F)}$$ uniform spin susceptibility

Pauli susceptibility

Again: Pauli susceptibility allows measurement of density of states.

magnetic, T.F. is electric

(p. 344 Ziman)

RKKY interactions

Assume single local magnetic impurity $\vec{m}_1$ ← example: localized f electrons in rare earth metals

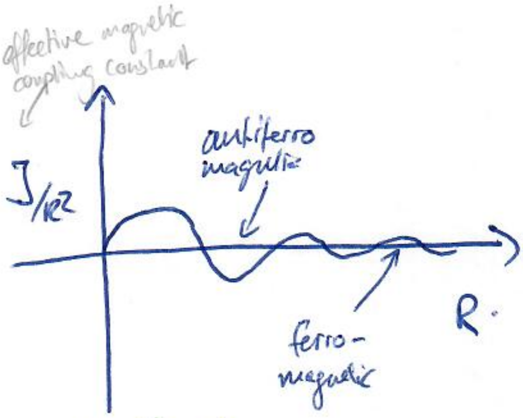
$$\Rightarrow \quad \vec{H}(\vec{r}) = \frac{1}{4\pi}\left[\underbrace{\frac{3\vec{r}(\vec{m}_1 \cdot \vec{r})}{r^5} - \frac{\vec{m}_1}{r^3}}_{\text{zero upon spherical average}} \underbrace{- \frac{4\pi}{3}\vec{m}_1 \delta(\vec{r})}_{\text{contact field}}\right]$$

no derivation

$$\Rightarrow \quad \vec{H}(\vec{r}) = A\vec{m}_1 \delta(\vec{r})$$

← coupling

$$\Rightarrow \quad \vec{M}(\vec{r}) = \vec{m}_1 A \chi(\vec{r})$$



Assume another localized magnetic moment $\vec{m}_2$ at $\vec{R}+\vec{r}$

$$\mathcal{H} = -A^2 \chi(\vec{R})\vec{m}_1 \cdot \vec{m}_2$$

Ruderman-Kittel-Kasui-Yosida interaction is the primary source of magnetic coupling in rare earth materials, where the magnetic electrons are very localized.

$$J_{RKKY}(R) \propto \chi(R)$$

Use NMR and impurities (ferromagnetic, here Fe) to induce a knight shift in neighbours to prove oscillatory behaviour of RKKY-Interactions.

Spin waves

goldstone p 407 Huang

linearized $\Rightarrow$ eOM

Only transverse spin component fluctuate.

Dispersion relation for spin wave excitation in ferromagnet:

$$\varepsilon(k) = \hbar\omega_{\vec{k}} = S\,|J(0) - J(\vec{k})| \overset{\text{cubic}}{\sim} k^2$$

ferromagnet: $Q=0$

circular precession ω

Spin waves are quantized