Definition

$p = \hbar k$

$k = \frac{2\pi}{\lambda}$

$E = \hbar\omega$

Operatoren

$\hat{\vec p} = -i\hbar\vec\nabla$

$T = -\frac{\hbar^2}{2m}\Delta$

$P\psi(x) = \psi(-x)$

$T\psi(x,t) = \psi^*(x,-t)$

Heisenberg

$\Delta x \Delta p \geq \hbar$

$[x,p] = i\hbar$

$[p,x] = -i\hbar$

Skalarprodukt

$\langle\varphi|\psi\rangle = \langle\psi|\varphi\rangle^*$

$\langle\varphi|\psi\rangle = \int d^n r\, \varphi^* \psi$

$\langle\varphi|A\psi\rangle = \langle A^\dagger\varphi|\psi\rangle$

Nullstellen

$\sin x = n\pi$

$\cos x = \pi(n+\frac{1}{2})$

Mechanik

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot x}\right) - \frac{\partial L}{\partial x} = 0$

$H = \frac{\partial L}{\partial \dot q}\dot q - L = T + V$

$\dot q = \frac{\partial H}{\partial p}, \quad \dot p = -\frac{\partial H}{\partial q}$

$\frac{df}{dt} = \{f,H\} + \frac{\partial f}{\partial t}$

δ-Funktion

$\lim_{\varepsilon\to 0} \frac{e^{-x^2/2\varepsilon}}{\sqrt{2\pi\varepsilon}}$

$\lim_{\varepsilon\to 0} \frac{\varepsilon/x}{x^2+\varepsilon^2}$

$\lim_{\varepsilon\to 0} \frac{1}{\pi}\frac{\sin(x/\varepsilon)}{x}$

$\lim_{\varepsilon\to 0} \frac{\varepsilon}{\pi}\frac{\sin^2(x/\varepsilon)}{x^2}$

10 Regeln

- Observablen sind hermitesch
- Erwartungswert: $\langle A\rangle = \langle\psi|A|\psi\rangle$
- $|\psi\rangle$, falls a von A gemessen $\Rightarrow |a\rangle$

$\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$

Schrödinger

$i\hbar\partial_t\psi = H\psi, \qquad E = \frac{\hbar^2 k^2}{2m}$

$V=0 \Rightarrow \psi = e^{i(kx-\omega t)}$

$\partial_t H = 0 \Rightarrow H\psi = E\psi$

$\Rightarrow \psi(r,t) = e^{-\frac{i}{\hbar}Et}\psi(r,0)$

Gauss-Integrale

$\int_{-\infty}^{\infty} dx\, e^{-ax^2+bx+c} = \sqrt{\frac{\pi}{a}}\, e^{\frac{b^2}{4a}+c}$

Residuum $\mathrm{Res}_a f = \frac{1}{2\pi i}\oint f$

1. Ord. $\mathrm{Res}_a f = \lim_{z\to a}(z-a)f(z)$

2. Ord. $\mathrm{Res}_a f = \lim_{z\to a}\frac{\partial}{\partial z}\left((z-a)^2 f(z)\right)$

Fourierreihe

$V(x) = \sum_{n=-\infty}^{\infty} V_n e^{ik_n x}, \quad k_n = \frac{2\pi}{L}n$

$V_n = \frac{1}{L}\int_0^L dx\, V(x)\, e^{-ik_n x}$

Fouriertransformation

Hin: $G(k,\omega) = \int d^n x\, dt\, e^{-i(kx-\omega t)} G(x,t)$

Rück: $G(x,t) = \frac{1}{(2\pi)^{n+1}}\int d^n k\, d\omega\, e^{i(kx-\omega t)} G(k,\omega)$

Fundamentallösung

$\left(\frac{\hbar^2}{2m}\nabla^2 + E\right)G(r,E) = \delta(r)$

$(i\hbar\partial_t - H)G(x,t) = i\hbar\delta(x)\delta(t)$

$G(k,\omega) = \frac{1}{\left(Le^{-i(kx-\omega t)}\right)/e^{-i(kx-\omega t)}}$

zuerst L ausführen

Integration Determinante

$\int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta \int_0^{\infty} r^2\, dr$

$\int_0^{2\pi} d\varphi \int_0^{\infty} r\, dr$

Integral

$\int_0^{\infty} dr\, r^2 e^{-xr^2} = 2x^{-3}$

$\int \sin^2(ax)dx = \frac{x}{2} - \frac{\sin(2ax)}{4a}$

$\int \cos^2(ax)dx = \frac{x}{2} + \frac{\sin(2ax)}{4a}$

Definition (Substitution)

$k = \frac{\sqrt{2mE}}{\hbar}$

$\alpha = \sqrt{2m(V-E)}/\hbar$

2dim rot.symm Potential

$\Delta_{2D} = \partial_r^2 + \frac{1}{r}\partial_r + \frac{1}{r^2}\partial_\varphi^2$

Ansatz: $R_l(r)e^{il\varphi}$

$R_l(r) = \frac{u(r)}{r^\alpha} \overset{\text{opt.}}{=} \frac{u(r)}{\sqrt r}$

$\Delta_r \frac{u(r)}{\sqrt r} = \frac{u''}{\sqrt r} + \frac{u}{4r^{5/2}}$

Sphärische Probleme

$\Delta_3 = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\Delta_S$

$\Delta_r, \quad \Delta_S Y = -l(l+1)$

Ansatz: $\psi(\vec r) = R(r)\, Y_{l,m}(\theta,\varphi)$

optimal: $R(r) = \frac{u(r)}{r}$

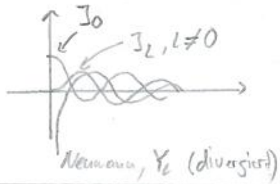
$\Delta_r \frac{u(r)}{r} = \frac{u''}{r}$

Bessel DGL $s = kr$

$r^2\frac{d^2R}{dr^2} + r\frac{dR}{dr} + (r^2 - l^2)R = 0$

$R = a_l J_l + b_l Y_l, \quad l\in\mathbb{Z}$

$R = a_l J_l + b_l J_{-l}, \quad l\notin\mathbb{Z}$



Modifizierte Bessel DGL

$r^2\frac{d^2R}{dr^2} + r\frac{dR}{dr} - (r^2 + l^2)R = 0$

$R = c_l I_l + d_l K_l, \quad l\in\mathbb{Z}$

$R = c_l I_l + d_l I_{-l}, \quad l\notin\mathbb{Z}$

I_l divergiert

Pauli Matrizen hermitesch, unitär, det = −1, tr = 0

$\sigma_1 = \begin{pmatrix}0&1\\1&0\end{pmatrix}, \quad \sigma_2 = \begin{pmatrix}0&-i\\i&0\end{pmatrix}, \quad \sigma_3 = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$

Kugelfunktionen

$Y_{0,0} = \frac{1}{\sqrt{4\pi}}, \quad Y_{1,0} = \sqrt{\frac{3}{4\pi}}\cos\theta$

$Y_{1,\pm1} = \mp\sqrt{\frac{3}{8\pi}}\, e^{\pm i\varphi}\sin\theta$

Schrödinger Matrix

Hols Matrix, $E = E I_n$

$\det(H - E) = 0$

Drehmatrix

$R = \begin{pmatrix}\sin\varphi & -\cos\varphi\\ \cos\varphi & \sin\varphi\end{pmatrix}$

Trigonometrie

$S = SC \pm CS$

$C = CC \mp SS$

$\sin x = \frac{e^{ix} - e^{-ix}}{2i} = x - \frac{x^3}{6}$

$\cos x = \frac{e^{ix} + e^{-ix}}{2} = 1 - \frac{x^2}{2}$

$\tan x = x + \frac{1}{3}x^3, \quad \arctan x = x - \frac{1}{3}x^3$

Definitheit

$\det S \geq 0, \quad S_{00} < 0$

Bessel/Neumann

$j_0(x) = \frac{\sin x}{x} \qquad j_1 = \frac{\sin x - x\cos x}{x^2}$

$n_0(x) = -\frac{\cos x}{x} \qquad n_1 = -\frac{\cos x + x\sin x}{x^2}$

Tiefe Energie

$j_l(\rho_0) = \frac{\rho_0^{\,l}}{(2l+1)!!}$

$n_l(\rho_0) = \frac{(2l-1)!!}{\rho_0^{\,l+1}}$

Pauli

$\exp(-i\,\theta/2\ \sigma\cdot n) = \cos\frac{\theta}{2}\mathbb{1} - i\sin\frac{\theta}{2}(\sigma\cdot n)$

Sphärische Hankel

$h_l^{(1)} = j_l + i y_l$ ← ausgehend

$h_l^{(2)} = j_l - i y_l$ ← eingehend

aus $(\Delta - \lambda)\psi = 0$ nach Separation von $(\theta\varphi)$

$\left[\partial_\rho^2 + \frac{2}{\rho}\partial_\rho + \left(1 - \frac{l(l+1)}{\rho^2}\right)\right]R_l(\rho) = 0$

$V=0 \Rightarrow R_l(r) = j_l(kr)$

$= \frac{1}{2}\left[h_l^{(2)}(kr) + h_l^{(1)}(kr)\right]$

$V\neq 0 \quad R_l(r) = \alpha_l\left(h_l^{(2)} + S_l h_l^{(1)}\right)$

einfallend, auslaufend, Faktor

$|S_l| = 1$ Teilchenerhaltung

Harm. Osz. $|n\rangle = \frac{1}{\sqrt{2^n n!\sqrt\pi}} H_n(x)\, e^{-\frac{x^2}{2}}$

$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 q^2$

$a = \frac{1}{\sqrt2}\left(\sqrt{\frac{m\omega}{\hbar}}\,q + \frac{i}{\sqrt{m\hbar\omega}}\,p\right)$

$a^\dagger = \frac{1}{\sqrt2}\left(\sqrt{\frac{m\omega}{\hbar}}\,q - \frac{i}{\sqrt{m\hbar\omega}}\,p\right)$

$q = \sqrt{\frac{\hbar}{2m\omega}}\,(a^\dagger + a)$

$p = i\sqrt{\frac{m\hbar\omega}{2}}\,(a^\dagger - a)$

$[q,p] = i\hbar$ (Heisenberg)

$[a,a^\dagger] = 1$, $[\cdot,\cdot]$ antisymm.

$N = a^\dagger a, \quad N|n\rangle = n|n\rangle$

$a|n\rangle = \sqrt{n}\,|n-1\rangle$

$a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$

$E_n = \hbar\omega(n + \frac{1}{2})$

Virial $2\langle T\rangle = \langle q\frac{dV}{dq}\rangle$

Drehimpuls

$L_i = \varepsilon_{ijk}x_j(-i\hbar)\partial_k$

$L_\pm = L_x \pm iL_y$

$L_x = \frac{L_+ + L_-}{2}$

$L_y = \frac{L_+ - L_-}{2i}$

$[L_i, L_j] = i\hbar\varepsilon_{ijk}L_k$

$[L_z, L_\pm] = \pm\hbar L_\pm$

$L^2|l,m\rangle = \hbar^2 l(l+1)|l,m\rangle$

$L_z|l,m\rangle = \hbar m|l,m\rangle, \quad L_z = \frac{\hbar}{i}(x\partial_y - y\partial_x)$

$L_\pm|l,m\rangle = \hbar\sqrt{l(l+1) - m(m\pm1)}\,|l,m\pm1\rangle$

Spin

$S^2|s,m\rangle = s(s+1)|s,m\rangle$

$S_z|s,m\rangle = m|s,m\rangle$

$j = l \pm s, \quad m = m_l + m_s$

$J^2|j,m\rangle = \hbar j(j+1)|j,m\rangle$

$j_z|j,m\rangle = \hbar m|j,m\rangle$

Elektromagnetisches Feld

$H = \frac{1}{2m}\left(p - \frac{q}{c}A\right)^2 + q\varphi = T + V$

Gauge: $A' = A + \nabla\chi = 0, \quad \varphi' = \varphi - \frac{1}{c}\frac{\partial\chi}{\partial t}$

$\psi' = \psi_0\exp\left(\frac{iq}{\hbar c}\chi\right)$, Randbed.

$B = \mathrm{rot}A, \quad E = -\nabla\varphi - \frac{1}{c}\frac{\partial A}{\partial t}$

Clebsch Gordan $H_l \otimes H_{1/2} = H_{l+\frac12} \oplus H_{l-\frac12}$

$H_{l+\frac12}: \langle l,\tfrac12, m\mp\tfrac12, \pm\tfrac12 | l,\tfrac12, l+\tfrac12, m\rangle = \sqrt{\frac{l\pm m + 1/2}{2l+1}}\ |\left(l,\tfrac12, m\mp\tfrac12, \pm\tfrac12\right)$

(l s m_l m_s l s j m)

$H_{l-\frac12}: \langle l,\tfrac12, m\mp\tfrac12, \pm\tfrac12 | l,\tfrac12, l-\tfrac12, m\rangle = \mp\sqrt{\frac{l\mp m + 1/2}{2l+1}}$

$|l+\tfrac12, m\rangle = \sqrt{\frac{l+m+\frac12}{2l+1}}\ |l, m-\tfrac12\rangle\otimes|\tfrac12,\tfrac12\rangle + \sqrt{\frac{l-m+\frac12}{2l+1}}\ |l, m+\tfrac12\rangle\otimes|\tfrac12,-\tfrac12\rangle$

$|l-\tfrac12, m\rangle = -\sqrt{\frac{l-m+\frac12}{2l+1}}\ |l, m-\tfrac12\rangle\otimes|\tfrac12,\tfrac12\rangle + \sqrt{\frac{l+m+\frac12}{2l+1}}\ |l, m+\tfrac12\rangle\otimes|\tfrac12,-\tfrac12\rangle$

minus

Shift in Position

$q \rightsquigarrow (q - q_0), \quad \psi' = \exp(-ipq_0/\hbar)\psi_0$

Wasserstoffatom $|n,l,m\rangle$ — s p d f: 0 1 2 3

$\psi_{n,l,m} = R_{n,l}\, Y_{l,m}, \quad 0 \leq l \leq n-1, \quad -l \leq m \leq l$

$R_{1,0} = 2\kappa^{3/2}\exp(-\kappa r)$

$R_{2,0} = 2\kappa^{3/2}(1-\kappa r)\exp(-\kappa r)$

$R_{2,1} = \frac{2\kappa^{3/2}}{\sqrt3}\kappa r\exp(-\kappa r)$

Entartung $n^2, \quad a_B = \frac{\hbar^2}{m_e e^2}$

$\kappa = \frac{Z}{a_B n}, \quad Z = \text{anz } p^+$

$E^0_{kin} = \frac{e^2}{2a_B}, \quad E^0_{pot} = \frac{-e^2}{a_B}$

$P\psi_{n,l,m} = (-1)^l\, Y_{l,m}$

$\psi_n = N_n H_n(x)\exp\left(-\frac{x^2}{2}\right)$

$x = \sqrt{\frac{m\omega}{\hbar}}\, q, \quad H_0 = 1, \quad H_1 = 2x$

$H_2 = 4x^2 - 2, \quad N_N = \frac{1}{\sqrt{\sqrt\pi\, 2^n n!}}$

Tiefe Energie

$l=0, \ u(0) = 0$

Tot. Wirkungsquerschnitt

$\sigma = \frac{4\pi}{k^2}\sum_l (2l+1)\sin^2\delta_l$

Energiedichte

$F(E) \rightsquigarrow \frac{dF}{dE}$, $\frac{dn(E)}{dE} = g(E)$

$G(E) = \int_0^E dE$

Topf

$\psi_n(x) = \begin{cases}\frac{1}{\sqrt a}\sin\left(\frac{n\pi}{2a}x\right), & n \text{ gerade } 0,2,\ldots \\ \frac{1}{\sqrt a}\cos\left(\frac{n\pi}{2a}x\right), & n \text{ ungerade } 1,3,\ldots\end{cases}$

grund state

Verschiebung: $x \mapsto x - a$, Erster angeregter Zustand: $n = 2$

$E = \frac{\hbar^2 k^2}{2m}$

Bloch $A=0$ mit Gauge, RB anpassen $|\psi'(x)e^{i\alpha} = \psi'(x+L)$ Bloch form

$\left[-\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2} + V(x)\right] = \psi_{n,\alpha}(x) = \Sigma_{n,\alpha}\psi'_{n,\alpha}(x), \quad \psi' = e^{i\alpha x/L}\psi$

$\alpha \in [-\pi,\pi], \quad \psi_{n,k} = e^{ikx}u(r), \quad u(r)$ periodisch

Drehimpuls Generatoren $R_z(\theta) = \begin{pmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0&0&1\end{pmatrix}$

$L_x = \begin{pmatrix}0&0&0\\0&0&-1\\0&1&0\end{pmatrix}, \quad L_y = \begin{pmatrix}0&0&1\\0&0&0\\-1&0&0\end{pmatrix}, \quad L_z = \begin{pmatrix}0&-1&0\\1&0&0\\0&0&0\end{pmatrix}$

Operatoren

$U_a = e^{-i\vec a\cdot\hat{\vec p}/\hbar}$ Translation um $\vec a$

$U_\omega = e^{-i\vec\omega\cdot\vec L/\hbar}$ Drehung um $\vec\omega$

$U_{\Delta t} = e^{iH\Delta t/\hbar}$ Zeittranslation um Δt

$R(\hat n,\theta) = e^{-i\theta\hat n\cdot\vec S/\hbar}$ Rotationsoperator

Propagator

$K(x,t) = \sqrt{\frac{m}{2\pi i\hbar t}}\, e^{\frac{imx^2}{2\hbar t}}$ Propagation des Teilchens mit Wahrsch. $P = |K|^2$ für ungestörte Bewegung durch Raum

Green Funktion für den $H = \frac{p^2}{2m}$

Heisenberg

$i\hbar\frac{d}{dt}A_H(t) = [A_H, H_H] + i\hbar\partial_t A_H$

Dichtematrix $\binom{?}{?} \to (0, -i)$

$\sum_i p_i|\psi_i\rangle\langle\psi_i| = \rho$

hermitesch, pos. definit, $\mathrm{tr}\,\rho = 1$

$\mathrm{tr}\,\rho^2 = \begin{cases}1 & \text{rein} \Leftrightarrow \det\rho = 0\\ <1 & \text{gemischt}\end{cases}$

$\langle A\rangle = \mathrm{tr}(A\rho), \quad i\hbar\partial_t\rho = [H,\rho]$

$\langle\varphi|A|\varphi\rangle = \mathrm{tr}(|\varphi\rangle\langle\varphi|A)$

$P_r[a_i] = \mathrm{tr}(\rho P_i) = \langle\psi_k|P_i|\psi_k\rangle$

Ring

$A = \frac{\phi}{2\pi R^2}\begin{pmatrix}-y\\x\\0\end{pmatrix}, \quad B = \frac{\phi}{\pi R^2}\hat e_z, \quad \hat x = R\hat e_\varphi$

$A_x(\varphi) = \frac{\phi}{2\pi R} = \frac{\phi}{L} \Rightarrow \chi = -\frac{\phi}{L}x$

Konst. Kraftfeld

$H = -Fq$

Ansatz: $(q - q_0)^2$

Operatoren

$U_a = e^{-i\vec a\cdot\vec p/\hbar}, \quad T_a^{-1}(r) = r - a$

$U_\omega = e^{-i\vec\omega\cdot\vec L/\hbar}, \quad R_\omega^{-1}(r) = r - \omega\times r$

$U_{\Delta t} = e^{iH\Delta t/\hbar}, \quad T_\tau^{-1}(t) = t - \tau$

Def ⇒ Taylor in δ_τ ⇒ Ableitung ⇒ DGL, $U_{(\theta)}$

Prob

$|\langle\varphi|+\rangle|^2, \quad |\langle\varphi|-\rangle|^2$

$|+\rangle\langle 1|, \quad |-\rangle\langle -1|$

+

Ansätze

Rotat. symmetr. 2dim: $\psi = R_l(r)e^{il\varphi}$

Rotat. symmetr. 3dim: $\psi = R\, Y_{l,m}$

3Dim Harm Osz: $\psi = XYZ$

V=0, freies Teilchen: $\psi = e^{i(kx-\omega t)}$

$[p_y, H]$: $\psi = X(x)\, e^{ik_y y}$

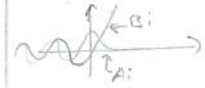
Airy

$(\partial_y^2 + y)\psi(y) = 0$

$\psi(y\to\infty) \to 0$

$\psi(y) = N_1 Ai(-y)$

$= N_2\, Bi(-y)$ unphysikal.



Transfermatrix $e^{i\kappa x}$-Ansatz

$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1+\frac{i\alpha}{k} & 1-\frac{i\alpha}{k} \\ 1-\frac{i\alpha}{k} & 1+\frac{i\alpha}{k} \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix}$$

$\underbrace{\qquad}_{M}$

$M^{-1}: \frac{i\alpha}{k} \rightsquigarrow \frac{k}{i\alpha}$, $\det M = \frac{i\alpha}{k}$

$$M = \begin{pmatrix} 1/t & r^*/t^* \\ r/t & 1/t^* \end{pmatrix}$$

$\psi_r = Ae^{ikx} + Be^{-ikx}$

Propagation: $\det \tilde{M} = 1$

$\tilde{M} = \begin{pmatrix} e^{-ikw} & 0 \\ 0 & e^{ikw} \end{pmatrix}$

$\tilde{M} = \begin{pmatrix} e^{\alpha w} & 0 \\ 0 & e^{-\alpha w} \end{pmatrix}$

Rayleigh-Ritz

$$\frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} \geq E_0 \Rightarrow \psi_0$$

$\psi = \alpha\psi_0 + \beta\psi_0^{\perp}$

$\rightarrow \frac{\partial}{\partial\alpha}\langle\psi|H|\psi\rangle \overset{!}{=} 0$

Stationär entartet 1. Ordn.

$H = H_0 + H'$, finde ψ_0 von H_0

Welche Entartung tritt auf?

Welche Operatoren kommutieren?

Bsp. $[l_z, H'] = 0 \Rightarrow m' \neq m \Rightarrow$ Einträge $= 0$

$M = \langle n', l', m' | H' | n, l, m\rangle$

Bsp. $n=2$)

$$\begin{array}{c|cccc} & 2s0 & 2p1 & 2p0 & 2p\text{-}1 \\ \hline 2s0 & & & & \\ 2p1 & & & & \\ 2p0 & & & & \\ 2p\text{-}1 & & & & \end{array}$$

Falls H' mit P kommutiert und H' ungerade

$\Rightarrow$ Diagonale $= 0$ Einträge $\neq 0$

Compakte Matrix $(\quad)$

Diese Diagonalisieren, identifiz. Vektoren mit Zustände.

0 Einträge sind noch entartet in grosser Matrix

Streumatrix

$\begin{smallmatrix} a \rightarrow & \leftarrow B \\ b \leftarrow & \rightarrow A \end{smallmatrix}$

$$\begin{pmatrix} b \\ A \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}\begin{pmatrix} a \\ B \end{pmatrix} = S\psi_{in} = \psi_{out}$$

Sunitär, falls symmetrisches S (P) Streupotential und Zeitumkehrinvar. (T)

$\Rightarrow S = \begin{pmatrix} r & t \\ t & r \end{pmatrix}$, $S_D = \begin{pmatrix} r+t & 0 \\ 0 & r-t \end{pmatrix}$

$r = |r|\exp(i2\varphi)$, $\varphi = \arctan\left(\frac{|r|}{|t|}\right)$

$\varphi = \begin{cases} \pi/4 & \text{linear ansteigen Pot} \\ \pi/2 & \text{unendlich hoch Pot.} \end{cases}$

δ-Potential

$[V_0] = \mathrm{J}\cdot\mathrm{m}$

$H = -\frac{\hbar^2}{2m}\partial_x^2 + V_0\,\delta(x)$

$$M = \begin{pmatrix} 1+iv/k & iv/k \\ -iv/k & 1-iv/k \end{pmatrix}$$

$v = mV_0/\hbar^2$, $k = \frac{\sqrt{2mE}}{\hbar}$

$V_0 < 0 \Rightarrow E = -\frac{mV_0^2}{2\hbar^2}$

$V_0 > 0 \Rightarrow t = \frac{1}{1+\frac{iv}{k}}$

$|t|^2 \approx \frac{2\hbar^2 E}{mV_0^2}$, V_0 gross

Stationär nicht entartet

$H = H_0 + H'$

Matrixelement: $\langle l|H'|k\rangle$

0^{th} order

$E_0 = E_0$ von H_0, $|\psi_0\rangle$ von H_0

1^{st} order

$E_1 = \langle n|H'|n\rangle$, $|n\rangle$ von H_0

$$|\psi_1\rangle = \sum_{l\neq n}\frac{\langle l|H'|n\rangle}{\varepsilon_n - \varepsilon_l}|l\rangle, \quad \varepsilon_n \text{ von } H_0$$

$\rightsquigarrow$ Umformen nach $|n\rangle$

2^{nd} order

$$E_2 = \sum_{l\neq n}\frac{|\langle l|H'|n\rangle|^2}{\varepsilon_n - \varepsilon_l}$$

$$|\psi_2\rangle = \sum_{l\neq n}\left(\sum_{k\neq n}\frac{\langle l|H'|k\rangle\langle k|H'|n\rangle}{(\varepsilon_n-\varepsilon_l)(\varepsilon_n-\varepsilon_k)} - \frac{\langle l|H'|n\rangle\langle n|H'|n\rangle}{(\varepsilon_n-\varepsilon_l)^2}\right)|l\rangle$$

Normierung

$$\langle\psi|\psi\rangle = 1 + \sum_{l\neq n}\frac{|\langle l|H'|n\rangle|^2}{(\varepsilon_n-\varepsilon_l)^2}$$

Ansatz: $\psi_2' = \psi_2 - \mu\psi_0 \Rightarrow \mu = -\frac{1}{2}\sum_{l\neq n}\frac{\langle n|H'|l\rangle\langle l|H'|n\rangle}{(\varepsilon_n-\varepsilon_l)^2}$

$$|\psi_2\rangle_{norm} = |\psi_2\rangle - \frac{1}{2}\sum_{l\neq n}\frac{\langle n|H'|l\rangle\langle l|H'|n\rangle}{(\varepsilon_n-\varepsilon_l)^2}|n\rangle$$

2te Ordn. Störungstheorie führt immer zu einer Erniedrigung der Energie des Grundzustand.

Lippmann Schwinger

$G(\vec r, E_k) = -\frac{m}{2\pi\hbar^2}\cdot\frac{e^{ikr}}{r}$

$$\psi_k = e^{i\vec k\cdot\vec r} + \int d^3r'\, G(\vec r - \vec r', E_k)\, V(\vec r')\,\psi_k(\vec r')$$

Born 1te Näherung: $\psi_k(\vec r) = e^{i\vec k\cdot\vec r}$

$\hookrightarrow f_k^{(1)}(\Omega) = -\frac{m}{2\pi\hbar}\int d^3r'\, V(\vec r')\, e^{i(\vec k - \vec k_e)\cdot\vec r'}$

$\frac{d\sigma}{d\Omega} = |f_k(\Omega)|^2$, $\sigma_{tot} = \frac{4\pi}{k^2}\sum_l (2l+1)|f_l|^2$

$f_l = \frac{e^{i\delta_l}\sin\delta_l}{k}$, $k = \frac{g_0}{r_0}$

Resonanzen

$\kappa = \frac{k}{l}$

$l = \sqrt{2m(E-V)}/\hbar$, $k = \frac{\sqrt{2mE}}{\hbar}$

$$t = \frac{1}{\cos wl - i\cosh y\sin wl}$$

$$|t|^2 = \frac{1}{1 + \frac{\sin^2(wl)V^2}{4E(E-V)}} \leq 1$$

$|t|^2 = 1 \Leftrightarrow l = \frac{n\pi}{w} \Rightarrow E_{res} = \frac{\hbar^2\pi^2}{2mw^2}n^2 + V$

$= E_0(n\pi)^2 + V$

geb Zustand: $n_{bound} = \left\lceil\sqrt{\frac{|V|}{E_0}}\right\rceil < n_{min}$

Transfer

$$M = \frac{1}{2}\begin{pmatrix} 1+\frac{l}{k} & 1-\frac{l}{k} \\ 1-\frac{l}{k} & 1+\frac{l}{k} \end{pmatrix}$$

Dirac

$$\langle p|x\rangle = \frac{1}{\sqrt{2\pi\hbar}}e^{-ipx/\hbar}$$

Partialwellenamplitude

$f_l = \frac{e^{i\delta_l}\sin\delta_l}{k}$

Partieller Wirkungsquerschnitt

$\sigma_l = 4\pi(2l+1)|f_l|^2$

Streutheorie

$> r_0$: $R_l(r) = \frac{1}{2}\left(h_l^{(2)}(kr) + e^{2i\delta_l(k)}h_l^{(1)}(kr)\right)$

$= e^{i\delta_l}\left(\cos(\delta_l)j_l(kr) - \sin(\delta_l)n_l(kr)\right)$

Streulänge a: $a = -\frac{\tan\delta_0}{k}$, $k\cot\delta_0 = -\frac{1}{a}$

$a>0$ repulsiv, $a<0$ attraktiv, $\rho = kr$, $\rho_0 = kr_0$

Tunneln

$\kappa = \frac{k}{\alpha}$, $y = -\log\kappa$

$$M = \begin{pmatrix} \cosh\alpha w + i\sinh y\sinh\alpha w & i\cosh y\sinh\alpha w \\ -i\cosh y\sinh\alpha w & \cosh\alpha w - i\sinh y\sinh\alpha w \end{pmatrix}$$

$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ r \end{pmatrix}$, $\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} t \\ 0 \end{pmatrix}$, $M = \begin{pmatrix} \frac{1}{t} & \frac{r^*}{t^*} \\ \frac{r}{t} & \frac{1}{t^*} \end{pmatrix}$

$$|t|^2 = \frac{1}{1 + \frac{V^2\sinh^2\alpha w}{4E(V-E)}}$$

$\alpha w \,\hat{=}$ wie verboten

$$t = \frac{2ik\alpha}{2ik\alpha\cosh\alpha w - (\alpha^2 - k^2)\sinh\alpha w}$$

Stationär, fast entartet

$|n\rangle$ & $|m\rangle$ beinahe Entartet oder vollst.

$H = H_0 + H' = \underbrace{H_0 + H_1'}_{=: H_1} + H_2'$

$\varepsilon_m' = \langle m|H_1|m\rangle = \varepsilon_m + \langle m|H'|m\rangle$

$\varepsilon_n' = \langle n|H_1|n\rangle = \varepsilon_n + \langle n|H'|n\rangle$

$\delta' = \langle m|H_1|n\rangle = \langle m|H'|n\rangle$

$\delta'^* = \langle n|H_1|m\rangle = \langle n|H'|m\rangle$

$l \neq m,n$: ε_l, $|l\rangle$ $\rightsquigarrow$ keine Entartung

$l = m,n$: $\varepsilon_\pm = \frac{\varepsilon_m' + \varepsilon_n'}{2} \pm \sqrt{\left(\frac{\varepsilon_m' - \varepsilon_n'}{2}\right)^2 + |\delta'|^2}$

$|\pm\rangle = \frac{1}{\sqrt 2}\left[\delta'|m\rangle + (\varepsilon_\pm - \varepsilon_m')|n\rangle\right]$

Startvektor für H_2'-Problem

Fermis Goldene Regel (Zeitabh. erste Ordn.)

Übergangsrate Γ von: $|i\rangle \longrightarrow |f\rangle$, $H(t) = H_0 + H'(t)$, $|n\rangle$ von H_0

$H_0|f\rangle = \varepsilon_f|f\rangle$, $H_0|i\rangle = \varepsilon_i|i\rangle$

$$P_{i\to f} = \frac{1}{\hbar^2}\left|\int_{t_0}^{t}dt'\, e^{i(\varepsilon_f - \varepsilon_i)t'/\hbar}\langle f|H'(t)|i\rangle\right|^2$$

Bsp. $H'(t) = V\Theta(t - t_0)$

$$\Gamma_t = \sum_f P_{i\to f}(t) = \int d\varepsilon_f\,\rho(\varepsilon_f)\frac{\sin^2(\omega_{if}t/2)}{(\hbar\omega_{if}/2)^2}|\langle f|V|i\rangle|^2$$

$= \frac{2\pi}{\hbar}t\rho(\varepsilon_f)|\langle f|V|i\rangle|^2\Big|_{\varepsilon_i = \varepsilon_f}$ $\leftarrow$ normales Skalarprodukt $\int\ldots dx$

emission absorpt., $\varepsilon_i < \varepsilon_f$

$\rho(\varepsilon_f) = \frac{1}{4}\left[\delta(\varepsilon_f - \varepsilon_i + \hbar\omega) + \delta(\varepsilon_f - \varepsilon_i - \hbar\omega)\right]$

$\Gamma = \frac{dP_{i\to f}}{dt} \overset{\text{hier}}{=} \frac{2\pi}{\hbar}|\langle f|V|i\rangle|^2\rho(\varepsilon_f)$ (Fermis Goldene Regel)

Topf (verschoben)

$k = \frac{n\pi}{d}$, $E = \frac{\hbar^2\pi^2 n^2}{2md^2}$

$Y_n(y) = \sqrt{\frac{2}{d}}\sin\left(\frac{n\pi}{d}y\right)\Theta(d-y)\Theta(y)$

Heisenberg

$H = \frac{P^2}{2m}$

$\partial_t H_H(t) = 0 \Rightarrow H_H(t) = H$

$[x_H(t), H] = i\hbar\frac{P}{m} \overset{HB}{\Rightarrow} i\hbar\frac{dx}{dt} = \frac{P}{m}$

$x_H(t) = \frac{P}{m}t + x_H(0)$

Fermi

$\omega_{if} = (\varepsilon_f - \varepsilon_i)/\hbar$

$H'(t) = V\Theta(t - t_0)$

$$P_{i\to f} = \left(\frac{\sin(\omega_{if}t/2)}{\hbar\omega_{if}/2}\right)^2|\langle f|V|i\rangle|^2$$

Bsp. Freies Teilchen

$\rho(E) = \frac{mL^3}{2\pi\hbar^3}\sqrt{2mE}$

$\Gamma = \frac{2\pi}{\hbar}|\langle f|V|i\rangle|^2\rho(\varepsilon_f)$

$H' = Ve^{\eta t}$, $\eta \to 0^+$

$$P_{i\to f}(t) = \frac{e^{2\eta t}}{(\hbar\omega_{if})^2 - (\hbar\eta)^2}|\langle f|V|i\rangle|^2$$

$H' = V\cos\omega t$

$\Gamma = \frac{2\pi}{\hbar}\frac{1}{4}|\langle f|V|i\rangle|^2\left[\delta(\varepsilon_f - \varepsilon_i - \hbar\omega) + \delta(\varepsilon_f - \varepsilon_i + \hbar\omega)\right]$

Streulänge Topf

$k^2 = \frac{2mE}{\hbar^2}$, $\tilde k^2 = k^2 + \frac{2m|V_0|}{\hbar^2}$

$u(r) \propto \begin{cases} \sin(kr + \delta_0) \\ \sin(\tilde k r) \end{cases}$ $k\cot(kr_0 + \delta_0) = \tilde k\cot(\tilde k r_0)$

$\eta = kr_0$, $\xi = \tilde k r_0 = \sqrt{\eta^2 + x_0^2}$, geb. Zustand falls $= 0$

$\Rightarrow V_0$ gefunden aus $\xi^2 = \pi^2(n + \frac{1}{2})^2$

$V_0 = -\frac{\hbar^2}{2mr_0}\left[(\pi/2)^2(2n+1)^2\right](1+\varepsilon)$

$\eta\cot\delta_0 = \frac{\pi^2(2n+1)^2}{8}\varepsilon + O(\varepsilon^2)$

$a = \frac{8r_0}{\pi^2(2n+1)^2}\frac{1}{\varepsilon}$