Shot noise

Marnik Metting van Rijn

November 30, 2019

Abstract

A microscopic consideration of an electrical current reveals that the charge is displaced in quantized portions of the size of the elementary charge. Using the fact that the current is quantized and therefore creating noise in the signal, the elementary charge can be determined. In this experiment a vacuum tube was connected to a resonant circuit in order to generate noise at a specific frequency. The generated signal was then rectified to obtain the mean current through the resistance of the resonant circuit. Using the Schottky formula the elementary charge was determined to be $1.73 \cdot 10^{-19}$ C with an error of $\pm 7\%$, where the literature value lies within 1σ standard deviation.

Contents

1 Introduction

In the early years of the 20th century, Walter Schottky, a German physicist, studied the noise of a vacuum tube (see Figure 1), which he then distinguished into two major types of noise:

- thermal noise and
- shot noise.

Thermal noise occurs in conductors at finite temperature [1] due to the thermal fluctuations of charged particles – they are not further discussed in this report. In contrast to thermal noise, shot noise can only be observed in a conductor with an electrical current flow. A current consists of seperate particles carrying the charge in quantized portions, which are usually of the size of the elementary charge $|e|$ [1]. Provided that the electrons evaporate independently, fluctuations are expected in the current [2]. A condition for shot noise to have a measurable impact is that the mean free path of a charged particle is of comparable order as the dimension of the conductor considered [1]. The gain of analysing shot noise in contemporary research is that the correlated motion of electrons, which arises from the Pauli exclusion principle and electron-electron interactions [1], can be studied by measuring macroscopic quantities. Historically this experiment is interesting since it offered a possibility to determine the elementary charge e with high precision using devices which were available 95 years ago [3].

2 Theory

Considering a voltage applied onto a device with finite resistance, the averaged current $\langle I \rangle$ can be defined as

$$\langle I \rangle = \lim_{T \to \infty} \frac{1}{T} \int_0^T dt I(t), \tag{1}$$

where $I(t)$ is the current at time t.
Instantaneous fluctuations $\Delta I(t)$ from $\langle I \rangle$ are called noise and are described by

$$\Delta I(t) = I(t) - \langle I \rangle. \tag{2}$$

Averaging the square of $\Delta I(t)$ gives the quantity

$$\langle \Delta I^2 \rangle = \lim_{T \to \infty} \frac{1}{T} \int_0^T dt \Delta I^2(t) \tag{3}$$

which is called the mean square fluctuation amplitude.
In 1918 Walter Schottky found a formula [1] which relates the elementary charge $|e|$ to the mean square fluctuation amplitude $\langle \Delta I^2 \rangle$, the averaged current $\langle I \rangle$ and the bandwidth Δf of the device used to measure the noise.

$$\langle \Delta I^2 \rangle = 2|e|\langle I \rangle \Delta f \tag{4}$$

Equation (4) bears a possibility to determine the elementary charge $|e|$ by measuring shot noise. A derivation of Equation (4) is provided in [1].

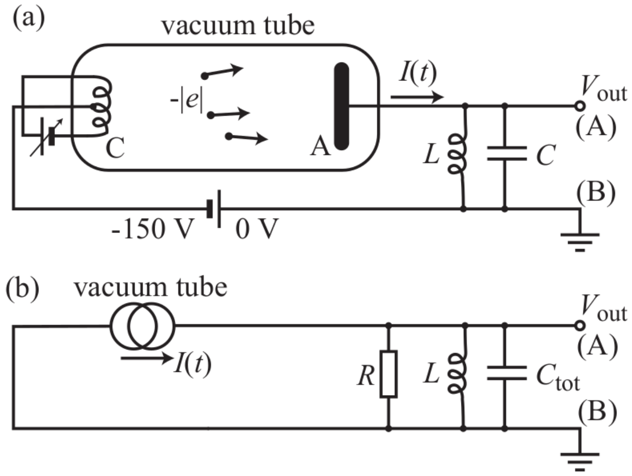

Figure 1: In (a) the schematic of the noise generator which is used to generate and filter shot noise at a specific frequency. In (b) the resistance of the LC-circuit is drawn as an ohmic resistance, whereat this only holds for signals at resonance frequency.

3 Experimental set-up

3.1 Overview

The goal of this experiment is to measure the mean square fluctuation amplitude $\langle \Delta I^2 \rangle$, the averaged current $\langle I \rangle$ and the bandwidth of the device and use Equation (4) to determine the elementary charge $|e|$. By applying a constant voltage over a diode with a heating current (see Figure 1 (a)) a current is induced through the diode. Using the fact that the electrons are emitted independently, the current through the diode has shot noise. This signal will then be led through a LC-circuit which shortens all the signals to ground which are not close to the resonance frequency of the LC-circuit. The remaining signal which ideally has the same frequency as the resonance frequency of the LC-circuit is then amplified and rectified. Finally the current of the signal is measured and Equation (4) can be used to determine the elementary charge $|e|$.

3.2 Connectors

All the devices used in this experiment are connected via BNC-connectors[1]. The connectors do not cause a measurable amplitude decrease of the signal even if more than 5 long (blue) cables are connected in series. This can be checked by measuring the amplitude of a signal using an oscilloscope when only one short (yellow) cable is connected and comparing this to the signal when 5 long (blue) cables are connected in series. It is important to assure that each cable does not have an influence on the signal when the cable is bent or moved. The outer conductor of a BNC connector is shielded and can be used as ground potential. Using BNC T pieces, the signal can be split and used for parallel measurements.

3.3 High Frequency Oscillator

To determine the resonance frequency of the LC-circuit, a high-frequency oscillator is used. For the calibration described in Section 4.5, a signal is needed as a reference for the signal generated by the LC-circuit. The reference signal is generated by the high frequency oscillator. In this experiment, the **Wavetek 5MHz Function Generator, Model FG-5000** is used and its specifications are provided in Appendix D.

[1]BNC is the abbreviation for Bayonet Neill-Concelman.

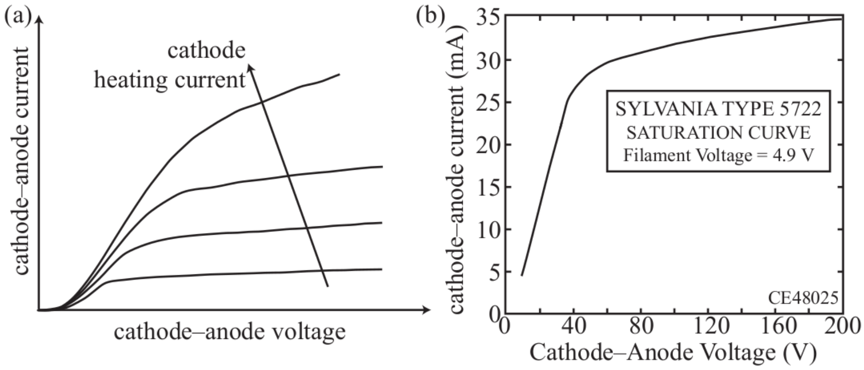

Figure 2: The cathode heating current is a dc-current and is shown in Figure 1 (a) at the cathode (C). In this experiment, the voltage applied on the diode is 150V which is in the saturation region of the diode, i.e., the cathode-anode current has negligible dependence on the cathode-anode voltage. Illustration from [1].

3.4 Noise Generator

Figure 1 shows two simplified schematics of the circuit used to generate the noise and to filter one specific frequency. A constant voltage of 150 V is applied on the diode. As shown in Figure 1 (a), the cathode is heated by a current in order to induce thermionic emission of electrons. The diode is connected parallel to a LC-circuit which turns into an ohmic resistance at the resonance frequency. All other frequency which are significantly not close to the resonance frequency are shorted to ground via the inductor L. Using a knob, the current $I(t)$ shown in Figure 1 (b) generated in the noise generator can be adjusted. As Figure 2 shows, the adjustment is done by varying the heating current which results in different cathode-anode current. The reading of the ammeter of the noise generator has an intrinsic resolution of 1 mA and a measurement range of 0-30 mA which results in a dynamic range[2] of 34 dB.

3.5 Voltage Amplifier

Provided that the signal exiting the noise generator is at resonance frequency, the signal is weak due to the high impedance of the circuit. Therefore the signal has to be amplified using an ac-coupled voltage amplifier which ensures that no dc-current is amplified [1]. A knob on the amplifier is used to adjust the amplification of the signal. Signals which are amplified to less than 3 V are outside the saturation region. In Appendix E Table 11 shows that the amplification factor A has no dependence on the amplitude of the incoming signal. The knob of the amplifier is not tight and therefore requires attention that the knob is not accidentally touched or the amplifier moved. The amplification factor A of the amplifier can be determined by measuring the amplitude of the incoming signal and comparing to the amplified amplitude of the signal.

3.6 Rectifier

In order to measure the current, a rectifier is used. After rectification, the signal is led through an internal resistance over which the voltage difference is measured. A RC low-pass filter is used for

[2] The dynamic range DR is defined as

$$DR = \frac{\delta f}{\Delta f}, \tag{5}$$

where δf is the intrinsic resolution, Δf the measurement range of the considered measurement device and DR given in dB (Decibel).

measuring the voltage since averaging is needed for oscillating signals. Similar to the voltage amplifier, the display on the ammeter on the rectifier has an upper bound for the possible measurement range. For the rectifier used in this experiment, the ammeter can not measure the current of signals with higher RMS voltages than 300 mV. By applying an oscillating signal from the high frequency oscillator on the rectifier, the rectified signal can be compared to the input signal using the oscilloscope. The reading of the ammeter has an intrinsic resolution of 1 μA and a measurement range of 0-50 μA which results in a dynamic range of 34 dB.

3.7 Oscilloscope

Throughout the whole experiment, the oscilloscope **Tektronix TBS 1072B-EDU** is used to check the quality of the waveform. If the amplifier goes into saturation for example, this can be seen clearly by checking the output signal of the amplifier. Due to the built-in impedance of 10 MΩ in the oscilloscope, it is possible to check the quality of the waveform in parallel using a T piece without having a significant influence on the signal.

4 Experimental Procedure and Measurements

4.1 Theory and Formula

Equation (4) is the fundamental formula used in this experiment since it relates the measurable quantity $\langle \Delta I^2 \rangle$ (the mean square fluctuation amplitude), $\langle I \rangle$ (the averaged current) and Δf (the bandwidth) of the device used to measure the noise. In order to measure $\langle \Delta I^2 \rangle$ a rectifier with a built-in ammeter is used. The current I_z measured on the ammeter can be described as

$$I_z = G_I \langle U_{amp}^2(t) \rangle, \tag{6}$$

where $\langle U_{amp}^2(t) \rangle$ is a squared quantity due to the rectification of the amplified signal and G_I a constant of the rectifier. The proportionality between $\langle U_{amp}^2(t) \rangle$ and I_z is justified since the internal resistance of the rectifier over which the voltage drop $\langle U_{amp}^2(t) \rangle$ is measured is an ohmic resistance. By the use of the fact that the current $I(t)$ through the diode is quantized of the order of the elementary charge $|e|$, a relation between the output voltage $\langle U_{amp}^2(t) \rangle$ and the elementary charge can be derived.

$$\langle U_{amp}^2(t) \rangle = A^2 \frac{|e||\langle I \rangle|R}{2C_{tot}} \tag{7}$$

The factor A is the amplification factor of the voltage amplifier, whereas R is the resistance and C_{tot} the total capacitance of the LC-circuit. A detailed derivation of Equation (7) can be found in the Appendix A. Inserting Equation (6) into Equation (7) yields a relation between the measured current I_z and the elementary charge.

$$I_z = \frac{G_I A^2 R |e||\langle I \rangle|}{2C_{tot}} \tag{8}$$

The bandwidth $\Delta f = 1/4RC_{tot}$ is introduced and inserted into Equation (8) which yields the final equation

$$I_z = G_I A^2 R^2 2|e||\langle I \rangle|\Delta f. \tag{9}$$

Equation (9) allows a determination of the elementary charge $|e|$ by measuring the quantities I_z and $|\langle I \rangle|$ and the current $|\langle I \rangle|$ dependant resistance R. All other constants such as G_I, A^2 and C_{tot} are fixed by the experimental set-up.

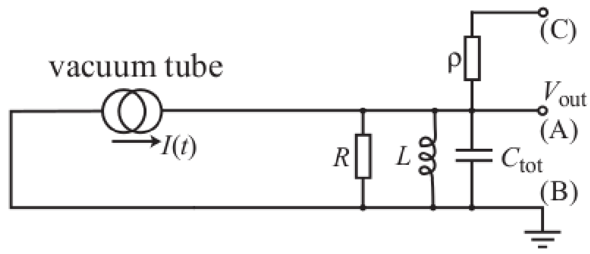

Figure 3: Schematic of the LC-circuit at resonance frequency. The second connection (C) which was not shown in Figure 1 is not used for the final measurement but for initial measurements such as the measurement of the resonance frequency, the total capacitance and the resistance. Image from [1].

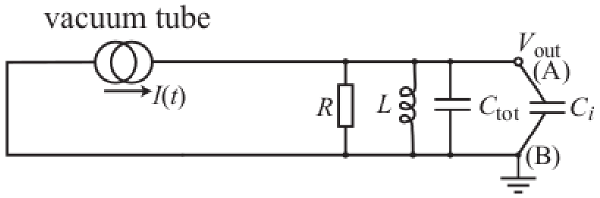

Figure 4: Schematic of the LC-circuit in order to measure the total capacitance C_{tot}. The second capacitance C_i is connected into (A). Image from [1].

4.2 Resonance Frequency

In Section 4.5 a signal is needed which is similar in frequency and amplitude to the signal excited by the noise generator. Therefore the resonance frequency of the LC-circuit has to be determined. The resonance frequency ω_1 is determined by applying a known signal from the high frequency oscillator into the LC-circuit (C) and measuring the output (A) with the oscilloscope. By varying the frequency of the incoming signal, the resonance frequency ω_1 is found by maximising the amplitude of the output signal. Alternatively the input signal into the noise LC-circuit can be measured at terminal (C) with the oscilloscope and compared to the output signal at terminal (A) using T pieces. The frequency can then be varied until the phase shift of the two signals vanish, which essentially means that the resonance frequency is found. At the resonance frequency the resistance R becomes ohmic and therefore does not induce a phase shift. The measured frequency using the oscilloscope deviates from the reading of the high frequency oscillator, which is described in Appendix D.

4.3 Total Capacitance

In order to determine the total capacitance C_{tot} another capacitor C_i is connected parallel to C_{tot}. The change in the resonance frequency of the LC-circuit, which the second capacitor C_i induces, can be measured using the same method as described in Section 4.2. The resonance frequency of an LC-circuit is given by

$$\omega = \frac{1}{\sqrt{LC}}. \tag{10}$$

The two measurements define a system of equations which can be solved for C_{tot} and L. The total capacitance C_{tot} is then given by

$$C_{tot} = \frac{\omega_2^2 C_i}{\omega_1^2 - \omega_2^2}, \tag{11}$$

where ω_2 is the resonance frequency of the LC-circuit with C_i connected parallel to C_{tot} and ω_1 the resonance frequency found in Section 4.2.

4.4 Resistance

At resonance frequency of the LC-circuit, the noise generator acts as an ohmic resistance. The resistance R of the noise generator can be measured by exciting the LC-circuit at (C) with a signal with given voltage U_1 and resonance frequency ω_1 and extracting the voltage U_2 at (A). Since the signal is very weak and difficult to measure, the output signal at (A) is amplified using the voltage amplifier. To determine the resistance R the signal is measured after amplification but then divided by amplification factor A. Further the amplifier has to be connected to the LC-circuit while measuring the resistance due to the fact that the amplifier has an input impedance of 1 MΩ and is ac-coupled via a 10 nF capacitor. Without the amplifier the resonance frequency drops by 14 kHZ. Figure 3 shows a further resistance ρ in the schematic. This has to be considered in the calculations of the resistance, since the input signal is initially led through ρ i.e. the high frequency oscillator is connected at the terminal (C). The resistance R has a dependence on the diode current $I(t)$ and must therefore be measured for several diode currents. It follows that

$$R = \frac{U_2}{U_1 - U_2}\rho, \tag{12}$$

since the current $I(t)$ through the resistance R and ρ is the same [1].

4.5 Calibration of the Rectifier

Equation (6) relates the current I_z measured by the built-in ammeter of the rectifier to the voltage $\langle U_{amp}^2(t)\rangle$ by using a proportionality factor G_I. By applying a known signal with RMS value $\langle U_{amp}^2(t)\rangle$ the rectifier and noting the corresponding current I_z, the factor G_I can be determined. Taking into account, that it is a linear proportionality G_I can be found as the gradient of the first order polynomial fit of I_z to $\langle U_{amp}^2(t)\rangle$.

4.6 Elementary Charge

Knowing all the constants of the experimental set-up, such as G_I, A^2 and C_{tot}, Equation (9) can be used to determine the elementary charge $|e|$. For a given diode current $\langle I\rangle$ varying from 10 mA to 30 mA, the output signal of the noise generator taken at terminal (A) is amplified by a known factor of A and led into the rectifier, which measures the corresponding current I_z. This measurement is done without a reference signal applied into the terminal (C), which would distort the noise of the generator.

5 Data

Throughout the whole experiment the uncertainties were predicted by linear error propagation theory. As proposed in [4] the python package *uncertainties* [6] was used to calculate the errors. For the histograms the number of bins k was chosen using the Sturges formula [5] given by

$$k = \lceil log_2 n \rceil + 1, \tag{13}$$

where $\lceil . \rceil$ is the ceil function and n the number of measurements. Rounding was done using the rule recommended by PDG [7]. Corollaries of the central limit theorem (CLT) were used in the data analysis which was further based on the frequentist approach. The pull in the histograms are defined by

$$pull = \frac{x_i - x_0}{\sigma_i}, \tag{14}$$

where x_i and σ_i are the measured values and the corresponding uncertainty and x_0 the expectation value.

Measurement	Value	Error
$\frac{\omega_1}{2\pi}$ [kHz]	290	$\pm$ 3
$\frac{\omega_2}{2\pi}$ [kHz]	127	$\pm$ 3
C_i [pF]	5365	$\pm$ 7
C_{tot} [nF]	1.27	$\pm$ 0.08
L [mH]	9.34	$\pm$ 0.55

Table 1: Measurements described in Section 4.2 and Section 4.3.

The mean value of the calculated elementary charge was calculated to be $(1.73 \pm 0.13)\ 10^{-19}$ As. The literature value is given by $1.602176634{\cdot}10^{-19}$ As [8].

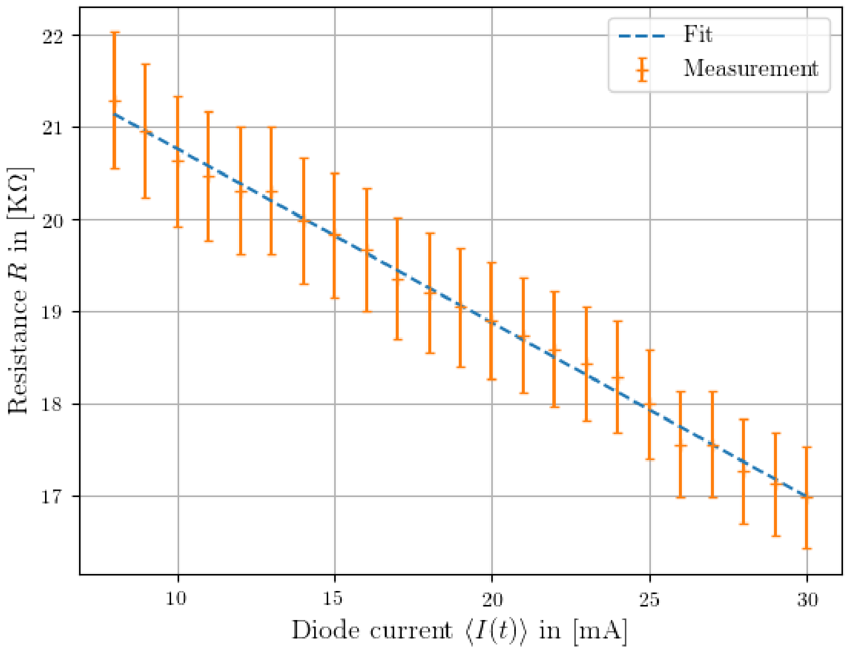

Figure 5: Scatter plot for the determination of the resistance R with the aquired data in Table 5.

Measurement	Value	Error
ρ [kΩ]	33	$\pm$ 0.5
U_1 [mV]	6.76	$\pm$ 0.02
$A \cdot 10^3$	0.79	$\pm$ 0.14

Table 2: The measurement described in Section 4.4 was conducted with these fixed values. The measured values for U_2 and $\langle I\rangle$ are shown in the Appendix C in Table 5.

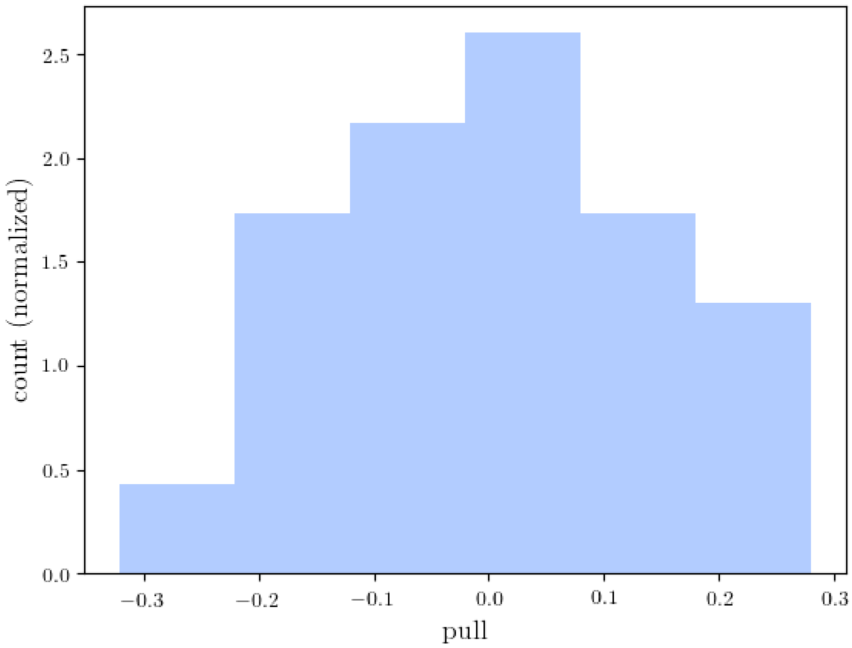

Figure 6: The theory proposes a linear dependency of the resistance R on the diode current $\langle I \rangle$. Using the method of least squares a first order polynomial curve was fitted in the data (See Figure 5). This histogram shows the deviation of the fitted curve from the measured values. The numbers of bins k was calculated using Equation (13) and was set to 6. The histogram was normalized to the area of 1.

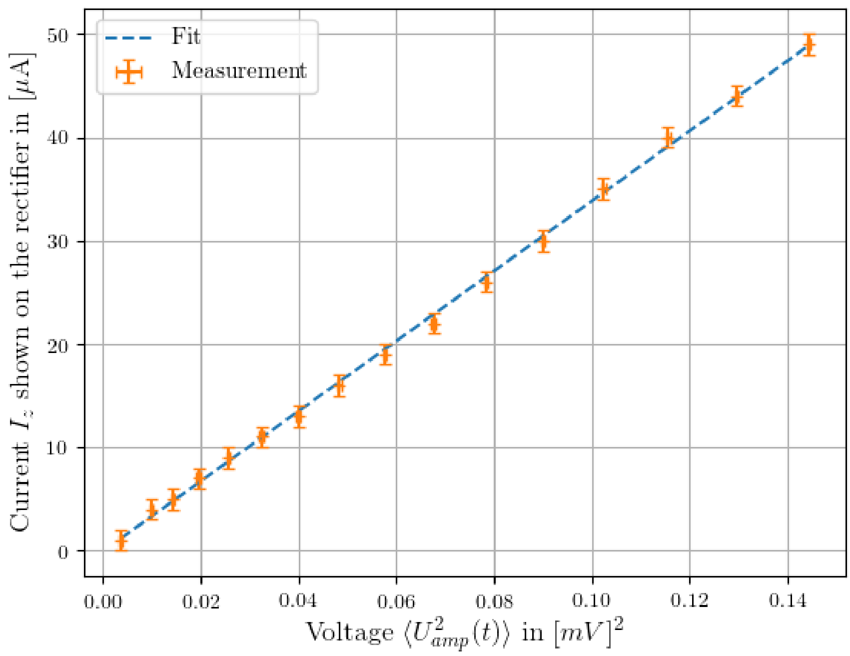

Figure 7: Scatter plot of the measured values shown in Table 6, whereupon the Voltage $\langle U_{amp}^2(t) \rangle$ was squared before plotting. The gradient of the linear regression is the quantity G_I. The error bars of the Voltage $\langle U_{amp}^2(t) \rangle$ are not clearly visible as they are smaller than the marker.

m $[\frac{A}{V^2}]$	340
q $[\mu A]$	-0.07
(1-p) $\cdot 10^{-12}$	7

Table 3: Calculated values for the calibration described in Section 4.5, where m is the slope of the fit and q the y-axis intercept of the fit shown in Figure 7. In the last row, the p corresponds to the p-value in the frequentist approach.

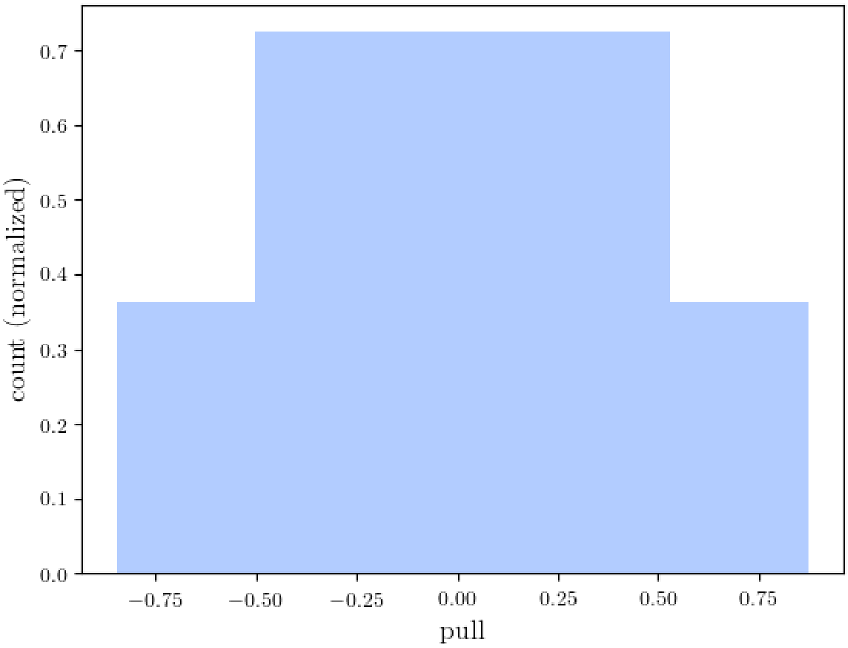

Figure 8: The theory proposes a linear dependency of the current I_z on the voltage $\langle U^2_{amp}(t)\rangle$. Using the method of least squares a first order polynomial curve was fitted in the data (See Figure 7). This histogram shows the deviation of the fitted curve from the measured values. The numbers of bins k was calculated using Equation (13) and was set to 5. The histogram was normalized to the area of 1.

$\langle I \rangle$ in [mA]	I_z in [μA]	e in 10^{-19}[As]	Error of e in 10^{-19}[As]	Deviation in %
Error: ± 1	Error: ± 1	-	-	-
10	5	1.8	± 0.4	12
11	5	1.7	±0.4	6
12	6	1.8	±0.4	12
13	6	1.7	±0.3	6
14	7	1.9	±0.3	19
15	7	1.8	±0.3	12
16	7	1.7	±0.3	6
17	8	1.8	±0.3	12
18	8	1.7	±0.3	6
19	8	1.7	±0.2	6
20	9	1.8	±0.2	12
21	9	1.7	±0.2	6
22	9	1.7	±0.2	6
23	10	1.8	±0.2	12
24	10	1.7	±0.2	6
25	10	1.7	±0.2	6
26	10	1.6	±0.2	-0.1
27	11	1.7	±0.2	6
28	11	1.7	±0.2	6
29	11	1.7	±0.2	6
30	11	1.6	±0.2	-0.1

Table 4: Aquired data from the measurement of the elementary charge described in Section 4.6.

6 Discussion

6.1 Resonance Frequency and Total Capacitance

The **Wavetek 5MHz Function Generator, Model FG-5000** has an internal display, which shows the frequency of the generated signal at the precision of ±1 kHz. Comparing to the frequency initially measured by the oscilloscope, a deviation of ±3 kHz was found. Using the oscilloscope to compare the input signal and output signal, the resonance frequency could be determined precisely with an error of ±0.1 kHz. Considering the initial error of ±3 kHz the propagated error for the resonance frequency was calculated to be ±3 kHz. The error of the capacitance C_i was written on the capacitor. Finally using Equation (4.3) C_{tot} was calculated to be 1.27 nF with an uncertainty of 6 %. The precision of the measurement of C_{tot} could be improved by decreasing the error of the resonance frequency ω. This however requires the knowledge whether the oscilloscope or the function generator is to be trusted for frequency measurement. The errors were propagated using Equation (23) in Section B in which the error calculation of the quantity C_{tot} is derived in detail.

6.2 Resistance

In order to increase the precision of the measurement of the resistance R, 23 measurements were taken. Figure 5 shows the scatter plot and the linear regression, which lies within the error bound for every measurement. Since the display of the noise generator has a resolution of 1 mA, the error of $\langle I \rangle$ was chosen to be ±1 mA. Table 2 shows the values which were left unchanged throughout the measurement. The value of A has an uncertainty due to the fact that it was calculated using a reference signal measured by the oscilloscope. The oscilloscope has an error which gives reason for the uncertainty of A. Figure 6 shows the deviation of the linear regression from the calculated values for

R. Since the deviation lies within a standard deviation of 1σ, this measurement is in accordance with the theory of a linear dependency of the resistance R on the diode current $\langle I \rangle$.

6.3 Calibration of the Rectifier

Equation (6) proposes a proportionality of the current I_z to the voltage $\langle U^2_{amp}(t)\rangle$ by the factor of G_I. In order to increase the precision 23 measurements were taken. The error bars of the $\langle U^2_{amp}(t)\rangle$ are not visible in Figure 7 as they are smaller than the marker. Nevertheless, Figure 8 shows that the deviation of the linear regression from the measured values lie within 1σ uncertainty. This is in accordance with the theory which proposes a linear dependency.

6.4 Elementary Charge

Table 4 shows the calculated values of the elementary charge with the propagated error. In contrast to [1] the values of $\langle I \rangle$ were varied between 10 mA and 30 mA and not as proposed between 5 mA to 25 mA. This correction was done due to the fact, that the rectifier did not show a linear proportionality between $\langle I \rangle$ and I_z, when diode currents of less than 10 mA were applied. This might arise from internal noise of the amplifier, which is measurable as soon as the diode current of the noise generator is set to 0 mA. In that case, the rectifier measures a current through the resistance R of 1 μA even though the theory proposes no current through the set-up. Since the model described in [1] predicts a linear model with no y-axis interception, the offset of 1 μA has to be deducted. The values calculated without the correction of 1 μA are shown in Table 7. Considering the values shown in Table 7 there is an inaccuracy in the final result which can be minimised by deducting the 1 μA as shown in Table 4. The mean value for the elementary charge e lies within 1σ standard deviation. The calculations of the elementary charge were done using a constant amplification factor A. The knob of the amplifier however, is not tight enough so that any minimal change can be excluded. Therefore the inaccuracy may arise from a changed amplification factor A.

7 Conclusion

For each part of the experiment, several measurement were done in order to increase the precision of the result. The final value for the elementary charge e was calculated to be $(1.73 \pm 0.13)\ 10^{-19}$ As. This value lies within one standard deviation. An improvement in accuracy of the set-up can be achieved by considering the noise of the amplifier and the resulting offset in the measured quantity I_z. Further, measuring devices with a higher dynamic range would increase the precision since it is better to have less measurements with a smaller uncertainty than several measurements with large uncertainty. Considering a possible change in amplification factor, an improvement of the measurement can be done be tightening the knob of the amplifier. Overall the Shot Noise experiment provides an interesting insight into the effects of the quantisation of charge and offers an opportunity to get more familiar with measuring devices.

A Derivation of Equation 6

Using Kirchhoff's circuit laws and the defining formulas of the capacitor the circuit shown in Figure 1 is mathematically described by

$$\ddot{U}_A + \frac{1}{\tau}\dot{U}_A + \omega_0^2 U_A = \frac{1}{C_{tot}}\dot{I}(t), \tag{15}$$

where

$$\omega_0 = \frac{1}{\sqrt{LC_{tot}}}$$

and

$$\tau = RC_{tot}.$$

Further f_{res} is defined by

$$\omega_0 = 2\pi f_{res}.$$

Considering the term $\frac{1}{\tau}$, Equation (15) describes a driven damped harmonic oscilattor, where its resonance frequency is f_{res} and the damping factor $\frac{1}{\tau}$. Equation (15) can be solved in either the time or frequency domain. In this derivation, the solution will be described in the time domain. As proposed in [1], the solution can be found by considering the auxiliary problem of the circuit response $G(t,t')$ to an excitation pulse at time t'. Following formula describes the response of the resonant circuit

$$\partial_t^2 G(t,t'I + \frac{1}{\tau}\partial_t G(t,t') + \omega_0^2 G(t,t') = \alpha\delta(t-t'), \tag{16}$$

where α corresponds to the strength of the excitation pulse. By causality $G(t,t')$ must be zero before the excitation pulse acts onto the resonant circuit. The Green's function is then given by

$$G(t,t') = \frac{\alpha}{\omega} e^{-(t-t')/2\tau} sin[\omega(t-t')]\theta(t-t'). \tag{17}$$

This Equation is derived in [1] and describes a damped oscillation, where its frequency is given by

$$\omega = \sqrt{\frac{1}{LC} - \frac{1}{(2RC)^2}} = \sqrt{\omega_0^2 - (\frac{1}{2\tau})^2}.$$

Since the Green function has been found, the solution for the initial differential equation is given by integration. The integral can then be partially integrated which leads to

$$U_A(t) = \frac{1}{C_{tot}}\sqrt{1 + \frac{1}{4\omega^2\tau^2}} \int_{-\infty}^{t} dt' e^{-(t-t')/2\tau} cos[\omega(t-t') + \beta] I(t'), \tag{18}$$

where β is defined using the equation

$$tan\beta = \frac{1}{\omega\tau}.$$

In the circuit of the experiment $\omega\tau \gg 1$, which means that damping is low. The action of the rectifier is mathematically described by

$$I_z = G_I \Gamma \int_{-\infty}^{t} dt' U_{amp}^2(t') e^{-\Gamma(t-t')}, \tag{19}$$

where I_z is the reading of the ammeter and Γ the inverse time constant of the low-pass filter. The quantity G_I is defined above in Equation (6). Considering that Γ is very small, the above integral represents a time average of U_{amp}^2 i.e.

$$I_z = G_I \langle U_{amp}^2(t) \rangle. \tag{20}$$

Assuming that the electrons are ejected randomly, which is fulfilled in the saturation region, and making use of the quantisation of the electrical charge the following expression can be found for $U_{amp}(t)$:

$$U_{amp}(t) = \frac{-|e|A}{C_{tot}} \sqrt{1 + \frac{1}{4\omega^2\tau^2}} \sum e^{-(t-t_i)/2\tau} cos[\omega(t-t_i) + \beta] \theta(t-t_i) \tag{21}$$

Finally Campbell's theorem can be applied which leads to [1]

$$I_z = \frac{G_I A^2 R |e| |\langle I \rangle|}{2C_{tot}}, \tag{22}$$

which is in accordance with Equation (9).

B Example of an Error Propagated Quantity

All the errors in this experiment were calculated according to linear error propagation theory.

$$\sigma_f = \sqrt{\sum_{i=1}^{N} (\frac{\partial f}{\partial x_i}\bigg|_{x_i = x_i^0} \sigma_{x_i})^2}, \tag{23}$$

where f is the function, σ_{x_i} the uncertainties of the measured values x_i and σ_f the uncertainty of the function evaluated at the measured quantities. As an example, the error of Equation (11) is calculated.

$$\Delta C_{tot} = \sqrt{(\frac{\omega_2^2}{\omega_1^2 - \omega_2^2} \Delta C_i)^2 + (\frac{2\omega_2 C_i(\omega_1^2 - \omega_2^2) + \omega_2^3 C_i}{(\omega_1^2 - \omega_2^2)^2} \Delta\omega_2)^2 + (\frac{2\omega_2^2 C_i \omega_1}{(\omega_1^2 - \omega_2^2)^2} \Delta\omega_1)^2} \tag{24}$$

C Measurements

$\langle I \rangle$ in [mA]	$\langle U_{amp}(t) \rangle \cdot A$ in [V]	R in [kΩ]	Residual in [kΩ]
Error: ± 1	Error: ± 0.01	-	-
8	2.10	21.3 ± 0.7	0.2
9	2.08	21.0 ± 0.7	0
10	2.06	20.6 ± 0.7	0.2
11	2.05	20.5 ± 0.7	0.1
12	2.04	20.3 ± 0.7	0.1
13	2.04	20.3± 0.7	0.1
14	2.02	20.0 ± 0.7	0
15	2.01	19.8 ± 0.7	0
16	2.00	19.7 ± 0.7	0.1
17	1.98	19.4 ± 0.6	0
18	1.97	19.2 ± 0.6	0.1
19	1.96	19.1± 0.6	0
20	1.95	18.9± 0.6	0
21	1.94	18.7± 0.6	0
22	1.93	18.6± 0.6	0.1
23	1.92	18.4± 0.6	0.1
24	1.91	18.3± 0.6	0.2
25	1.89	18.0± 0.6	0.1
26	1.86	17.6± 0.6	0.1
27	1.86	17.6± 0.6	0
28	1.84	17.3± 0.6	0.1
29	1.83	17.1± 0.6	0.1
30	1.82	17.0± 0.6	0

Table 5: Aquired data for the measurement of the resistance R. This was done as described in Section 4.4. The residual corresponds to the deviation of the measurement to the fit in absolute value.

I_z in [μA]	RMS of $\langle U_{amp}(t) \rangle$ in [mV]
Error: ± 1	Error: ± 0.01
1	0.06
2	0.10
3	0.12
7	0.14
9	0.16
11	0.18
13	0.20
16	0.22
19	0.24
22	0.26
26	0.28
30	0.30
35	0.32
40	0.34
44	0.36
49	0.38

Table 6: Values used for the calibration of the set up as described in Section 4.5. The root mean square (RMS) value of the signal U_2 was measured using the oscilloscope.

$\langle I \rangle$ in [mA]	I_z in [μA]	e in 10^{-19}[As]	Error of e in 10^{-19}[As]	Deviation in %
Error: ± 1	Error: ± 1	-	-	-
10	6	2.2	± 0.4	37
11	6	2.0	±0.4	24
12	7	2.1	±0.4	31
13	7	2.0	±0.3	24
14	8	2.0	±0.3	24
15	8	2.0	±0.3	24
16	8	2.0	±0.3	24
17	9	2.0	±0.3	24
18	9	2.0	±0.3	24
19	9	1.9	±0.2	18
20	10	1.9	±0.2	18
21	10	2.0	±0.2	24
22	10	1.9	±0.2	18
23	11	1.8	±0.2	12
24	11	1.8	±0.2	12
25	11	1.8	±0.2	12
26	11	1.8	±0.2	12
27	12	1.9	±0.2	18
28	12	1.9	±0.2	18
29	12	1.8	±0.2	12
30	12	1.8	±0.2	12

Table 7: Aquired data from the measurement described in Section 4.6 without the correction of 1 μA in I_z.

D High Frequency Oscillator

Min. frequency	0.001 Hz
Max. frequency	1.5 Mhz

Table 8: General information of the frequency oscillator.

Min. output voltage	1.5 mV
Max. output voltage	14.2 V
Frequency deviation at 286 kHz	+3 kHz
Frequency deviation at 100 kHz	+3 kHz
Frequency deviation at 400 kHz	0 kHz
Frequency deviation at 300 kHz	+3 kHz

Table 9: Measurement of the output voltages were done at 286 kHz. The frequency of the generated signal (57.3 mV, 40dB attenutation) was measured using the oscilloscope **Tektronix TBS 1072B-EDU** and compared to the displayed frequency on the high frequency oscillator.

Frequency deviation at 400 kHz	+3 kHz
Frequency deviation at 286 kHz	+1 kHz

Table 10: Properties of the **Wavetek 5MHz Function Generator, Model FG-5000**. All the measurements were done right after turning the function generator on and at 400 kHz, 6.12 mV and at 60dB attenuation.

E Voltage Amplifier

Voltage [mV]	Frequency [kHz]	Amplification factor [-]
1.43	286	34.5
6.03	286	31.8
7.22	286	31.7
9.08	286	32
15.1	286	31.8
4.48 (10μA)	286	32.1
5.47 (15μA)	286	31.8
6.32 (20μA)	286	31.8
7.02 (25μA)	286	31.8
7.66 (30μA)	286	31.9
8.13 (35μA)	286	31.9
8.7 (40μA)	286	31.8
9.2 (45μA)	286	31.9

Table 11: A signal generated by the high frequency oscillator was connected to the voltage amplifier and then into the rectifier to achieve the same condition as in the final measurement. For a given signal measured with the oscilloscope, the amplification factor was determined of the voltage amplifier, while the knob was not touched throughout the measurements. It shows no voltage dependence at the voltage range, at which the amplifier is operating in the experiment. The current correspond to the measured currents shown on the rectifier. Even though the knob was not touched it can not be excluded that the knob was changed since it is not tight.

References

[1] Physikpraktikum 3 + 4, *Shot noise*,[Online], 6 Oct 2019, https://vp.phys.ethz.ch/Experimente/pdf/Shot_Noise.pdf

[2] W. Schottky, Ann. Phys. (Leipzig) 57, 541 (1918).

[3] A.W. Hull and N.H. Williams, *Determination of Elementary Charge E from Measurements of Shot-Effect.*, Phys.Rev. 25, 147 (1925)

[4] M. Donega and S. Gvasaliya, *Exercise 1 - Problems.ipynb* https://moodle-app2.let.ethz.ch/pluginfile.php/813110/mod_folder/content/0/Exercise%201%20-%20Problems.ipynb?forcedownload= 1 (9. 11.2019)

[5] Sturges, H. A. (1926). *The choice of a class interval.* Journal of the American Statistical Association. 21 (153): 6566.

[6] Uncertainties: a Python package for calculations with uncertainties, Eric O. LEBIGOT, http://pythonhosted.org/uncertainties/ (9.11.2019)

[7] Particle Data Group: pdg.lbl.gov

[8] Bureau International des Poids et Mesures - SI BASE UNITS, https://www.bipm.org/utils/en/pdf/si-promotion/SI-Base-Units.pdf (22.11.2019)