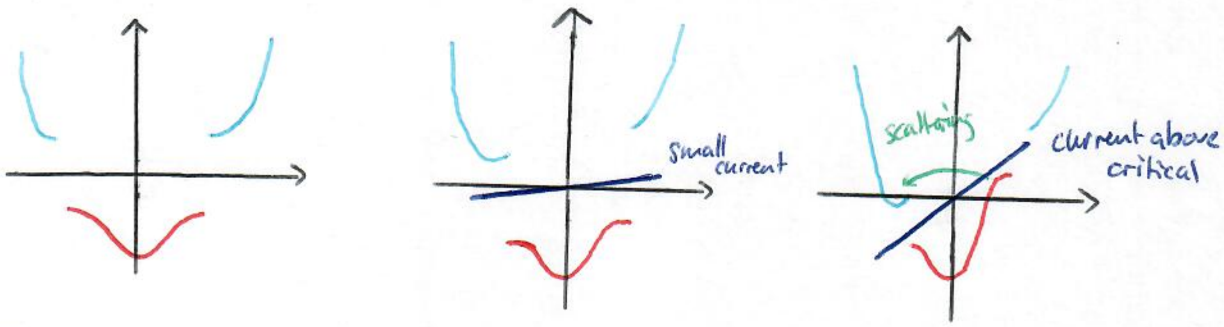
# Superconductivity

**Definition:**

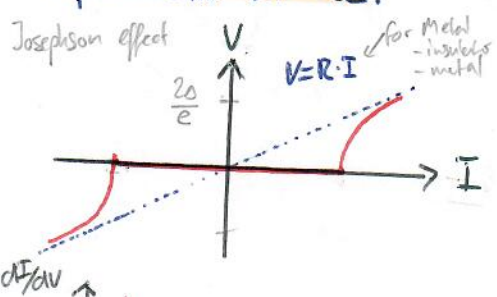
Superconductivity is a set of physical properties observed in certain materials where electrical resistance vanishes and magnetic flux fields are expelled from the material. It is further characterized by a gap in exitation and a critical current and field.

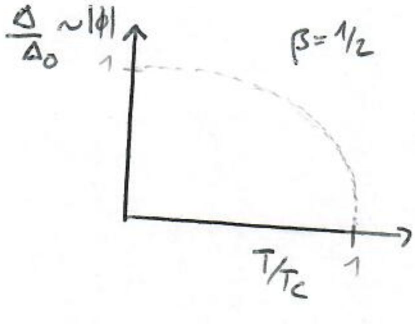
**Derivation:**

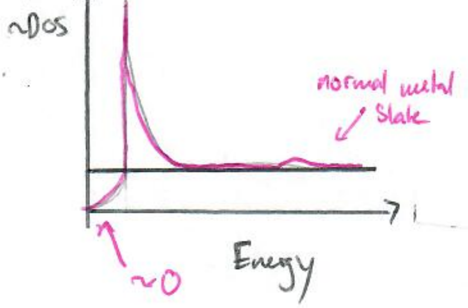
Superconductivity arises from the interaction between electrons resulting from virtual exchange of phonons which is attractive when the energy difference between the electrons states involved is less than the phonon energy, $\hbar\omega_D$. It is favourable to form a superconducting phase when this attractive interaction dominates the repulsive screened Coulomb interaction.

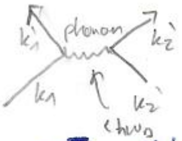



**Experimental evidence:**





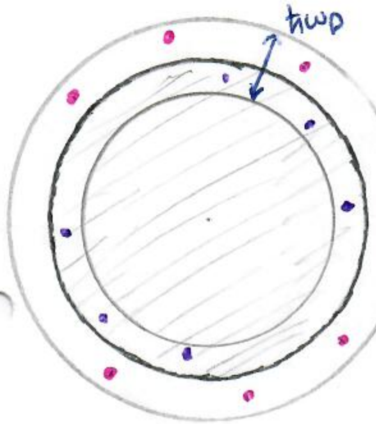

## Phonon-mediated electron attraction

$e^-$ attracts ions, $e^{-'}$ sees charge density $\Rightarrow$ effective attraction

$e^-$ travel at $v_F$, create phonons, another $e^{-'}$ absorbs phonon

s. 582 ascroft

$$|\varepsilon(\vec{k}_1) - \varepsilon(\vec{k}_1')| = |\varepsilon(\vec{k}_2) - \varepsilon(\vec{k}_2')| < \hbar\omega_D \qquad \text{(Debye energy)}$$




New ground state must have holes / excitations, within a band $\hbar\omega_D$. $k_B T \ll \hbar\omega_D$

$$V_{k_1,k_2;k_1',k_2'} = \begin{cases} -V & \text{if}^{\text{all}} \text{ in ring} \\ 0 & \text{else} \end{cases}$$

## Cooper pairs

The ones benefiting most from V are arranged in Cooper pairs with net zero momentum.

$$\vec{q} = \vec{k}_1 + \vec{k}_2 \quad \xRightarrow[\text{argument}]{\text{geometrical}} \quad \vec{q} \to 0$$

$\to$ momentum cons. , spinless, bosons

$\Delta \sim$ complex amplitude of condensate

## Parametrization using pair-states

$$|\lambda_{\vec{k}}\rangle = u_{\vec{k}} |\phi_{\vec{k}}\rangle + v_{\vec{k}} |\psi_{\vec{k}}\rangle \quad , \quad u_k^2 + v_k^2 = 1$$

$u_k^2 = n_k$ ~ prob. full

slot $\pm k$ full

slot $\pm k$ empty

$$\Rightarrow \quad |\Psi\rangle = \prod_{\vec{k}} |\lambda_{\vec{k}}\rangle$$

## The Energy of the superconducting ground state

$$|\Psi\rangle = \prod_{\vec{k}} |\lambda_{\vec{k}}\rangle$$

Ground state degenerate

different phase in $\Delta$

summation inside two layer

$$\hat{K} = \sum_{\vec{k}}{}' \epsilon_{\vec{k}} \hat{n}_{\vec{k}} + E_0 \qquad \text{(Kinetic energy)}$$

$\epsilon = \varepsilon - \varepsilon_F$

$$E_{kin} = \langle\Psi|\hat{K}|\Psi\rangle = 2 \sum_{\vec{k}}{}' \epsilon_{\vec{k}} u_{\vec{k}}^2 + E_0$$

$$\hat{V} = -V \sum_{\vec{k},\vec{k}'}{}' |\phi_{\vec{k}}\rangle\langle\varphi_{\vec{k}'}|$$

$$E_{e-e} = -V \sum_{\vec{k},\vec{k}'}{}' u_{\vec{k}} v_{\vec{k}} u_{\vec{k}'} v_{\vec{k}'}$$

$$\frac{\partial E}{\partial u_k} \overset{v_k = \sqrt{1-u_k^2}}{\doteq} 0 \Rightarrow \quad 4\epsilon_{\vec{k}} u_k - 2V\left(v_k - \frac{u_k^2}{v_k}\right) \underbrace{\sum_{\vec{k}'}{}' u_{\vec{k}'} v_{\vec{k}'}}_{} = 0$$

$$\Delta = V \sum_{k'}{}' u_{k'} v_{\vec{k}'} \overset{\text{For the Fermi sea}}{=} 0$$

$$\Rightarrow \quad E_k^2 = \epsilon_k^2 + \Delta^2$$

$$\lambda = V\nu(\varepsilon_F)$$

calculation

$$\Rightarrow \quad \Delta \sim 2\hbar\omega_D e^{-\frac{1}{V\nu(\varepsilon_F)}} = 2\hbar\omega_D e^{-\frac{1}{\lambda}}$$

magnitude

see. CDW band gap

relative phase $v_k, u_k$ are not arbitrary! Minimization leads to relative phase equal for all $\vec{k}$ and equal to phase of $\Delta$.

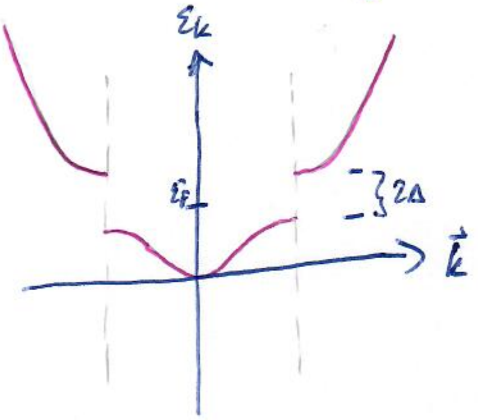
OP = $\Delta$, $\Delta_{\text{Fermi sea}} = 0$, phase of $\Delta$ cannot be measured unlike in CDW.

→ no conjugate field. Gauge freedom: $u_k$ and $v_k$ phase unrelated as $v_k = 0$

→ in superconducting state $(u_k - v_k)$ phase fixed by $\Delta$ $SO(2) \sim U(1)$

# Excitations superconductivity

$$\tilde{E}_{\vec{k}} = \pm\sqrt{\epsilon_{\vec{k}}^2 + \Delta^2}$$




dispersion of non-interacting fermionic quasiparticles

$\hookrightarrow N_{qp} = N_{e^-}$

excitation $\Leftrightarrow$ breaking apart Cooper pair

lowest excited state has an energy $2\Delta$

visible in specific heat or thermal conductivity

## Phase transition

Assume heating from $T < T_c \rightarrow T = T_c$.

if $k_B T \sim 2\Delta \Rightarrow$ quasiparticles proliferate, destroy cooper pairs

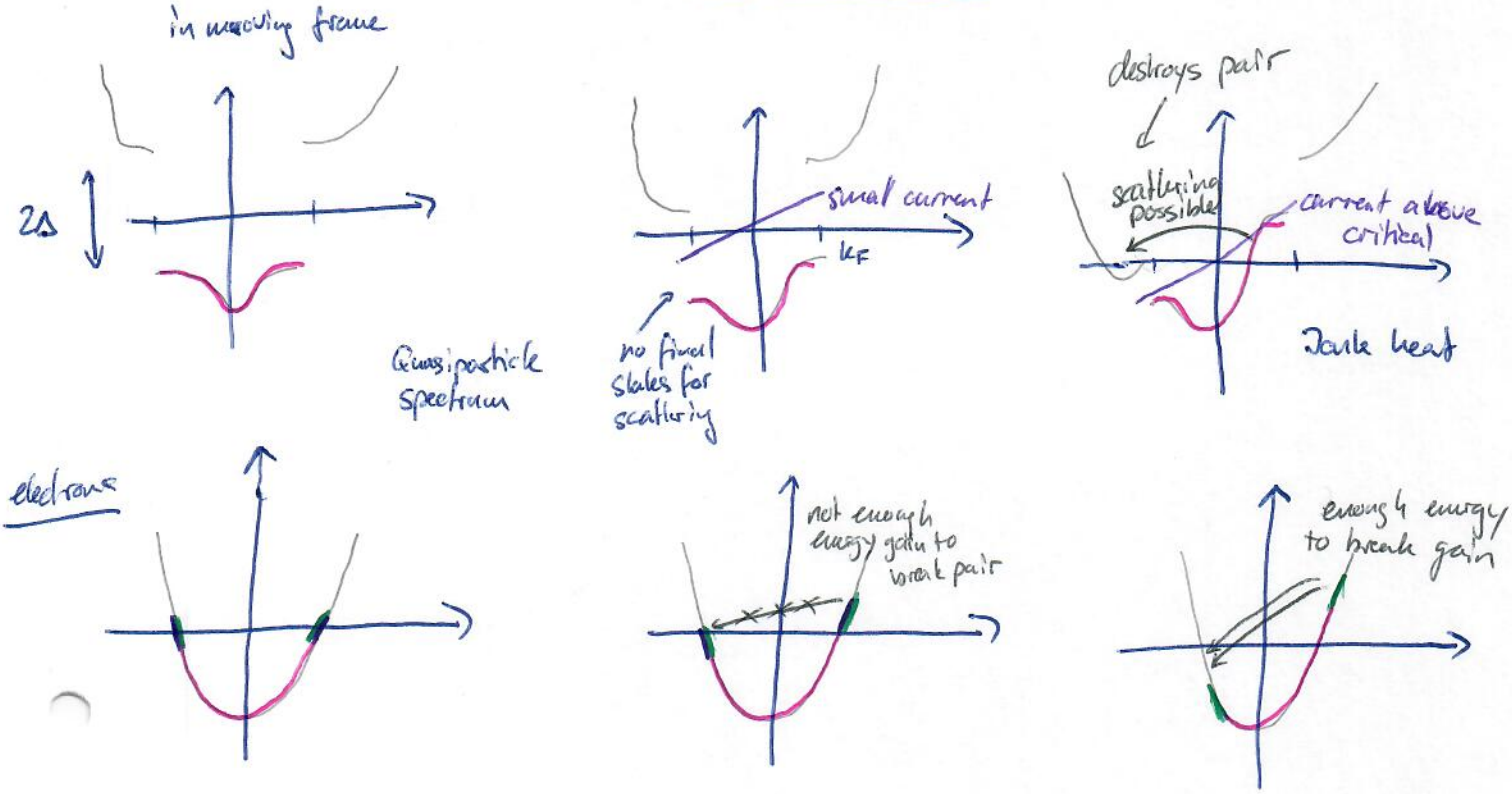
can no longer lower the ground state $\Rightarrow \Delta$ decreases $\overset{?}{h=0}$

if $\Delta = 0 \Leftrightarrow T = T_c$ (continuous phase transition)*

$$k_B T_c = 1{,}14\, \hbar\omega_D\, e^{-\frac{1}{\lambda}}$$

$$\Rightarrow \quad 2\Delta = 3{,}52\, k_B T_c$$

# Current in a superconductor




Compare: gap prevents dissipative scattering, no sliding lattice distortion as in CDW, no dissipation nor pinning

Gap is intrinsic feature of electrons, not imposed by external periodic boundary.

Zero friction!, can't however be too fast

carrier concentration

$$j_c = \frac{ne\Delta}{\hbar k_F}$$