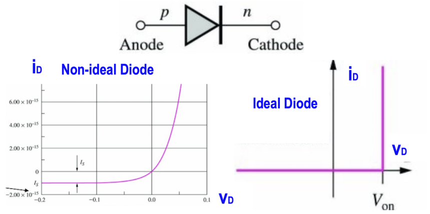
Diodes



Diode i-v Equation

$$i_D = I_S\left[\exp\left(\frac{qv_D}{nkT}\right) - 1\right] = I_S\left[\exp\left(\frac{v_D}{nV_T}\right) - 1\right]$$

$I_s =$	reverse saturation current	A
$v_D =$	voltage applied to diode	V
$n =$	non-ideality factor	dimensionless

Diode current for Reverse, Zero, and Forward Bias

Reverse bias: $i_D \approx I_s(0-1) = -I_s$
Zero bias: $i_D \approx I_s(1-1) = 0$
Forward bias: $i_D \approx I_s \exp\left(\frac{v_D}{nV_T}\right)$

Constant Voltage Drop Model for Diode

The ideal diode is either on or off:

Forward-biased:	$v_D = V_{on} = 0.7\text{V}$	$i_D > 0$	**on**
Reverse-biased:	$v_D < V_{on}$	$i_D = 0$	**off**

Diode Circuit Analysis Procedure

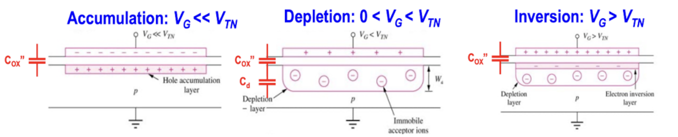
1. Assume operation region of all the diodes (either on or off).
2. Analyze circuit using constant voltage drop model.
3. Check results to check consistency with assumptions:

   $v_D < 0.7\text{V}$ for all off diodes and $i_D > 0$ for on diodes
4. May need to iterate this process.
5. Obtain diode operating point = Q-point (i_D, v_d)

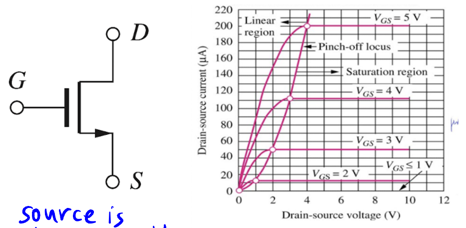
Field-Effect Transistors (FET)

Horizontal devices. Source and Drain are symmetric. $i_G = 0$.

MOS Capacitors



NMOS Structure and Qualitative I-V Behavior



source is always at the lower potential $\rightarrow$ D, S switches, when $V_S > V_D$

NMOS Operating Regions

Op. Region	Condition	Equation
Cut-off	$V_{GS} < V_{TN}$	$i_D = 0$
Triode (linear)	$V_{GS} > V_{TN}$; $V_{GS} - V_{TN} > V_{DS}$; $(V_{DS} > 0)$	$i_D = K_n\left(V_{GS} - V_{TN} - \frac{V_{DS}}{2}\right)V_{DS}$
Saturation (pinch-off)	$V_{GS} > V_{TN}$; $V_{DS} > V_{GS} - V_{TN}$; $(V_{GS} - V_{TN} > 0)$	$i_D = \frac{1}{2}K_n(V_{GS} - V_{TN})^2(1 + \lambda V_{DS})$

$K_n = K_n' W/L = \mu_n C_{ox}'' W/L$ $\quad K_n' = \mu_n C_{ox}''$ (A/V^2)
$C_{ox}'' = \varepsilon_{ox}/T_{ox}$ $\quad \varepsilon_{ox} =$ oxide permittivity (F/cm)
$T_{ox} =$ oxide thickness (cm)

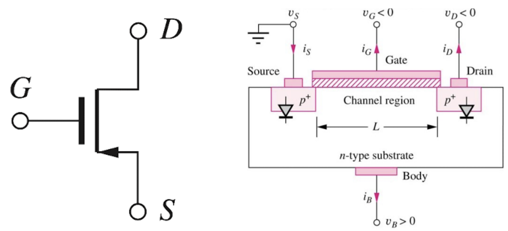
NMOS: Transconductance

Relates the change in drain current to a change in gate-source voltage:

$$g_m = \left.\frac{\partial i_D}{\partial V_{GS}}\right|_{Q-pt} \qquad [g_m] = \text{S}$$

In Saturation mode: $g_m = K_n(V_{GS} - V_{TN}) = \mu_n C_{ox}'' \frac{W}{L}(V_{GS} - V_{TN})$
With channel length modulation: $\mu_n C_{ox}'' \frac{W}{L}(V_{GS} - V_{TN})(1 + \lambda V_{DS})$

PMOS Transistors



Source is always at higher potential

PMOS Operating Regions

Op. Region	Condition	Equation
Cut-off	$V_{SG} < \lvert V_{TP}\rvert$	$i_D = 0$
Triode (linear)	$V_{SG} > \lvert V_{TP}\rvert$; $V_{SG} - \lvert V_{TP}\rvert > V_{SD}$; $(V_{SD} > 0)$	$i_D = K_p\left(V_{SG} - \lvert V_{TP}\rvert - \frac{V_{SD}}{2}\right)V_{SD}$
Saturation (pinch-off)	$V_{SG} > \lvert V_{TP}\rvert$; $V_{SD} > V_{SG} - \lvert V_{TP}\rvert$; $(V_{SG} - \lvert V_{TP}\rvert > 0)$	$i_D = \frac{1}{2}K_p(V_{SG} - \lvert V_{TP}\rvert)^2(1 + \lambda V_{SD})$

MOSFET and BJT Biasing Circuits

Transistor Bias sets the DC operating point around which the device operates (off, triode, saturation). It determines the transistor small-signal behaviors (Gain, BW, noise, ...) and large-signal behaviors (linearity, compression points, slew rate, ...).

Bias Analysis Approach

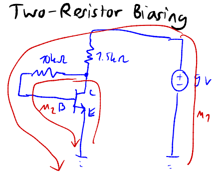
1. Assume an operating region
   (mostly saturation for MOSFETs, forward-active for BJTs
2. Use circuit analysis to find V_{GS} (biasing voltage) for MOSFETs or I_B for BJTs. (Often assume $V_{BE} = 0.7\text{V}$)

C	open circuit	L	short circuit
AC current source	open circuit	AC voltage source	short circuit

3. MOSFETs: Use V_{GS} to calculate I_D and I_D to find V_{DS}.
   $\rightarrow$ NMOS Q-Point (I_{DS}, V_{DS}, V_{GS})
   $\rightarrow$ PMOS Q-Point (I_{SD}, V_{SD}, V_{SG})
4. BJTs: Use I_B to calculate I_C and I_E and with these find V_{CE}
   $\rightarrow$ NPN Q-Point (I_C, V_{CE})
   $\rightarrow$ PNP Q-Point (I_C, V_{EC})
5. Check validity of operating region assumptions.
6. Change assumptions and analyze again if required.

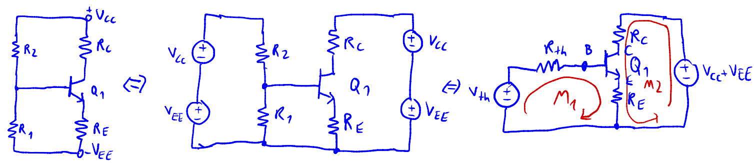
Two-Resistor and Four-Resistor Biasing for FET and BJT Examples:

Two-Resistor Biasing



M1 yields I_B $\Rightarrow$ can calculate I_E, I_C
M2 yields V_{CE}

Four-Resistor Biasing:



M1 yields $I_B \Rightarrow I_C, I_E$ determinable M2 yields V_{CE} $\Rightarrow$ check assumptions with $V_{BC} = V_{BE} - V_{CE}$

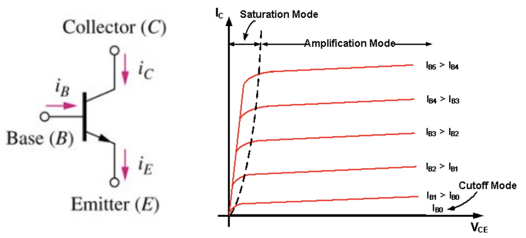
Bipolar Junction Transistors (BJT)

Vertical devices. Collector and Emitter are not symmetric. $i_B \neq 0$.

NPN: Operating Regions

Base-Emitter Junction	Base-Collector Junction: Reverse biased $v_{BC} < 0$	Base-Collector Junction: Forward biased $v_{BC} > 0$
Forward biased $v_{BE} > 0$	Forward-active Region (good amplifier)	Saturation Region (closed switch)
Reverse biased $v_{BE} < 0$	Cutoff Region (open switch)	Reverse-active Region (poor amplifier)

NPN: Forward and Reverse Characteristics



NPN Transistor: Forward Characteristics

$$i_C = i_F = I_S\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right]$$

$$i_B = \frac{i_C}{\beta_F} = \frac{I_S}{\beta_F}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right]$$

$$i_E = i_C + i_B = \frac{I_S}{\alpha_F}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right] = (\beta_F + 1)I_B$$

In forward-active region: $\beta_F = \frac{i_C}{i_B}$ and $\alpha_F = \frac{i_C}{i_E} = \frac{\beta_F}{\beta_F + 1}$

NPN Transistor: Reverse Characteristics

$$i_E = -i_R = -I_S\left[\exp\left(\frac{v_{BC}}{V_T}\right) - 1\right]$$

$$i_B = \frac{i_R}{\beta_R} = \frac{I_S}{\beta_R}\left[\exp\left(\frac{v_{BC}}{V_T}\right) - 1\right], \qquad \beta_R = \frac{\alpha_R}{1 - \alpha_R}$$

$$i_C = i_B - i_E = -\frac{I_S}{\alpha_R}\left[\exp\left(\frac{v_{BC}}{V_T}\right) - 1\right], \qquad \alpha_R = \frac{\beta_R}{\beta_R + 1}$$

$i_C =$ Collector current $\quad i_B =$ Base current
$i_E =$ Emitter current $\quad V_T =$ 25mV at room temp
$I_S =$ BJT saturation current, $10^{-18}\text{A} \leq I_S \leq 10^{-9}\text{A}$
$\beta_F =$ forward common-emitter current gain, $20 \leq \beta_F \leq 500$ (as high as possibe)
$\alpha_F =$ forward common-base current gain, $0.95 \leq \alpha_F \leq 1$
$\beta_R =$ reverse common-emitter current gain, $0 \leq \beta_R \leq 0.95$ (as low as possible)
$\alpha_R =$ reverse common-base current gain, $0 \leq \alpha_R \leq 0.95$

NPN Transistor: Complete Transport Model for any bias

$$i_C = I_S\left[\exp\left(\frac{v_{BE}}{v_T}\right) - \exp\left(\frac{v_{BC}}{V_T}\right)\right] - \frac{I_S}{\beta_R}\left[\exp\left(\frac{v_{BC}}{V_T}\right) - 1\right]$$

$$i_E = I_S\left[\exp\left(\frac{v_{BE}}{v_T}\right) - \exp\left(\frac{v_{BC}}{V_T}\right)\right] + \frac{I_S}{\beta_F}\left[\exp\left(\frac{v_{BE}}{V_T}\right) - 1\right]$$

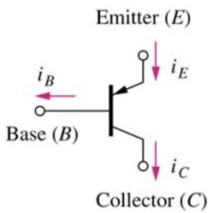
$$i_B = \frac{I_S}{\beta_F}\left[\exp\left(\frac{v_{BE}}{v_T}\right) - 1\right] + \frac{I_S}{\beta_R}\left[\exp\left(\frac{v_{BC}}{V_T}\right) - 1\right]$$

First term in both emitter and collector current expression gives current transported completely across base region.

NPN: Simplified Operating Regions Models

Re.	Cutoff	Forward active	Reverse active
Co.	$v_{BE} \leq -4V_T$ $V_{BC} \leq -4V_T$	$v_{BE} = 0.7\text{V} \geq 4V_T$ $V_{BC} \leq -4V_T = -0.1\text{V}$	
i_C	$\frac{I_S}{\beta_R}$	$I_S \exp\left[\frac{v_{BE}}{V_T}\right] = \beta_F I_B$	$-\frac{I_S}{\alpha_R}\exp\left[\frac{v_{BC}}{V_T}\right]$
i_E	$-\frac{I_S}{\beta_F}$	$\frac{I_S}{\alpha_F}\exp\left[\frac{v_{BE}}{V_T}\right] = (\beta_F + 1)I_B$	$-I_S\exp\left[\frac{v_{BC}}{V_T}\right]$
i_B	$-\frac{I_S}{\beta_F} - \frac{I_S}{\beta_R}$	$\frac{I_S}{\beta_F}\exp\left[\frac{v_{BE}}{V_T}\right]$	$\frac{I_S}{\beta_R}\exp\left[\frac{v_{BC}}{V_T}\right]$

PNP Transistor



$V_{BC} = V_{BE} - V_{CE}$

For the PNP Transistor we can take the same I-V-equations and the same conditions for the operating regions as with the NPN, we just have to replace v_{BE} with v_{EB} and v_{BC} with v_{CB} everywhere.

BJT Transconductance

$$g_m = \left.\frac{\partial i_C}{\partial V_{BE}}\right|_{Q-pt} = \left.\frac{\partial}{\partial V_{BE}}\left(I_S \exp\left(\frac{v_{BE}}{V_T}\right)\right)\right|_{Q-pt} = \frac{I_C}{V_T}$$

BJT Early Effect

In a practical BJT, the output characteristics have a positive slope in forward-active region; collector current is not independent of v_{CE}. **Early effect:** When the output characteristics are extrapolated back to point of zero i_C, the curves intersect at a common point $v_{CE} = -V_A$ (between 15 - 150V). Simplified equations (Early effect):

$$i_C = I_S \exp\left(\frac{v_{BE}}{V_T}\right)\left(1 + \frac{v_{CE}}{V_A}\right), \qquad r_o = \frac{\partial v_{CE}}{\partial i_C} = \frac{V_A}{I_C}$$

$$\beta_F = \beta_{FO}\left(1 + \frac{v_{CE}}{V_A}\right), \qquad i_B = \frac{I_S}{\beta_{FO}}\exp\left(\frac{v_{BE}}{V_T}\right)$$

$V_T = \frac{kT}{q}$

Transistor Amplifiers

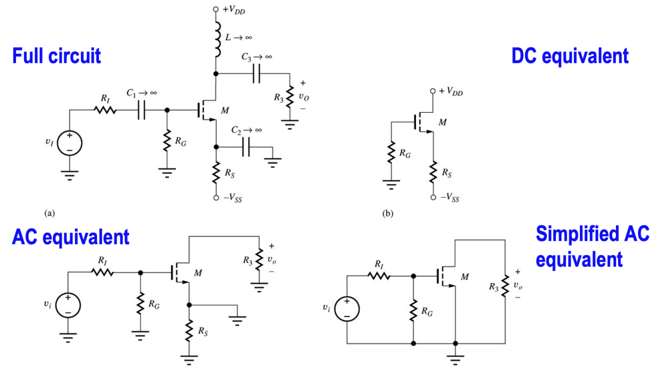
Amplifiers usually use electronic devices operating in the active region: Forward-active for BJT and saturation for MOSFET. The FET triode region. (BJT saturation region should NOT be used as amplifiers.)

DC and AC Analysis

Coupling Capacitor: DC blocking capacitor; their reactance at signal frequency is designed to be negligible i.e. $|Z| = |1/(\omega C)| << R_I + R_{in}$

Bypass Capacitor: AC blocking capacitor; provides low impedance path for AC current sources to the transistor terminals.

Since impedance of a capacitor increases with decreasing frequency, coupling and bypass capacitors reduce amplifier gain at low frequency.



Step 1: DC analysis

C	open circuit	L	short circuit
AC current source	open circuit	AC voltage source	short circuit

1. Find DC equivalent circuit with these simplifications.
2. Find Q-Point fron DC equivalent circuit by using large-signal transistor model.

Step 2: AC analysis

C	short circuit	L	open circuit
DC current source	open circuit	DC voltage source	GND

1. Find AC equivalent circuit with these simplifications.
2. Replace transistor by its small-signal model.
3. Use small-signal AC equivalent to analyze AC characteristics of amplifier.
4. Combine end results of DC and AC analysis to yield total voltages and currents in the network.

Small Signal Modeling

Diode Small Signal Model

Diode Small Signal Conductance: (Slope of I-V Char. at Q-Pt)

$$g_d = \left.\frac{\partial i_D}{\partial v_D}\right|_{\text{Q-Pt}} = \frac{I_S}{V_T}\exp\left[\frac{V_D}{V_T}\right] = \frac{I_D + I_S}{V_T} \approx \frac{I_D}{V_T} \approx 40 I_D, \quad (I_D >> I_S)$$

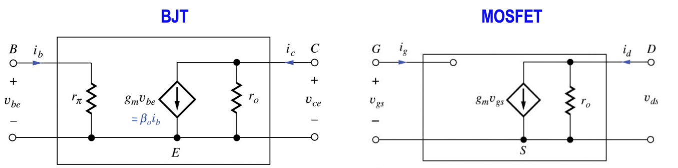
Diode Small Signal Resistance: $r_d = \frac{1}{g_d}$

Small-signal (linear) operation of the diode is valid for $v_d \leq 5\text{mV}$

Hybrid-Pi Model

The hybrid-pi model assumes **active region of operation**.

We assume $\beta_0 = \beta_f$ in this lecture for simplicity.

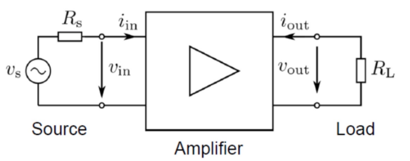


	BJT	MOSFET
Transconductance g_m	$\frac{I_C}{V_T} \approx 40 I_C$	$\frac{2I_D}{V_{GS}-V_{TN}} = \sqrt{2K_n I_D}$
Input Res. r_π	$\frac{\beta_0 V_T}{I_C} = \frac{\beta_0}{g_m}$	$r_\pi \to \infty$
Output Res. r_o	$\frac{V_A+V_{CE}}{I_C} \approx \frac{V_A}{I_C}$	$\frac{1+\lambda V_{DS}}{\lambda I_D} \approx \frac{1}{\lambda I_D}$
Intrinsic Voltage Gain μ_f	$g_m r_o = \frac{V_A+V_{CE}}{V_T} \approx \frac{V_A}{V_T}$	~~(∗)~~ $g_m r_o = \frac{1}{\lambda}\sqrt{\frac{2K_n}{I_D}}$
Small signal op. valid for (⋆)	$v_{be} \leq 5\text{mV}$ or $\frac{i_c}{I_c} = \frac{v_{be}}{V_T} \leq 0.2$	$\frac{i_d}{I_D} = \frac{g_m}{I_D} v_{gs} \leq 0.4$

(∗) ~~We ignore channel-length modulation here~~.

(⋆) We find the signal range by taylorexpanding i_d with respect to v_d (Diodes)/with respect to v_{gs} (FETs) resp. i_c with respect to v_{be} (BJTs) and stating that quadratic term << linear term

Single Transistor Amplifiers (Terminology)



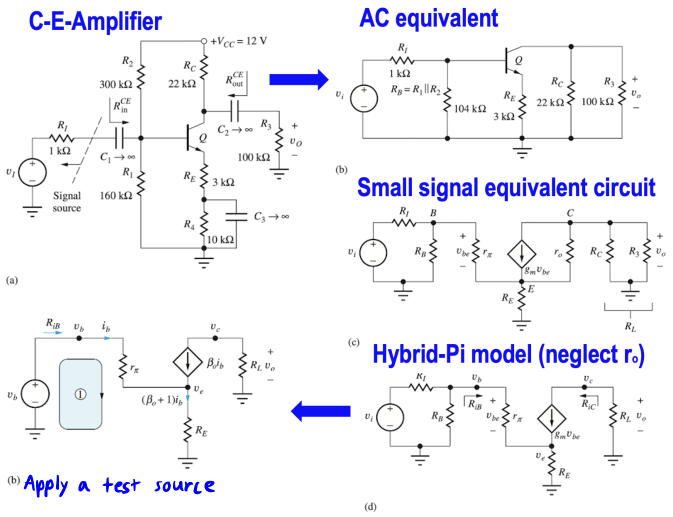
Impedance $\quad Z_{in} = \left.\frac{v_{in}}{i_{in}}\right|_{v_{out}=0} \qquad Z_{out} = \left.\frac{v_{out}}{i_{out}}\right|_{v_s=0}$

Gain $\quad A_V = \left.\frac{v_{out}}{v_s}\right|_{i_{out}=0} \qquad A_i = \left.\frac{i_{out}}{i_s}\right|_{v_{out}=0}$

Transconductance $\quad G_m = \left.\frac{i_{out}}{v_{in}}\right|_{v_{out}=0}$

Transresistance $\quad R_m = \left.\frac{v_{out}}{i_{in}}\right|_{i_{out}=0}$

Common-Emitter Amplifier (Inverting Amplifier)



Emitter is common to input and output signals → C-E-Amplifier

Terminal voltage gain from base to collector terminal:

$$A_{vt}^{CE} = \frac{v_c}{v_b} = -\frac{\beta_0 R_L}{r_\pi + (\beta_0+1)R_E} \approx -\frac{g_m R_L}{1+g_m R_E}, \qquad R_L = R_C \parallel R_3$$

Input resistance looking into the base terminal:

$$R_{iB} = \frac{v_b}{i_b} = r_\pi + (\beta_0+1)R_E \approx r_\pi(1+g_m R_E), \qquad (\beta_0 >> 0)$$

Overall input resistance looking into the amplifier at C_1:

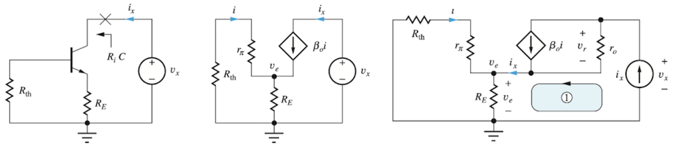
$$R_{in}^{CE} = R_B \parallel R_{iB}$$

Signal source voltage gain (overall voltage gain):

$$A_v^{CE} = A_{vt}^{CE}\left(\frac{v_b}{v_i}\right) \approx \left(\frac{-g_m R_L}{1+g_m R_E}\right)\left[\frac{R_B \parallel R_{iB}}{R_I + (R_B \parallel R_{iB})}\right] \approx \begin{cases} A_{vt}^{CE} & (*) \\ -g_m R_L & (\star) \end{cases}$$

Approx. (∗) holds if $R_I << R_B \parallel R_{iB}$ and (⋆) if additionally $R_E = 0$

Resistance at the collector:



$R_{th} = R_B \parallel R_I, \quad v_x$ is a test source, $\quad v_e = (\beta_0+1)iR_E, \quad i_x = \beta_0 i,$

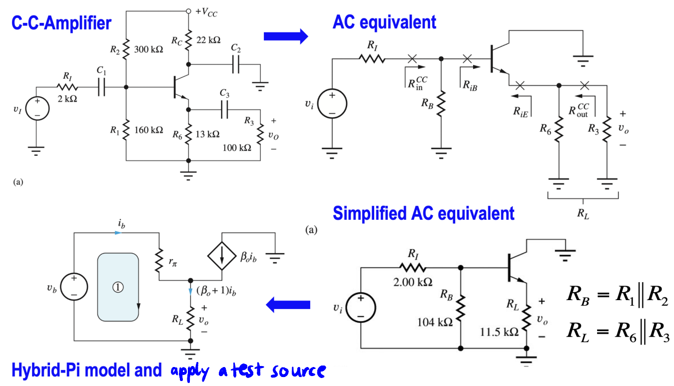
$$R_{iC} = \left[r_O + \frac{r_O\beta_0 R_E}{R_{th}+r_\pi+R_E}\right] + (R_{th}+r_\pi)\parallel R_E \approx \left[r_O + \frac{r_O\beta_0 R_E}{R_{th}+r_\pi+R_E}\right]$$

$$\approx r_O\left[1+g_m(R_E \parallel r_\pi)\right] = \begin{cases} \approx r_O g_m (R_E \parallel r_\pi), \text{ if } r_\pi >> R_E \\ \leq \beta_0 r_O \approx \mu_f r_\pi, \text{ if } R_E >> r_\pi \end{cases}$$

Overall output resistance:

$$R_{\text{out}}^{CE} = R_C \parallel R_{iC} = R_C \parallel \left[r_O + \frac{r_O\beta_0 R_E}{R_{th}+r_\pi+R_E}\right]$$

Common-Collector Amplifier (Follower Amplifier)



Collector is common to input and output signals → C-C-Amplifier

Terminal voltage gain from base to emitter terminal:

$$A_{vt}^{CC} = \frac{v_e}{v_b} = -\frac{(\beta_0+1)R_L}{r_\pi + (\beta_0+1)R_L} \approx \frac{g_m R_L}{1+g_m R_L}, \qquad R_L = R_6 \parallel R_3$$

Input resistance looking into the base terminal:

$$R_{iB} = \frac{v_b}{i_b} = r_\pi + (\beta_0+1)R_L \approx r_\pi(1+g_m R_L), \qquad (\beta_0 >> 0)$$

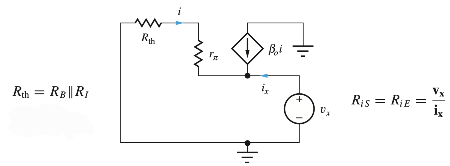
Overall input resistance looking into the amplifier at C_1:

$$R_{in}^{CC} = R_B \parallel R_{iB}$$

Signal source voltage gain (overall voltage gain):

$$A_v^{CC} = A_{vt}^{CC}\left(\frac{v_b}{v_i}\right) = A_{vt}^{CC}\left[\frac{R_B \parallel R_{iB}}{R_I + (R_B \parallel R_{iB})}\right]$$

Resistance at the emitter:



v_x is a test source, $\quad i_x = -i - \beta_0 i = \frac{v_x}{r_\pi R_{th}}(-1-\beta_0)$

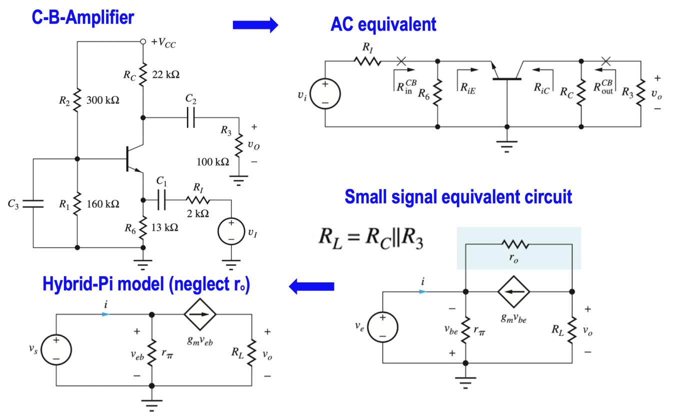
$$R_{iE} = \frac{r_\pi + R_{th}}{\beta_0+1} \approx \frac{1}{g_m} + \frac{R_{th}}{\beta_0}, \qquad (\beta_0 >> 1)$$

Overall output resistance:

$$R_{\text{out}}^{CC} = R_6 \parallel R_{iE} \approx \frac{1}{g_m}$$

Power: $P_D = V\cdot I = \overbrace{V_{CE} I_C + V_{BE} I_B}^{BJT}$, $\overbrace{V_{DS} I_D + V_{GS} I_G}^{FET}$ ← Use DC circuits for this

Common-Base Amplifier (Non-Inverting Amplifier)



Base is common to both input and output signals → C-B-Amplifier

Terminal voltage gain from emitter to collector terminal:

$$A_{vt}^{CB} = \frac{v_o}{v_e} = g_m R_L$$

Input resistance looking into the emitter terminal:

$$i = \frac{v_e}{r_\pi} + g_m v_e, \qquad R_{iE} = \frac{v_e}{v_i} = \frac{r_\pi}{\beta_0 + 1} \approx \frac{1}{g_m}, \qquad (\beta_0 >> 1)$$

Overall input resistance looking into the amplifier at C_1:

$$R_{in}^{CB} = R_6 \parallel R_{iE}$$

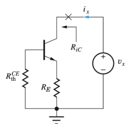
Signal source voltage gain (overall voltage gain):

$$A_v^{CB} = \frac{g_m R_L}{1 + g_m R_{th}} \left(\frac{R_6}{R_I + R_6} \right) \approx \begin{cases} \frac{g_m R_L}{1+g_m R_I} & (*) \\ \frac{R_L}{R_{th}} & (\star) \\ g_m R_L & (\bullet) \end{cases} \qquad (R_{th} = R_6 \parallel R_I)$$

Approx. $(*)$ holds if $R_6 >> R_I$ and $(\star)$ if additionally $g_m R_{th} >> 1$.

Approx. $(\bullet)$ holds if $g_m R_I << 1$.

Resistance at the collector:



$$R_{iC} = r_o + \frac{r_o \beta_0 R_E}{R_{th}^{CE} + r_{pi} + R_E} = r_o + \frac{r_o \beta_0 R_{th}}{r_\pi + R_{th}} \approx r_o + r_o g_m (R_{th} \parallel r_\pi)$$

Overall output resistance:

$$R_{out}^{CB} = R_C \parallel R_{iC} = R_C \parallel r_o[1 + g_m(R_6 \parallel R_I \parallel r_\pi)]$$

MOSFET Single Transistor Amplifiers

We can take the same formulas as for the BJT single transistor amplifiers and just let:

$R_E \to R_S$, $R_B \to R_G$, $R_C \to R_D$

$v_c \to v_d$, $v_b \to v_g$, $v_e \to v_s$

$r_\pi \to \infty$, $\beta_0 \to \infty$, $i_G = 0$

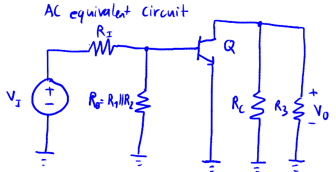
The resulting equations will look like this:

	C.-Source (C-S) Amp	C.-Drain (C-D) Amp	C.-Gate (C-G) Amp
BJT eq	like C-E	like C-C	like C-B
Terminal Volt. Gain	$A_{vt}^{CS} = \frac{v_o}{v_1} = -\frac{g_m R_L}{1+g_m R_S}$	$A_{vt}^{CD} = \frac{v_o}{v_1} = +\frac{g_m R_L}{1+g_m R_L} \approx 1$	$A_{vt}^{CG} = \frac{v_o}{v_1} = g_m R_L$
Signal Src Volt. Gain	$A_v^{CS} = \frac{v_o}{v_i} = A_{vt}^{CS} \frac{R_G}{R_I+R_G}$	$A_v^{CD} = \frac{v_o}{v_i} = A_{vt}^{CD} \frac{R_G}{R_I+R_G}$	$A_v^{CG} = \frac{v_o}{v_i} = \frac{g_m R_L}{1+g_m R_{th}} \left[\frac{R_6}{R_I+R_6}\right]$
Inp. Term. Res.	$R_{iG} \to \infty$	$R_{iG} \to \infty$	$R_{iS} = \frac{1}{g_m}$
Outp. Term. Res.	$R_{iD} = r_o(1 + g_m R_S)$	$R_{iS} = \frac{1}{g_m}$	$R_{iD} = r_o(1 + g_m R_{th})$
Amp Inp. Res.	$R_{in}^{CS} = R_G$		
Amp Outp. Res.	$R_{out}^{CS} = R_D \parallel R_{iD}$		
Inp. Signal Range	$0.2(V_{GS}-V_{TN}) \cdot (1 + g_m R_S)$	$0.2(V_{GS}-V_{TN}) \cdot (1 + g_m R_L)$	$0.2(V_{GS}-V_{TN}) \cdot [1+g_m(R_I \parallel R_6)]$
Term. Curr. Gain	$A_i \to \infty$	$A_i \to \infty$	$+1$

$R_{th} = (R_I \parallel R_6)$ in the C-G Amplifier

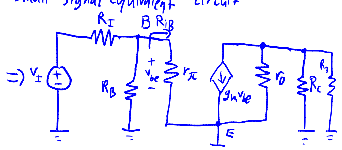
C-E / C-S Amplifier Analysis without R_E resp. R_S (simple)

AC equivalent circuit

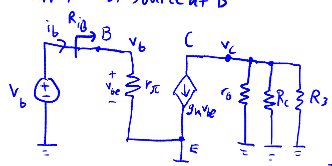


$R_L = R_C \| R_3$

Small signal equivalent circuit



Apply test source at B



$v_b = r_\pi \cdot i_b$

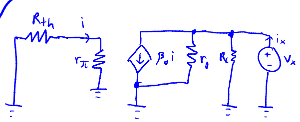
$\Rightarrow R_{iB} = \frac{v_b}{i_b} = r_\pi$

$R_{in}^{CE} = R_B \| R_{iB} = (R_1 \| R_2 \| R_{iB})$

$A_{vt}^{CE} = \frac{v_c}{v_b}$, $v_b = v_{be}$, $v_c = -g_m v_{be} \cdot (R_C \| R_3 \| r_o)$

$\Rightarrow A_{vt}^{CE} = -g_m (R_C \| R_3 \| r_o)$

$A_v^{CE} = -g_m (r_o \| R_C \| R_L) \cdot \frac{R_{in}^{CE}}{R_I + R_{in}^{CE}}$



v_x, $i_x = \beta_0 i$

$R_{th} = R_B \| R_I$

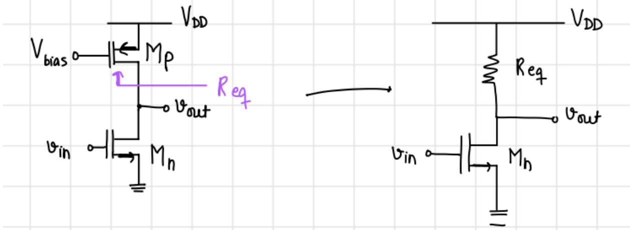
$i = 0 \Rightarrow \beta_0 i = 0$

$\Rightarrow v_x = i_x \cdot (r_o \| R_C)$

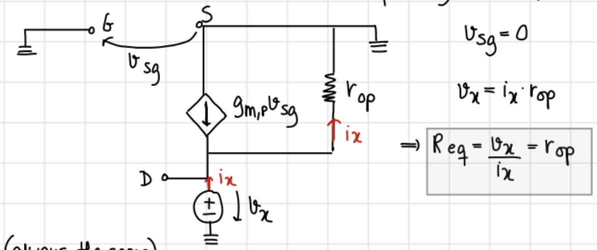
$\Rightarrow R_{out}^{CE} = r_o \| R_C$

Amplifiers with active loads

Common Source Amplifier with PMOS active load



1) Look at active load & determine R_{eq} using small signal



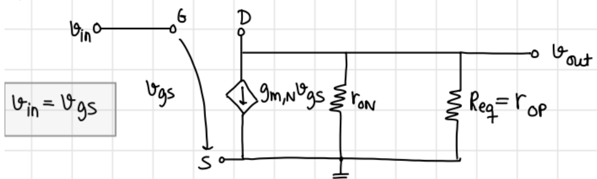
$v_{sg} = 0$

$v_x = i_x \cdot r_{op}$

$\Rightarrow \boxed{R_{eq} = \frac{v_x}{i_x} = r_{op}}$

(always the same)

2) Now look at whole circuit & replace M_n with small signal

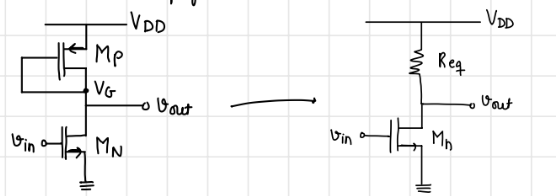


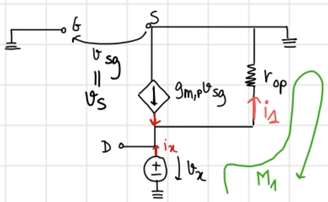
$\boxed{v_{in} = v_{gs}}$

$v_{out} = -g_{m,N} v_{gs} (r_{oN} \| R_{eq}) = -g_{m,N} \cdot v_{in} (r_{oN} \| r_{op})$

$\Rightarrow A = \frac{v_{out}}{v_{in}} = -g_{m,N} (r_{oN} \| r_{op})$

Common Source Amplifier with diode-connected PMOS active load





$v_{sg} = -v_x$

$i_1 = i_x + g_{mp} \cdot v_{sg} = i_x - g_{mp} \cdot v_x$

M_1: $i_1 r_{op} - v_x = 0$

$\Rightarrow v_x = r_{op} (i_x - g_{mp} v_x)$

$\Rightarrow R_{eq} = \frac{v_x}{i_x} = \frac{r_{op}}{1 + g_{mp} \cdot r_{op}} = r_{op} \| \frac{1}{g_{m,P}}$

Differential Amplifiers

Any two signals can be decomposed into their **differential-mode** signal and its **common-mode** signal.

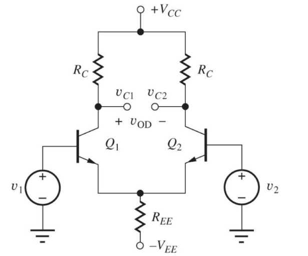
Common-mode signal = 0 → two signals are called fully differential

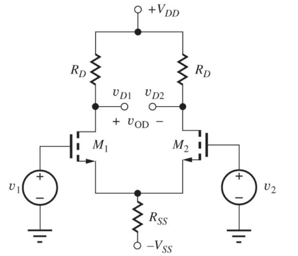
Differential-mode signal component: $V_{id} = V_{i+} - V_{i-}$

Common-mode signal component: $V_{icm} = \frac{V_{i+}+V_{i-}}{2}$

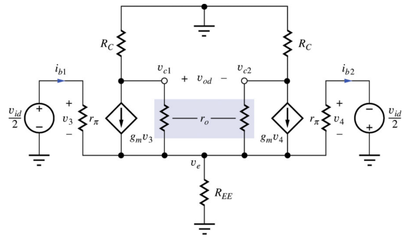
$v_1 = v_{ic} + \frac{v_{id}}{2}$ $\quad v_{id} = v_1 - v_2$ $\quad v_{od} = v_{c1} - v_{c2}$ $\quad$ (FET: $v_{d1} - v_{d2}$)

$v_2 = v_{ic} - \frac{v_{id}}{2}$ $\quad v_{ic} = \frac{v_1+v_2}{2}$ $\quad v_{oc} = \frac{v_{c1}+v_{c2}}{2}$ $\quad$ (FET: $\frac{v_{d1}+v_{d2}}{2}$)





BJT Differential Mode

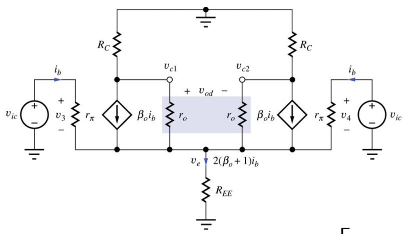


$$A_{dd} = \left.\frac{v_{od}}{v_{id}}\right|_{v_{ic}=0} = -g_m R_C$$

If v_{c1} or v_{c2} is used alone as output, output is called single-ended.

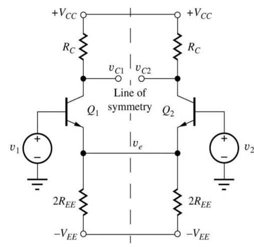
$A_{dd1} = \left.\frac{v_{c1}}{v_{id}}\right|_{v_{ic}=0} = -\frac{g_m R_C}{2} = \frac{A_{dd}}{2}$, $\qquad A_{dd2} = +\frac{g_m R_C}{2} = -\frac{A_{dd}}{2}$

BJT Common Mode



$$A_{cc} = \left.\frac{v_{oc}}{v_{ic}}\right|_{v_{id}=0} = -\frac{\beta_0 R_C}{r_\pi + 2(\beta_0+1)R_{EE}} \approx -\frac{R_C}{2R_{EE}}$$

Half Circuit Analysis

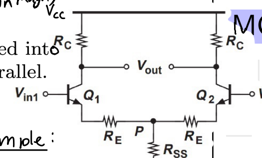


Half circuits must be fully symmetric:

Q-pt of Q_1 = Q-Pt of Q_2

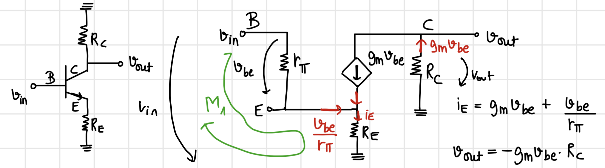
Power supplies: split into 2 equal halves in parallel. (halves in magn)

Emitter resistor: seperated into two equal resistors in parallel. (doubles in magn)



Differential Mode Half Circuits → Example:

Points on the line of symmetry are virtual grounds and connected to ground for AC analysis.



$i_E = g_m v_{be} + \frac{v_{be}}{r_\pi}$

$v_{out} = -g_m v_{be} \cdot R_C$

M_1: $v_{in} = v_{be} + i_E R_E = v_{be} + R_E \left[g_m v_{be} + \frac{v_{be}}{r_\pi} \right]$

$\Rightarrow v_{be} = \frac{v_{in}}{1 + g_m R_E + \frac{R_E}{r_\pi}} \quad \Rightarrow A_{dd} = \frac{-g_m R_C}{1 + g_m R_E + \frac{R_E}{r_\pi}}$

If $R_E = 0$, following equations hold:

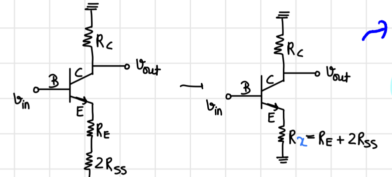
$v_{c1} = -g_m(R_C \parallel r_o)\frac{v_{id}}{2}$, $\qquad v_{c2} = +g_m(R_C \parallel r_o)\frac{v_{id}}{2}$

$v_{od} = v_{c1} - v_{c2} = -g_m(R_C \parallel r_o)v_{id}$

$R_{id} = \frac{v_{id}}{i_b} = 2r_\pi$, $\qquad R_{od} = 2(R_C \parallel r_o)$

Common Mode Half Circuits

Points on the line of symmetry are replaced by open circuits.



→ small signal model

$A_{cc} = \frac{-g_m R_C}{1 + g_m R_x + \frac{R_x}{r_\pi}}$

If $R_E = 0$, following equations hold:

$v_{c1} = v_{c2} = -\frac{\beta_0 R_C}{r_\pi + 2(\beta_0+1)R_{EE}} v_{ic} \approx -\frac{R_C}{2R_{EE}} v_{ic}$

$v_e = v_{ic}\frac{2(\beta_0+1)i_b R_{EE}}{r_\pi + 2(\beta_0+1)R_{EE}} \approx v_{ic}$

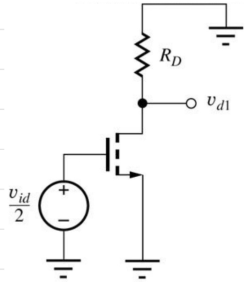
$R_{ic} = \frac{v_{ic}}{2i_b} = \frac{r_\pi + 2(\beta_0+1)R_{EE}}{2} = \frac{r_\pi}{2} + (\beta_0+1)R_{EE}$

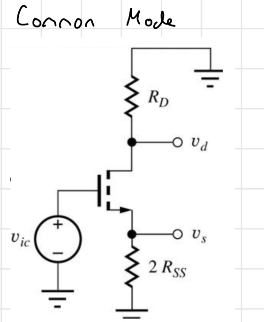
Common Mode Rejection Ratio

$$\text{CMRR} = \frac{A_{dd}}{2}\frac{1}{A_{cc}} = \frac{1 + g_m R_x + \frac{R_x}{r_\pi}}{2\left(1 + g_m R_E + \frac{R_E}{r_\pi}\right)} \overset{R_E=0}{\approx} g_m R_{EE}$$

$$\text{CMRR}_{\text{dB}} = 20 \cdot \log_{10}(\text{CMRR})$$

MOSFET Differential Amplifiers





$v_{d1} = -g_m(R_D \| r_o)\frac{v_{id}}{2}$

$v_{d2} = +g_m(R_D \| r_o)\frac{v_{id}}{2}$

$\therefore v_{od} = -g_m(R_D \| r_o)v_{id}$

$A_{dd} = \left.\frac{v_{od}}{v_{id}}\right|_{v_{ic}} = -g_m(R_D \| r_o) \simeq -g_m R_D$

Gain for single-ended output is

$A_{dd1} = \left.\frac{v_{d1}}{v_{id}}\right|_{v_{ic}} = -\frac{g_m(R_D \| r_o)}{2} = \frac{A_{dd}}{2}$

$A_{dd2} = \left.\frac{v_{d1}}{v_{id}}\right|_{v_{ic}} = +\frac{g_m(R_D \| r_o)}{2} = -\frac{A_{dd}}{2}$

$R_{id} = \infty \qquad R_{od} = 2(R_D \| r_o)$

Input Resistance

$v_{d2} = v_{d1} = -\frac{g_m R_D}{1+2g_m R_{SS}} v_{ic}$

$v_s = +\frac{2g_m R_{ss}}{1+2g_m R_{SS}} v_{ic} \cong v_{ic}$

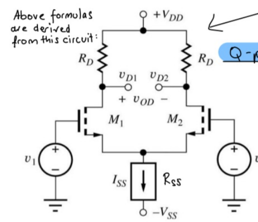
$A_{cc} = \left.\frac{v_{oc}}{v_{ic}}\right|_{v_{id}=0} = -\frac{g_m R_D}{1+2g_m R_{SS}} \cong -\frac{R_D}{2R_{SS}}$

$R_{ic} = \infty$

Single ended CMRR = $\frac{|A_{dd}/2|}{|A_{cc}|}$

$\text{CMRR} \cong (g_m R_{SS}) = \frac{1 + 2 g_m R_{SS}}{2}$

Above formulas are derived from this circuit:



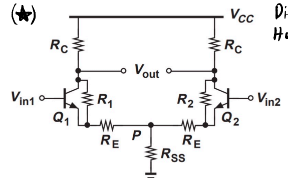
Q-point: we make 2 observations

1. If the circuit is perfectly symmetric, then the Q-point of $M_1 = M_2$
2. If the circuit is perfectly symmetric, then each branch carries $I_{SS}/2$.

Exercise type:

1) Find Q-pt. → Make common mode half circuit and use DC analysis to find Q-pt.
2) Find Add → Draw diff mode half circuit small signal model & solve for $A_{dd} = \frac{v_o}{v_{in}}$ using (✱) ($R_E \to 0$, $R_1 \to r_o$, $R_C \to R_D$)
3) Find Acc → Draw common mode half circuit small signal model & solve for $A_{cc} = v_o/v_{in}$ using (✱) ($R_E \to 2R_{EE}$, $R_1 \to r_o$, $R_C \to R_D$)

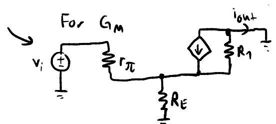
(✱)



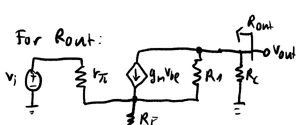
Diff mode Half-circuit ⇒ common emitter amplifier with R_E and $r_o = R_1$ both non-neglected!

Diff mode Half circuit ⇒ $A_{DD} = \underbrace{\frac{r_\pi + R_E - r_\pi(1+g_m R_1)}{r_\pi R_E(1+g_m R_1) + R_1(r_\pi + R_E)}}_{G_M = \frac{i_{out}}{v_{in}}} \cdot \underbrace{\left\{R_C \,\|\, \left[R_1 + (1+g_m R_1)(r_\pi \| R_E)\right]\right\}}_{R_{out}}$

For G_M



For R_{out}:



Simplifications:
- $R_1 \to \infty$ ⇒ normal BJT circuit
- $R_1 = r_o$, $r_\pi \to \infty$, $R_E = 2R_{SS}$ } MOSFET circuit

Frequency Response

Bode Plot Guide

1. Factorize: $\underline{F}_{tot}(s) = K_0 s^r \underbrace{\underline{F}_1(s)\cdot\underline{F}_2(s)\cdot\ldots\underline{F}_n(s)}_{\underline{F}^*_{tot}(s)}$

Singularity	$\mathbf{F_i(s)}$	**Amplitude**	**Phase**
LHP Zero	$1+sT_{z,i}$	+20dB/dec	$+\frac{\pi}{2}$ (over $0.1s_i$ & $10s_i$)
RHP Zero	$1-sT_{z,i}$	+20dB/dec	$-\frac{\pi}{2}$ (over $0.1s_i$ & $10s_i$)
LHP Pole	$\frac{1}{1+sT_{p,i}}$	−20dB/dec	$-\frac{\pi}{2}$ (over $0.1s_i$ & $10s_i$)
RHP Pole	$\frac{1}{1-sT_{p,i}}$	−20dB/dec	$+\frac{\pi}{2}$ (over $0.1s_i$ & $10s_i$)
Zero at $s=0$	$\frac{s}{s_0}$	+20dB/dec 0dB at $s=s_0$	const. $\frac{\pi}{2}$
Pole at $s=0$	$\frac{s_0}{s}$	−20dB/dec 0dB at $s=s_0$	const. $-\frac{\pi}{2}$

2. Sort partial systems by ascending corner frequencies: $s_i = 1/T_{n,i}$ resp. $s_i = 1/T_{p,i}$, (s_1 =smallest corner freq.)

Amplitude Plot (log-log plot)

3. Starting point at s_1: $20\cdot\log_{10}(|\underline{F}_{tot}(js_1)|)$
Don't forget the $\cdot j$ and the absolute value!

← or calculate $\omega\to0$ & $\omega\to\infty$
If $\omega\to0$ is negative
⇒ we start at $\varphi=\pm180°$

4. Starting point to the left: Line with slope $r\cdot 20$dB/dec (For $r=0$ horizontal)

5. Starting point to the right: At each corner frequency s_i, the amplitude slope changes according to the corresponding partial system $F_i(s)$. Higher order poles/zeros: Change slope multiple (= order of this pole) times.

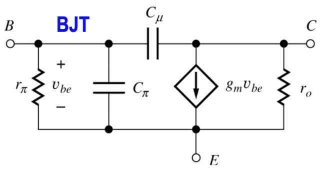
Phase Plot (logarithmic x-axis)

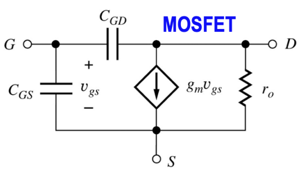
6. Starting frequency s_1 to the left:
$$\varphi(0) = \begin{cases} r\cdot\frac{\pi}{2}, & \text{if } K_0F^*_{tot}(0) > 0 \\ -\pi + r\cdot\frac{\pi}{2}, & \text{if } K_0F^*_{tot}(0) < 0 \end{cases}$$

7. Starting frequency to the right: At each corner frequency s_i, the phase changes according to the corresponding partial system $F_i(s)$

8. $s\to\infty$: Phase ϕ_{tot} approaches $(m-n)\cdot\frac{\pi}{2}$
(n=deg(denominator), m=deg(numerator) of $\underline{F}_{tot}(s)$)

Frequency Dependent Hybrid-Pi Model





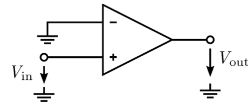
Remember the hybrid-pi model assumes active region operation!

$C_\mu = C_{\mu 0}/\sqrt{1+(V_{CB}/\phi_{jc})}$, $\quad C_{GS} = 2/3C_{GC} + C_{GSO}W$,
$C_{GD} = 2/3C_{GDO}W$, $\quad C_{GC} = C''_{ox}WL$

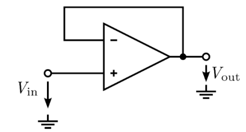
Operational Amplifiers (Op-Amps)

Basic Op-Amp Circuits

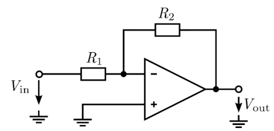
Voltage Comparator
$V_{out} = sign(V_{in})\cdot V_{cc}$



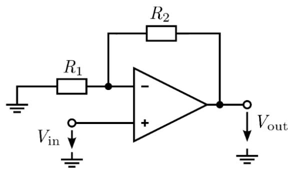
Voltage Follower
$V_{out} = V_{in}$



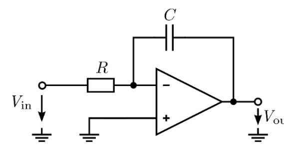
Inverting
$V_{out} = -\frac{R_2}{R_1}\cdot V_{in}$



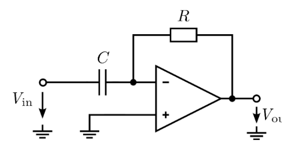
Non-Inverting
$V_{out} = \left(1+\frac{R_2}{R_1}\right)V_{in}$



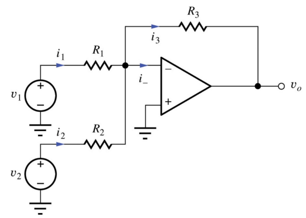
Integrator
$V_{out} = -\frac{V_{in}(s)}{sRC}$
$= V_{out}(0) - \frac{1}{RC}\int_0^t V_{in}(\tau)d\tau$



Diffentiator
$V_{out} = V_{in}(s)sRC =$
$-RC\frac{dV_{in}(t)}{dt}$



Summing Amplifier
$v_o = -\frac{R_3}{R_1}v_1 - \frac{R_3}{R_2}v_2$



Assumptions for Ideal Op-Amps

Ideal Op-Amp	Non-ideal	Practical
Infinite input impedance	Finite	10^7 to $10^12\Omega$
Zero output impedance	Non-Zero	1 to 100Ω
Infinite differential open loop gain	Finite	80 to 120dB
Infinite (OL) bandwidth	Finite	1 to 1000MHz
Infinite CMRR	Finite	70 to 120dB

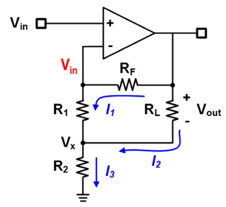
Contstraints for ideal Op-Amps with a negative feedback loop

Input Voltage Constraint: $V_{id} = 0 \Leftrightarrow V_+ = V_-$

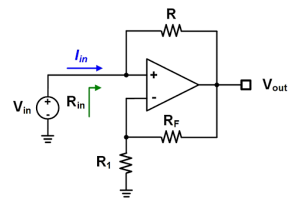
Input Current Constraint: $I_+ = I_- = 0$

More Op-Amp Circuits

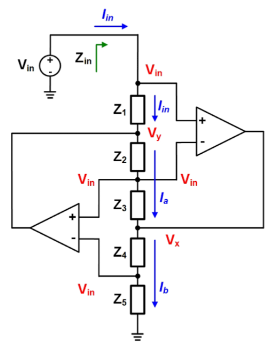
Floating Resistor Output
$A_V = \frac{V_{out}}{V_{in}}$
$= \frac{R_L(R_1+R_F)}{R_L(R_1+R_2)+R_2(R_1+R_F)}$



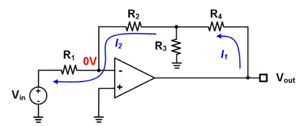
Negative R Converter
$R_{in} = \frac{V_{in}}{I_{in}} = -\frac{R_1}{R_F}R$



General Impedance Converter
$R_{in} = \frac{V_{in}}{I_{in}} = +\frac{Z_1Z_3Z_5}{Z_2Z_4}$



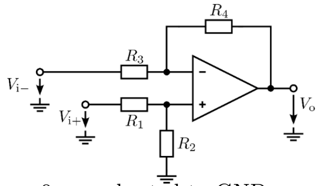
Resistive Ladder Feedback
$V_{in} = -R_1I_2,\ \ I_1 = \frac{V_{out}}{R_4+(R_2\|R_3)}$
$I_2 = \frac{R_3}{R_2+R_3}I_1$
$R_{in} = R_1,\ \ R_{out} = 0$
$A_V = \frac{V_{out}}{V_{in}} = -\frac{R_4R_2}{R_1R_3} - \frac{R_4+R_2}{R_1}$



Instrumentation Amplifiers (InAmps)

Used for precise amplification of weak sensor signals in the presence of distortion and noise, typically at microvolt level.
Linear circuit → Analysis by **signal superposition**

Basic Instrumentation Amplifier



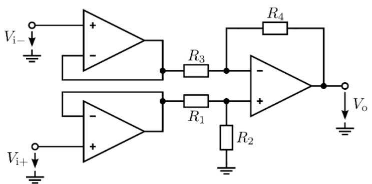
1. Enable Vi+, Vi- = 0 resp shorted to GND
2. This is a non-inverting Amp with input $= V_{i+} \cdot R_2/(R_1 + R_2)$
$$V_O(V_{i+}) = \frac{R_2}{R_1 + R_2} \cdot \frac{R_3 + R_4}{R_3} V_{i+}$$
3. Enable Vi-, Vi+ = 0 resp shorted to GND
4. This is a simple inverting amp with input $= V_{i-}$
$$V_O(V_{i-}) = -\frac{R_4}{R_3} V_{i-}$$

$$V_O = V_O(V_{i+}) + V_O(V_{i-}) = \frac{R_2(R_3 + R_4)}{R_3(R_1 + R_2)} V_{i+} - \frac{R_4}{R_3} V_{i-}$$

For a fully differential amplifier, we need
$$R_2/R_1 = R_4/R_3 = G \implies V_O = G(V_{i+} - V_{i-})$$

However, the input impedance of the two inputs are not the same:
$$R_{V_{i+}} = R_1 + R_2, \quad R_{V_{i-}} = R_3$$

Buffered Instrumentation Amplifier



Goal: Obtain high input impedance

$$V_0 = V_{icm}\left(\frac{R_2(R_3 + R_4)}{R_3(R_1 + R_2)} - \frac{R_4}{R_3}\right) + \frac{V_{id}}{2}\left(\frac{R_2(R_3 + R_4)}{R_3(R_1 + R_2)} + \frac{R_4}{R_3}\right)$$

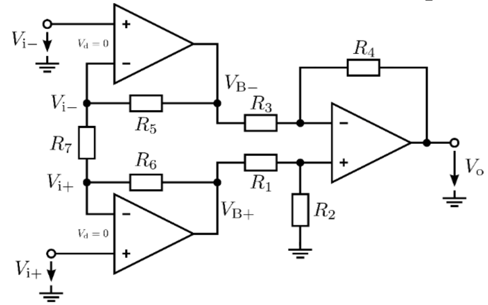
$$A_{cm} = \frac{V_O}{V_{icm}} = \frac{R_2(R_3 + R_4)}{R_3(R_1 + R_2)} - \frac{R_4}{R_3}$$

$$A_d = \frac{V_O}{V_{id}} = \frac{1}{2}\left(\frac{R_2(R_3 + R_4)}{R_3(R_1 + R_2)} + \frac{R_4}{R_3}\right)$$

$$CMRR = \frac{A_d}{A_{cm}} = \frac{V_0/V_{id}}{V_0/V_{icm}} = \frac{R_2(R_3 + R_4) + R_4(R_1 + R_2)}{2(R_2R_3 - R_4R_1)}$$

Input Stage Gain

$$v_{id} = v_{i+} - v_{i-} \qquad v_{i+} = v_{icm} + \frac{v_{id}}{2}$$
$$v_{BD} = v_{B+} - v_{B-} \qquad v_{i-} = v_{icm} - \frac{v_{id}}{2}$$



Goal: Achieve higher CMRR

Input Stage: (Differential and common mode gain)

$$V_{B-} = \frac{R_5 + R_7}{R_7} V_{i-} - \frac{R_5}{R_7} V_{i+}, \qquad V_{B+} = \frac{R_6 + R_7}{R_7} V_{i+} - \frac{R_6}{R_7} V_{i-}$$

$$A_B = \frac{V_{BD}}{V_{id}} = \frac{V_{B_+} - V_{B_-}}{V_{i_+} - V_{i_-}} = \frac{R_5 + R_6 + R_7}{R_7}$$

$$A_{cm,B} = \frac{V_{B_+} + V_{B_-}}{V_{i_+} + V_{i_-}} = 1$$

(No current through R_5, R_6, R_7)

Total: (Differential and common mode gain)

$$A_{d'} = \frac{V_O}{V_{id}} = \frac{A_B}{2}\left(\frac{R_2(R_3 + R_4)}{R_3(R_1 + R_2)} + \frac{R_4}{R_3}\right)$$

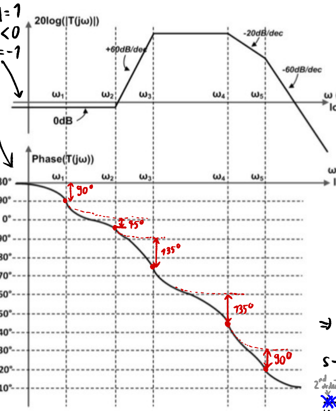
$$A_{cm} = A_{cm,B}\left(\frac{R_2(R_3 + R_4)}{R_3(R_1 + R_2)} - \frac{R_4}{R_3}\right)$$

$$\text{CMRR} = \frac{A_d'}{A_{cm}} = A_B \frac{A_d}{A_{cm}}$$

→ CMRR increased by factor A_B due to an input stage!

Intermezzo: Bode plot transfer fct reconstruction:

$|H_{0c}| = 1$
$H_{0c} < 0$
$\Rightarrow H_{0c} = -1$



ω_1: $-180°$, $+0\,dB/dec$

$\Rightarrow @\omega_1 \begin{cases} 1^{st} \text{ order RHP zero} \to -90°, +20dB/dec \\ 1^{st} \text{ order LHP pole} \to -90°, -20dB/dec \end{cases}$

ω_2: $-90°$, $+60\,dB/dec$

$\Rightarrow @\omega_2$: 3 zeros ← 2 RHP zeros $-90°$, $+20\,db/dec$; ← 1 LHP zero $+90°$, $+20\,dB/dec$

ω_3: $-270°$, $-60\,dB/dec$

$\Rightarrow @\omega_3$: 3 poles ← 3 LHP poles $-90°$, $-20\,dB/dec$

ω_4: $-270°$, $-20\,dB/dec$

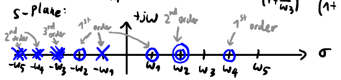
$\Rightarrow @\omega_4 \begin{cases} 2 \text{ LHP poles: } -90°, -20dB/dec \\ 1 \text{ RHP zero: } -90°, +20dB/dec \end{cases}$

ω_5: $-180°$, $-40\,dB/dec$

$\Rightarrow @\omega_5$: 2 LHP poles: $-90°$, $-20\,dB/dec$

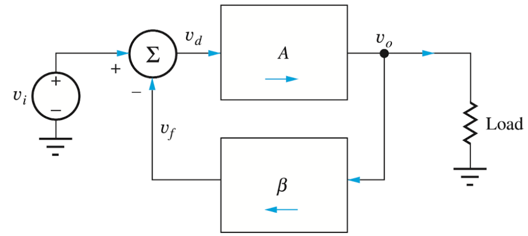
$$\Rightarrow H(s) = (-1) \cdot \frac{1 - \frac{s}{\omega_1}}{1 + \frac{s}{\omega_1}} \cdot \left(1 - \frac{s}{\omega_2}\right)^2 \cdot \left(1 + \frac{s}{\omega_2}\right) \cdot \frac{1}{\left(1 + \frac{s}{\omega_3}\right)^3} \cdot \frac{1 - \frac{s}{\omega_4}}{\left(1 + \frac{s}{\omega_4}\right)^2} \cdot \frac{1}{\left(1 + \frac{s}{\omega_5}\right)^2}$$

s-Plane:



Non-Ideal Operational Amplifiers Circuit Analysis

Negative Feedback Theory



Closed Loop Gain Analysis

$$A_v = \frac{v_O}{v_i} = \frac{A}{1 + A\beta} = \frac{1}{\beta}\left(\frac{A\beta}{1 + A\beta}\right) = A_v^{\text{ideal}}\left(\frac{T}{1 + T}\right)$$

A = open loop gain | $T = A\beta$ = loop gain/loop transmission
A_v = closed loop gain | $A_v^{\text{ideal}} = \frac{1}{\beta}$ ideal gain (if $T \to \infty$)

Gain Error: (ideal gain)-(actual gain)

$$\text{GE} = A_v^{\text{ideal}} - A_v = A_v^{\text{ideal}}\left(1 - \frac{T}{1 + T}\right) = \frac{A_v^{\text{ideal}}}{1 + T}$$

Fractional Gain Error: (ideal gain)-(actual gain)/(ideal gain)

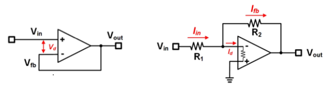
$$\text{FGE} = \frac{A_v^{\text{ideal}} - A_v}{A_v^{\text{ideal}}} = \frac{1}{1 + T} \approx \frac{1}{T}, \quad (T >> 1)$$

Different Kinds of Feedback

- **At the Input**

(1) Voltage → Series Feedback. "Series" connection means the feedback voltage signal and the input voltage signal are connected in series. *Because input voltage subtraction needs series connection.*

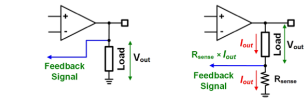
(2) Current → Shunt Feedback. "Shunt" connection means the feedback current signal and the input current signal are connected in series. *Because input current subtraction needs series connection.*



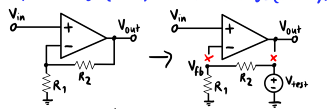
- **At the Output**

(1) Voltage → Shunt Feedback. To measure the load output voltage, "Shunt" connection means the feedback voltage signal is sensed in parallel with the load. *Measuring a voltage, you need a voltage meter placed in shunt.*

(2) Current → Series Feedback. To measure the load output current, "Series" connection means the feedback current signal is sensed as a voltage across a sensing resistor in series with the load. *Measuring a current, you need an Ampere meter placed in series.*



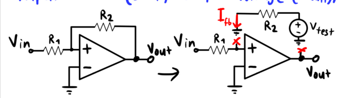
Input Voltage (Series) - Output Voltage (Shunt):



Open the Feedback, apply test voltage source at the output & measure the feedback voltage:

$$\beta = \frac{V_{fb}}{V_{test}} = \frac{R_1}{R_1 + R_2} \Rightarrow A_v = \frac{V_{out}}{V_{in}} = \frac{1}{\beta} = 1 + \frac{R_2}{R_1}$$

Input Current (Shunt) - Output Voltage (Shunt):

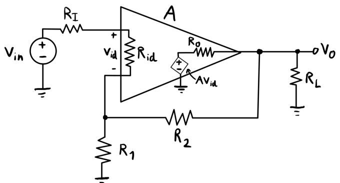


Open the Feedback, apply test voltage source at the output & measure the feedback current:

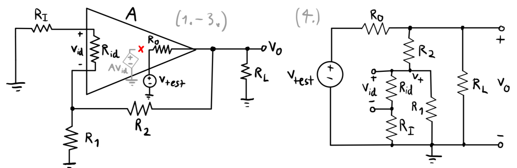
$$\beta = \frac{I_{fb}}{V_{test}} = \frac{1}{R_2} \Rightarrow \frac{V_{out}}{I_{in}} = \frac{1}{\beta} = R_2$$

$$\frac{V_{in}}{I_{in}} = -R_1 \Rightarrow \frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_{in}} \cdot \frac{V_{in}}{I_{in}} = R_2 \cdot \left(\frac{-1}{R_1}\right) = -\frac{R_2}{R_1}$$

Non-Ideal Non-Inverting Amplifier

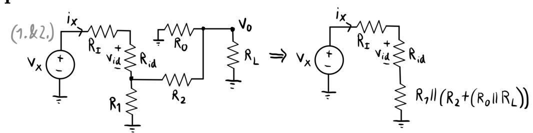


Closed Loop Voltage Gain Calculation:



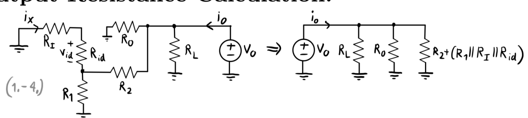
1. Set all input sources to zero
2. Cut the loop open between Av_{id} and R_O
3. Insert a test source v_{test} where the loop was cut
4. Find Av_{id} using standard circuit analysis
   $v_O = v_{test}\cdot\frac{R_L\|(R_2+(R_1\|(R_{id}+R_I)))}{R_O+(R_L\|(R_2+(R_1\|(R_{id}+R_I))))} = K_1\cdot v_{test}$
   $v_+ = v_O\cdot\frac{(R_{id}+R_I)\|R_1}{R_2+((R_{id}+R_I)\|R_1)} = K_2\cdot K_1\cdot v_{test}$
   $v_{id} = v_+\cdot\frac{R_{id}}{R_I+R_{id}} = K_3\cdot K_2\cdot K_1\cdot v_{test} = K\cdot v_{test}$
5. Calculate Loop Gain $T = \frac{Av_{id}}{v_{test}} = A\cdot K$
6. Calculate $\beta = \frac{R_1}{R_1+R_2}$
7. Calculate $A_v = A_v^{ideal}\left(\frac{T}{1+T}\right)$, where $A_v^{ideal} = \frac{1}{\beta}$

Input Resistance Calculation:



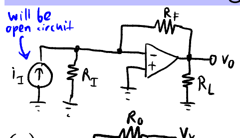
1. Apply a test source V_x at V_{in}
2. Break the loop between Av_{id} and R_O & ground R_O
3. Use property of input voltage (series) feedback & calculate R_{in}
   $R_{in} = R_{in,ol}(1+T)$, ($R_{in,ol}$ is input resistance with broken loop)
   $R_{in,ol} = \frac{v_x}{i_x} = R_I + R_{id} + (R_1 \parallel (R_2 + (R_O \parallel R_L)))$

Output Resistance Calculation:

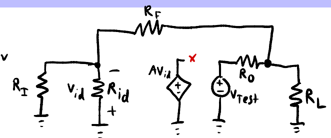


1. Apply a test source V_o at the output
2. Break the loop between Av_{id} and R_O & ground R_O
3. Set the input voltage source to zero (ground) & simplify circuit
4. Use property of output voltage (shunt) feedback & calculate R_{out}
   $R_{out} = \frac{R_{out,ol}}{1+T}$, ($R_{out,ol}$ is output resistance with broken loop)
   $R_{out,ol} = \frac{v_O}{i_O} = R_L \parallel R_O \parallel (R_2 + (R_1 \parallel R_I \parallel R_{id}))$

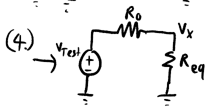
Non-Ideal Inverting Amplifier



(1.-3.) from non-inv $\longrightarrow$



(4.)

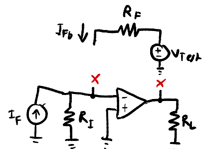


$R_{eq} = R_L \parallel (R_F + (R_I \parallel R_{id}))$

solve for v_{id}: $v_{id} = -V_{Test}\frac{R_{eq}}{R_{eq}+R_O}\cdot\frac{(R_I\|R_{id})}{R_F+(R_I\|R_{id})}$

(5.) $T = \frac{Av_{id}}{V_{test}}$, (6.) $A_v = A_{v,ideal}\left(\frac{T}{1+T}\right)$

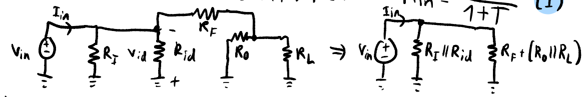
$\frac{V_o}{I_{in}}$ $\quad$ $\frac{V_o}{I_{in,ideal}}$



$\beta = \frac{I_{FB}}{V_{test}} = \frac{1}{R_F} \Rightarrow A_{v,ideal} = \frac{V_o}{I_{in}} = -\frac{V_{test}}{I_{FB}} = -R_F$

R_{in}: Use property of input current (shunt) feedback $R_{in} = \frac{R_{in,OL}}{1+T}$ (I)

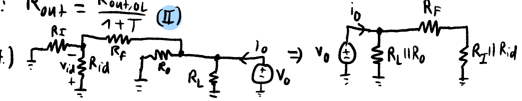
(1.)&(2.) $\Rightarrow$



$\Rightarrow V_{in}/I_{in} = R_{in,OL} = (R_I \| R_{id}) \| (R_F + (R_O \| R_L))$ $\Rightarrow$ calculate R_{in} using (I)

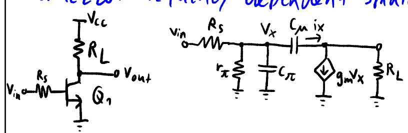
R_{out}: $R_{out} = \frac{R_{out,OL}}{1+T}$ (II)

(1.-4.)



$\Rightarrow R_{out,OL} = \frac{V_o}{i_o} = (R_L \| R_O) \| (R_f + (R_I \| R_{id})) \Rightarrow R_O = \frac{R_{out,OL}}{1+T}$ (II)

Intermezzo: Frequency dependent small signal model:



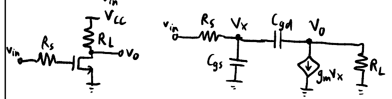
① $i_x = g_m V_x + \frac{V_o}{R_L}$

② $i_x = \frac{V_x - V_o}{\frac{1}{sC_\mu}} = sC_\mu(V_x - V_o)$

③ $v_{in} - V_x = \frac{R_S(1+sr_\pi C_\pi)v_x}{r_\pi} + R_S i_x$

Substitute ② into ① & solve for V_x. Substitute back into ③ and solve for $V_{in} = V_o \cdot \square$ (sth.)

$\Rightarrow H(s) = \frac{V_o}{V_{in}}(s)$



① $\frac{V_{in}-V_x}{R_S} = sC_{gs}V_x + (V_x - V_o)sC_{gd}$

solve for $V_{in} = V_x(1+sR_SC_{gs}+sR_SC_{gd}) - sR_SC_{gd}V_o$

② $sC_{gd}(V_x - V_o) = g_m V_x + \frac{V_o}{R_L}$

solve for $V_x = \frac{V_o(1+sR_LC_{gd})}{sR_LC_{gd} - g_m R_L}$

solve for v_o/v_{in}

TABLE 11.3
Inverting and Noninverting Amplifier Frequency Response Comparison

$\beta = \frac{R_1}{R_1+R_2}$	NONINVERTING AMPLIFIER	INVERTING AMPLIFIER
dc gain	$A_v(0) = 1 + \frac{R_2}{R_1}$	$A_v(0) = -\frac{R_2}{R_1}$
Feedback factor	, $T = A\beta$ $\beta = \frac{1}{A_v(0)}$	$\beta = \frac{1}{1+\|A_v(0)\|}$
Bandwidth	$f_B = \beta f_T$	$f_B = \beta f_T$
Input resistance	$R_{ic}\|R_{id}(1+A\beta)$	$R_1 + \left(R_{ID}\|\|\frac{R_2}{1+A}\right)$
Output resistance	$\frac{R_o}{1+A\beta}$	$\frac{R_o}{1+A\beta}$
Fractional Gain error	$\frac{1}{1+T}$	$\frac{1}{1+T}$

Frequency Response and Bandwidth of Op Amps

Most general purpose op-amps are low-pass amps designed to have high gain at DC and a single-pole frequency response described by:

$$A(s) = \frac{A_0\omega_B}{s+\omega_B} = \frac{\omega_T}{s+\omega_B} = \frac{A_0}{1+\frac{s}{\omega_B}}$$

A_0 = DC open loop gain, ω_B = open loop bandwidth ($f_B = \frac{\omega_B}{2\pi}$),
ω_T = unity gain frequency at which $|A(j\omega)| = 1$ (0dB)

$$|A(j\omega)| = \frac{A_0\omega_B}{\sqrt{\omega^2+\omega_B^2}} = \frac{A_0}{\sqrt{1+\frac{\omega^2}{\omega_B^2}}} \approx \begin{cases}\text{const. for } \omega << \omega_B \\ \frac{\omega_T}{\omega} \text{ for } \omega >> \omega_B\end{cases}$$

Gain Bandwidth Product: GBW $= |A(j\omega)|\omega \approx \omega_T$

Non-Inverting Amplifier

$$A_v(s) = \frac{A(s)}{1+A(s)\beta} = \frac{\frac{A_0\omega_B}{s+\omega_B}}{1+\frac{A_0\omega_B}{s+\omega_B}\beta} = \frac{A_0\omega_B}{s+\omega_B(1+A_0\beta)} = \frac{A_v(0)}{\frac{s}{\omega_H}+1}$$

$$\beta = \frac{R_1}{R_1+R_2}, \quad \omega_H = \omega_B(1+A_0\beta) = \omega_T\frac{(1+A_0\beta)}{A_0} = \frac{\omega_T}{A_v(0)}$$

ω_H = upper cutoff frequency = closed loop bandwidth $\xrightarrow{A_0\beta>>1} \beta\omega_T$

$A_v(0) = \frac{A_0}{1+A_0\beta}$ = closed loop DC gain $\xrightarrow{A_0\beta>>1} 1/\beta$

Inverting Amplifier

$$A_v = \left(-\frac{R_2}{R_1}\frac{A(s)\beta}{1+A(s)\beta}\right), \text{ where } \beta = \frac{R_1}{R_1+R_2}$$

$$A_v(s) = \left(-\frac{R_2}{R_1}\right)\frac{\frac{A_0\omega_B}{s+\omega_B}\beta}{1+\frac{A_0\omega_B}{s+\omega_B}\beta} = \left(-\frac{R_2}{R_1}\right)\frac{A_0\beta\omega_B}{s+\omega_B(1+A_0\beta)}$$

$$A_v \xrightarrow{A_0\beta>>1} \left(-\frac{R_2}{R_1}\right)\frac{1}{\frac{s}{\omega_H}+1}, \quad \omega_H = \frac{\omega_T}{\frac{A_0}{1+A_0\beta}} \xrightarrow{A_0\beta>>1} \beta\omega_T$$

$\approx \omega_B(1+A_0\beta)$

Non-Ideal Op Amp Characteristics

Output Voltage & Current Limitations

V_{out} is limited by the supply voltage: $\max(V_{out}) \leq V_DD$

$I_{out} = \frac{V_{out}}{R_L} \leq I_{out,max}$

$\Longrightarrow R_{L,min} = \frac{V_{DD}}{I_{out,max}}$

Slew Rate: Large Signal Dynamic Limitation

maximum rate of change of the output voltage of a physical circuit

$$\text{SL} = \max\frac{dv_O}{dt}$$

Ex.: $v_O = \sin(\omega t) \Longrightarrow \text{SL} = \max(\omega\cos(\omega t)) = \omega$

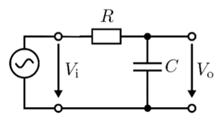
Common Mode Rejection Ratio

CMRR $= \frac{A}{A_{cm}}$ which is typically around 60 ∼ 120dB

Filters

First Order Passive Filters

Low Pass Filter

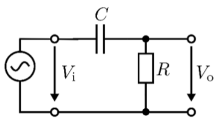


$$T(s)=\frac{V_o}{V_i}=\frac{1}{1+sRC}$$

Amplitude Response: $\left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$

Phase Response: $\angle\left(\frac{V_o(j\omega)}{V_i(j\omega)}\right) = -\arctan(\omega RC)$

High Pass Filter

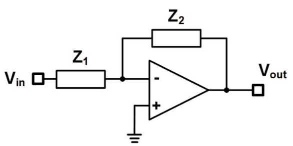


$$T(s)=\frac{V_o}{V_i}=\frac{sRC}{1+sRC}$$

Amplitude Response: $\left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}}$

Phase Response: $\angle\left(\frac{V_o(j\omega)}{V_i(j\omega)}\right) = \frac{\pi}{2}-\arctan(\omega RC)$

First Order Active Filters



$$H(s)=\frac{V_{out}(s)}{V_{in}(s)}=-\frac{Z_2(s)}{Z_1(s)}$$

Active Low Pass Filter

Set $Z_1(s)=R_1$ and $Z_2(s)=R_2 \parallel \frac{1}{sC} = \frac{R_2\frac{1}{sC}}{R_2+\frac{1}{sC}} = \frac{R_2}{sR_2C+1}$

$$A_v(s)=-\frac{R_2}{R_1}\frac{1}{1+sR_2C}=-\frac{R_2}{R_1}\frac{1}{1+\frac{s}{\omega_H}}, \qquad \left(\omega_H=2\pi f_H=\frac{1}{R_2C}\right)$$

Active High Pass Filter

Set $Z_1(s)=R_1+\frac{1}{sC}=\frac{sR_1C+1}{sC}$ and $Z_2(s)=R_2$

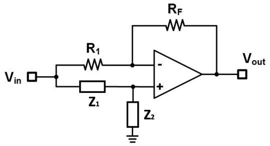
$$A_v(s)=-\frac{R_2}{R_1}\frac{s}{s+\omega_L}=\frac{A_0}{1+\frac{\omega_L}{s}}, \quad \left(A_0=-\frac{R_2}{R_1},\ \omega_L=2\pi f_L=\frac{1}{R_1C}\right)$$

Other Configurations → Other Transfer Functions

$Z_1=R_1 \parallel \frac{1}{sC_1}$, $Z_2=R_2 \parallel \frac{1}{sC_2}$ $H(s)=-\frac{C_1}{C_2}\frac{s+\frac{1}{R_1C_1}}{s+\frac{1}{R_2C_2}}=-k\frac{s+z}{s+p}$	$Z_1=R_1+\frac{1}{sC_1}$, $Z_2=R_2+\frac{1}{sC_2}$ $H(s)=-\frac{R_2}{R_1}\frac{s+\frac{1}{R_2C_2}}{s+\frac{1}{R_1C_1}}=-k\frac{s+z}{s+p}$

Bilinear Transfer Function Synthesis

With the previous Filters we can only realize LHP zeros and poles. To realize RHP zeros and poles, we can use following circuits:



Set $Z_1=C$, $Z_2=R_G$ $H(s)=\frac{sC-G_G\frac{G_1}{G_F}}{sC+G_G}$ Zero: $+\frac{G_1}{G_F}\frac{G_G}{C}$, Pole: $-\frac{G_G}{C}$	Set $Z_1=R_a$, $Z_2=C$ $H(s)=\frac{G_a-sC\frac{G_1}{G_F}}{G_a+sC}$ Zero: $+\frac{G_1}{G_F}\frac{G_a}{C}$, Pole: $-\frac{G_a}{C}$

By setting $\frac{G_1}{G_F}=1$ we achieve an all pass filter.

Second Order Transfer Functions

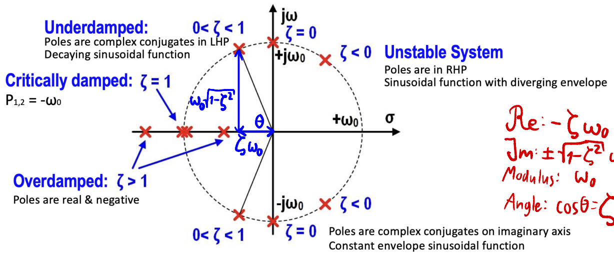
$$H(s)=\frac{N(s)}{\left(\frac{s}{\omega_0}\right)^2+2\zeta\left(\frac{s}{\omega_0}\right)+1}=\frac{N(s)}{\left(\frac{s}{\omega_0}\right)^2+\frac{1}{Q}\left(\frac{s}{\omega_0}\right)+1}$$

$N(s)$ = polynomial with deg$(N)\leq 2$. Defines the zeros of the system

ω_0 = natural frequency of the second order system

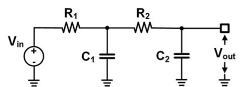
$Q=\frac{1}{2\zeta}$ = Quality factor, where ζ = dimensionless damping factor

$P_{1,2}=(-\zeta\pm\sqrt{\zeta^2-1})\omega_0$ are the two poles of the system.



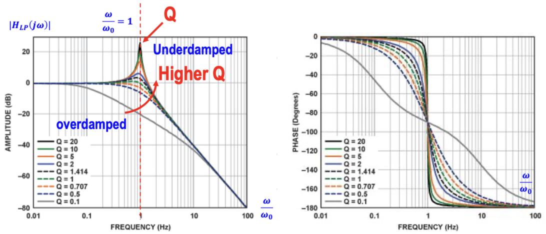
Re: $-\zeta\omega_0$
Im: $\pm\sqrt{1-\zeta^2}\,\omega_0$
Modulus: ω_0
Angle: $\cos\theta=\zeta=\frac{1}{2Q}$

Second Order Low Pass Filter (LP)



Set $N(s)=1 \to H_{LP}(\omega)=\frac{1}{1-\left(\frac{\omega}{\omega_0}\right)^2+\frac{1}{Q}\left(\frac{j\omega}{\omega_0}\right)}$

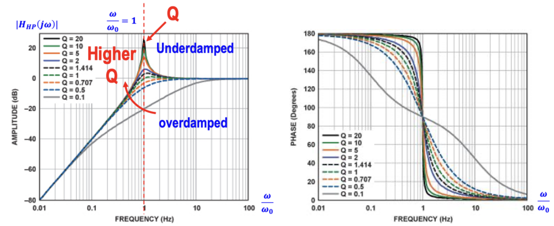
low freq $\frac{\omega}{\omega_0}<<1$ $H_{LP}\approx 1 = 0$dB	high freq $\frac{\omega}{\omega_0}>>1$ $\lvert H_{LP}\rvert$: -40dB/dec	characteristic freq $\frac{\omega}{\omega_0}=1$ $H_{LP}=-jQ \to -\frac{\pi}{2}$ phase shift



Second Order High Pass Filter (HP)

Set $N(s)=\left(\frac{s}{\omega_0}\right)^2 \to H_{HP}(\omega)=\frac{-\left(\frac{\omega}{\omega_0}\right)^2}{1-\left(\frac{\omega}{\omega_0}\right)^2+\frac{1}{Q}\left(\frac{j\omega}{\omega_0}\right)}$

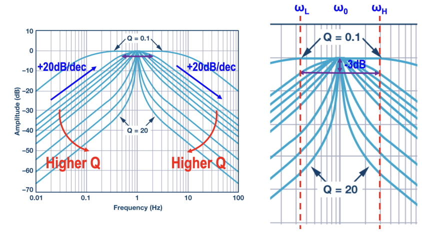
low freq $\frac{\omega}{\omega_0}<<1$ $\lvert H_{LP}\rvert$: $+40$dB/dec	high freq $\frac{\omega}{\omega_0}>>1$ $H_{LP}\approx 1 = 0$dB	characteristic freq $\frac{\omega}{\omega_0}=1$ $H_{LP}=jQ \to \frac{\pi}{2}$ phase shift



Second Order Band Pass Filter (BP)

Set $N(s)=\frac{1}{Q}\left(\frac{s}{\omega_0}\right) \to H_{BP}(\omega)=\frac{\frac{1}{Q}\left(\frac{\omega}{\omega_0}\right)}{1-\left(\frac{\omega}{\omega_0}\right)^2+\frac{1}{Q}\left(\frac{j\omega}{\omega_0}\right)}$

low freq $\frac{\omega}{\omega_0}<<1$ $\lvert H_{LP}\rvert$: $+20$dB/dec	high freq $\frac{\omega}{\omega_0}>>1$ $\lvert H_{LP}\rvert$: -20dB/dec	characteristic freq $\frac{\omega}{\omega_0}=1$ $\lvert H_{LP}\rvert\approx 1 = 0$dB



Band-Stop Notch Filter

Set $N(s)=1+\left(\frac{s}{\omega_0}\right)^2 \to H_N(\omega)=1-H_{BP}(\omega)$

low freq $\frac{\omega}{\omega_0}<<1$ $H_N(j\omega)\approx 1$	high freq $\frac{\omega}{\omega_0}>>1$ $H_N(j\omega)\approx 1$	characteristic freq $\frac{\omega}{\omega_0}=1$ $H_N=-j\left[1-\left(\frac{\omega}{\omega_0}\right)^2\right]Q$

Second Order All Pass Filter (AP)

Set $N(s)=1+\left(\frac{s}{\omega_0}\right)^2-\frac{1}{Q}\left(\frac{s}{\omega_0}\right) \to H_{AP}(j\omega)=\frac{1-\left(\frac{\omega}{\omega_0}\right)^2-\frac{1}{Q}\left(\frac{j\omega}{\omega_0}\right)}{1-\left(\frac{\omega}{\omega_0}\right)^2+\frac{1}{Q}\left(\frac{j\omega}{\omega_0}\right)}$

One pair of complex poles (LHP)

One pair of complex zeros (RHP)

They are symmetric

$|H_{AP}(j\omega)|=1$ for any frequency

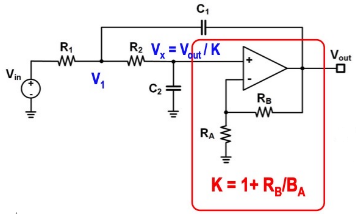
$$\angle H_{AP}(j\omega)=-2\arctan\left[\frac{\frac{1}{Q}\left(\frac{j\omega}{\omega_0}\right)}{1-\left(\frac{\omega}{\omega_0}\right)^2}\right]\in[-\pi,+\pi]$$

Sallen Key Filters (KRC Filters)

Problem with 2^{nd} order passive filters: $Q<0.5 \Rightarrow$ Two real poles $\Rightarrow$ overdamped $\Rightarrow$ no complex peaking & no sharp roll off

Goal: Increase Q to boost amplitude response at ω_0

Sallen Key Lowpass (LP) Filters



$$V_{out} = V_x \cdot K = K \frac{1}{R_2C_2s+1} V_1$$

$$KCL@V_1: \frac{V_{in}-V_1}{R_1} + \frac{V_{out}/K - V_1}{R_2} + \frac{V_{out}-V_1}{1/sC_1} = 0$$

$$H(s) = \frac{V_{out}}{V_{in}} = K \cdot \frac{1}{\left(\frac{s}{\omega_0}\right)^2 + \frac{1}{Q}\left(\frac{s}{\omega_0}\right) + 1}$$

$$= K \cdot \frac{1}{R_1C_1R_2C_2s^2 + [(1-K)R_1C_1 + R_1C_2 + R_2C_2]s + 1}$$

$H_{LP}(s=0)$ $=$ $H_{0LP} = K = 1 + \frac{R_B}{R_A}$, $\quad \omega_0 = \frac{1}{\sqrt{R_1C_1R_2C_2}}$, $\quad Q = \frac{1}{(1-K)\sqrt{\frac{R_1C_1}{R_2C_2}} + \sqrt{\frac{R_1C_2}{R_2C_1}} + \sqrt{\frac{R_2C_2}{R_1C_1}}}$

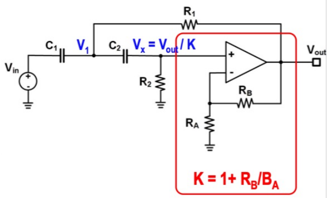
$\Rightarrow$ 5 unknowns K, R_1, C_1, R_2, C_2 but 3 equations $H_{LP}(s=0)$, ω_0, $Q \Rightarrow$ under-determined design problem

$\rightarrow$ Equal-component KRC Filter: Set $R_1 = R_2 = R$, $C_1 = C_2 = C$

$H_{0LP} = K = 1 + \frac{R_B}{R_A}$, $\omega_0 = \frac{1}{RC}$, $Q = \frac{1}{3-K}$ $\Rightarrow$ Q is completely set by K

However, $Q = \frac{1}{3-(1+\frac{R_B}{R_A})} = \frac{1}{2-\frac{R_B}{R_A}}$ $\Rightarrow$ Small variation in $\frac{R_B}{R_A}$ may lead to large Q variation $\rightarrow$ Possible oscillation

Sallen Key Highpass (HP) Filters

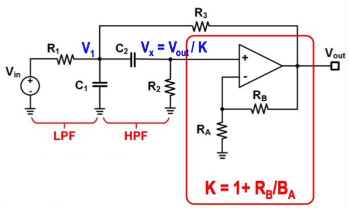


$$H(s) = K \cdot \frac{\left(\frac{s}{\omega_0}\right)^2}{\left(\frac{s}{\omega_0}\right)^2 + \frac{1}{Q}\left(\frac{s}{\omega_0}\right) + 1}$$

$H_{0LP} = K = 1 + \frac{R_B}{R_A}$, $\quad \omega_0 = \frac{1}{\sqrt{R_1C_1R_2C_2}}$

$$Q = \frac{1}{(1-K)\sqrt{\frac{R_2C_2}{R_1C_1}} + \sqrt{\frac{R_1C_1}{R_2C_2}} + \sqrt{\frac{R_1C_2}{R_2C_1}}}$$

Sallen Key Bandpass Filters



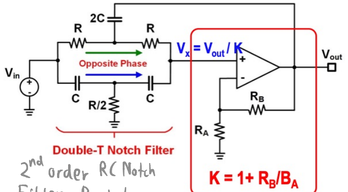
$$H(s) = H_{0HP} \frac{\frac{1}{Q}\left(\frac{s}{\omega_0}\right)}{\left(\frac{s}{\omega_0}\right)^2 + \frac{1}{Q}\left(\frac{s}{\omega_0}\right) + 1}$$

$$H_{0HP} = \frac{K}{1 + (1-K)\frac{R_1}{R_3} + \left(1 + \frac{C_1}{C_2}\right)\frac{R_1}{R_2}}, \quad \omega_0 = \frac{\sqrt{1 + \frac{R_1}{R_3}}}{\sqrt{R_1C_1R_2C_2}}$$

$$Q = \frac{\sqrt{1 + \frac{R_1}{R_3}}}{\left(1 + (1-K)\frac{R_1}{R_3}\right)\sqrt{\frac{R_2C_2}{R_1C_1}} + \sqrt{\frac{R_1C_2}{R_2C_1}} + \sqrt{\frac{R_1C_1}{R_2C_2}}}$$

Equal-component design: $R_1 = R_2 = R_3 = R$, $C_1 = C_2 = C$

Sallen Key Notch Filters



2^{nd} order RC Notch Filter Prototype

Equal-component KRC notch filter design yields:

$$H(s) = K \cdot \frac{\left(\frac{s}{\omega_0}\right)^2 + 1}{\left(\frac{s}{\omega_0}\right)^2 + \frac{1}{Q}\left(\frac{s}{\omega_0}\right) + 1}$$

$H_{0N} = K = 1 + \frac{R_B}{R_A}$

$\omega_0 = \frac{1}{RC}$, $\quad Q = \frac{1}{4-2K}$

Cascade of Op-Amp Based Active Filters

Any high order fct. can be decomposed into multiplication of 1^{st} and 2^{nd} order fct: 1^{st} order filters $\leftrightarrow$ Bilinear op-amp filters

2^{nd} order filters $\leftrightarrow$ KRC op-amp filters

$$T(s) = \frac{N(s)}{D(s)} = \frac{\sum_{i=0}^{m} b_i s^i}{\sum_{j=0}^{n} a_j s^j} = K \frac{\prod_{i=0}^{m_1}(s-z_i)\prod_{p=0}^{m_2}(s-z_p)(s-z_p^*)}{\prod_{j=0}^{n_1}(s-p_j)\prod_{q=0}^{n_2}(s-p_q)(s-p_q^*)}$$

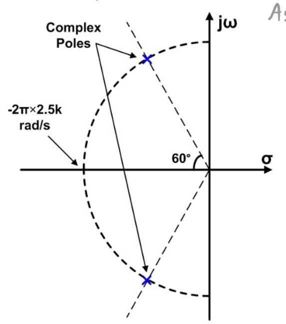
K is real, z_i are real, either positive or negative (RHP or LHP zeros)

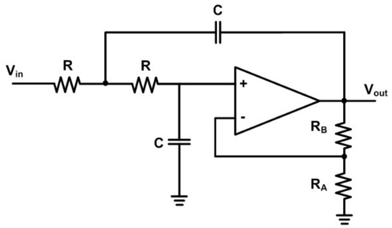
z_p, z_p^* are complex conjugate, either on LHP or RHP

p_j real, must be negative (only LHP), p_q, p_q^* complex conjugate, must be on LHP

Example: KRC/Sallen-Key Active Filters

Task: Design Sallen-Key LP Filter to realize following complex poles. Assume $C = 5nF$ and $R_A = 10k\Omega$ given





$$\omega_0 = 2\pi \cdot 2.5 krad/s \Rightarrow R = \frac{1}{\omega_0 C} = \frac{1}{2\pi \cdot 2.5 \cdot 10^3 \cdot 5 \cdot 10^{-4}} = 12.7k\Omega$$

$$\theta = \cos^{-1}\left(\frac{1}{2Q}\right) = 60° \Rightarrow \frac{1}{2Q} = \cos 60° = \frac{1}{2} \Rightarrow Q = 1$$

$$K = 3 - \frac{1}{Q} = 3 - 1 = 2 = 1 + \frac{R_B}{R_A}$$ Since $R_A = 10k\Omega \Rightarrow R_B = 10k\Omega$

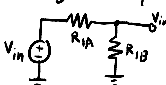
How would you modify the circuit's input without using any additional op-amp, to realize a total DC-gain of 3dB and still maintain the desired filter response (location of the complex poles unchanged)? (Assuming ideal op-amps)

The Sallen-key LP Filter has DC gain $= K = 2$ before modification

The desired gain is $3dB = \sqrt{2}$

$\Rightarrow$ We need to attenuate the signal by $\sqrt{2}$

Using the equivalent voltage source concept

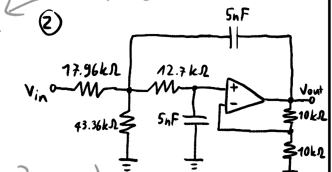


$V'_{in} = \frac{1}{\sqrt{2}} V_{in} = \frac{R_{1B}}{R_{1A} + R_{1B}} V_{in} \Rightarrow \frac{R_{1B}}{R_{1A} + R_{1B}} = \frac{1}{\sqrt{2}}$ ①

$R_{1A} \parallel R_{1B} = 12.7k\Omega$ ② The R we got in previous task

Solve Syst. of eq.: $R_{1A} = 17.96k\Omega$

$R_{1B} = 43.36k\Omega$



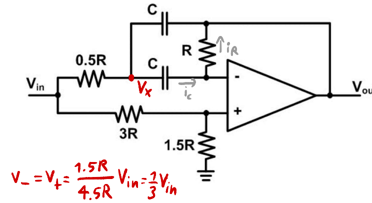
solve with HP-prime:

$$\text{solve}\left(\left\{\frac{x \cdot y}{x+y} = 12.7 \cdot 10^3, \frac{1}{\sqrt{2}} = \frac{y}{x+y}\right\}, \{x, y\}\right)$$

This yields: {[17960.5122421 43360.5122421]}

first entry is $x = R_{1A}$, second entry is $y = R_{1B}$

Example: Multiple Feedback All-Pass Filter



$V_- = V_+ = \frac{1.5R}{4.5R} V_{in} = \frac{1}{3} V_{in}$ (voltage divider at V_+)

① $KCL@V_x$: $\frac{V_{in}-V_x}{0.5R} = (V_x - V_{out})sC + \left(\frac{1}{3}V_{in} - V_{out}\right)\frac{1}{R}$

since $i_- = 0 \Rightarrow i_C = i_R$

② $KCL@V_-$: $\left(V_x - \frac{1}{3}V_{in}\right)sC = \left(\frac{1}{3}V_{in} - V_{out}\right)\frac{1}{R}$

② $\Rightarrow V_x = \frac{1}{3}V_{in} + \frac{1}{sRC}\left(\frac{1}{3}V_{in} - V_{out}\right)$ ③

① $\Rightarrow V_{in} - V_x = (V_x - V_{out})\frac{sRC}{2} + \left(\frac{1}{3}V_{in} - V_{out}\right) \cdot \frac{1}{2}$

$\Rightarrow V_x = \frac{1}{\left(\frac{sRC}{2}+1\right)}\left(\frac{5}{6}V_{in} + \frac{1}{2}V_{out} + \frac{sRC}{2}V_{out}\right)$ ④

③ = ④ $\Rightarrow \frac{1}{3}V_{in} + \frac{1}{sRC}\left(\frac{1}{3}V_{in} - V_{out}\right) = \frac{1}{\left(\frac{sRC}{2}+1\right)}\left(\frac{5}{6}V_{in} + \frac{1}{2}V_{out} + \frac{sRC}{2}V_{out}\right)$

$\Rightarrow \frac{1}{6}sRC\,V_{in} + \frac{1}{3}V_{in} + \left(\frac{1}{2} + \frac{1}{sRC}\right)\left(\frac{1}{3}V_{in} - V_{out}\right) = \frac{5}{6}V_{in} + \frac{1}{2}V_{out} + \frac{sRC}{2}V_{out}$

$\Rightarrow \frac{1}{6}sRC\,V_{in} + \frac{1}{3}V_{in} + \frac{1}{6}V_{in} + \frac{1}{3sRC}V_{in} - \frac{1}{2}V_{out} - \frac{1}{sRC}V_{out} = \frac{5}{6}V_{in} + \frac{1}{2}V_{out} + \frac{sRC}{2}V_{out}$

$\Rightarrow \frac{1}{6}sRC\,V_{in} - \frac{1}{3}V_{in} + \frac{1}{3sRC}V_{in} = V_{out} + \frac{1}{sRC}V_{out} + \frac{sRC}{2}V_{out}$

$$\Rightarrow H(s) = \frac{V_{out}}{V_{in}} = \frac{\frac{1}{6}sRC - \frac{1}{3} + \frac{1}{3sRC}}{\frac{sRC}{2} + 1 + \frac{1}{sRC}} = \frac{1}{3}\frac{s^2R^2C^2 - 2sRC + 2}{s^2R^2C^2 + 2sRC + 2} = \frac{1}{3}\cdot\frac{s^2 - \frac{2}{RC}s + \frac{2}{R^2C^2}}{s^2 + \frac{2}{RC}s + \frac{2}{R^2C^2}}$$

General all-pass transfer function: $H_{AP}(s) = H_{AP0} \cdot \frac{s^2 - \frac{s\omega_0}{Q} + \omega_0^2}{s^2 + \frac{s\omega_0}{Q} + \omega_0^2}$

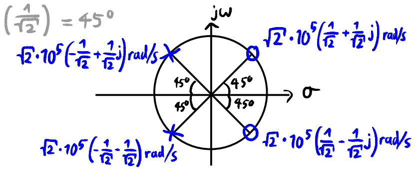
$\Rightarrow$ The given circuit is an all-pass filter with $H_{AP0} = \frac{1}{3}$, $\omega_0 = \sqrt{\frac{2}{R^2C^2}} = \frac{\sqrt{2}}{RC}$, $Q = \left(\frac{\frac{2}{RC}}{\omega_0}\right)^{-1} = \left(\frac{2}{RC}\cdot\frac{RC}{\sqrt{2}}\right)^{-1} = \frac{1}{\sqrt{2}}$

Let $R = 10k\Omega$, $C = 1nF$ $\Rightarrow \omega_0 = \sqrt{2} \cdot 10^5$ rad/s

$Q = \frac{1}{\sqrt{2}} > \frac{1}{2}$ $\Rightarrow$ complex zeros and poles

$\theta = \cos^{-1}\left(\frac{1}{2Q}\right) = \cos^{-1}\left(\frac{1}{2 \cdot \frac{1}{\sqrt{2}}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$

$\Rightarrow$ Pole-zero plot in s-Plane:



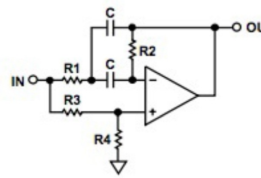
$\sqrt{2}\cdot 10^5\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j\right)$ rad/s

$\sqrt{2}\cdot 10^5\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j\right)$ rad/s

$\sqrt{2}\cdot 10^5\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j\right)$ rad/s

$\sqrt{2}\cdot 10^5\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}j\right)$ rad/s

Multiple feedback All-pass filter (general form)



$$H(s) = \frac{R_4}{R_3 + R_4} \cdot \frac{s^2 - s\frac{2}{R_2C} + \frac{1}{R_1R_2C^2}}{s^2 + s\frac{2}{R_2C} + \frac{1}{R_1R_2C^2}}$$

Must satisfy: $\frac{R_2R_3}{R_1R_4} = 4$

For given frequency ω_0, Quality factor Q and chosen value C R_1 & R_2:

Must satisfy: $R_1 = \frac{1}{2Q\omega_0 C}$ $\quad R_2 = \frac{2Q}{\omega_0 C}$

$\Rightarrow$ filter Gain $g = \frac{R_4}{R_3 + R_4} = \frac{Q^2}{1 + Q^2}$