Disclaimer:

- I can neither guarantee correctness nor completeness.
- for suggestions, reach me under msteinkel@ethz.ch

this summary is just a commented version of Leandro Treu's summary, with additional stuff about Amplifier design and smith-charts.

HFDT Summary

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HS2022 - Prof. Bolognesi

version of 25.12.22

2 General

HF: $300Mhz - 3GHz$ $\lambda: 1m - 1mm \rightarrow$ distributed circuits
$\lambda = \frac{c}{f} = \frac{c_0}{\sqrt{\epsilon_r}f}$; $\varepsilon_0 = 8{,}85 \cdot 10^{-12}\frac{As}{Vm}$; $\mu_0 = 1{,}26\cdot 10^{-6}\frac{Vs}{Am}$

consider stuff as HF $\Leftrightarrow$ $l < 0{,}1\lambda$

2.1 Skin effect

$\delta_s = \frac{1}{\sqrt{\pi f \mu_0 \mu_r \sigma}}$, $\sigma = conductivity = \frac{1}{\rho}$
Cylindrical conductor: $R_{DC} = \frac{l}{\pi r^2 \sigma}$ $R_{AC} = \frac{l}{2\pi r \delta_s \sigma}$
$\frac{R_{AC}}{R_{DC}} = \frac{r}{2\delta_s}$

Skin affect is apparent for $\delta_s \leq r$

2.2 Ohm's Law

What	Symbol
Resistance	R
Reactance	X
Impedance	$\boxed{Z = R + j \cdot X}$
Conductance	$G = \frac{Re(Z)}{Re(Z)^2 + Im(Z)^2}$
Susceptance	$B = \frac{-Im(Z)}{Re(Z)^2+Im(Z)^2}$
Admittance	$\boxed{Y = \frac{1}{Z} = G + j \cdot B}$

Attention: $G + jB = \frac{1}{R + jX}$ but in general $G \neq 1/R$

	Imp.[Z]	Adm.[Y]	Differential	Energy
Res	R	$\frac{1}{R}$	$U = R \cdot I$	-
Cap	$\frac{1}{j\cdot\omega\cdot C}$	$j\cdot\omega\cdot C$	$i(t) = C \cdot \frac{du(t)}{dt}$	$\frac{1}{2}CU^2$
In	$j \cdot \omega \cdot L$	$\frac{1}{j\cdot\omega\cdot L}$	$u(t) = L \cdot \frac{di(t)}{dt}$	$\frac{1}{2}LI^2$

2.3 Decibels & Neper

$\frac{P}{P_0}(dB) = 10 \cdot log(\frac{P}{P_0}) = 20 \cdot log(\frac{V}{V_0})$
$dB \rightarrow P_0 = 1W, dBm \rightarrow P_0 = 1mW$ $\rightarrow P(dBm) = 10\log\left(\frac{P(mW)}{1mW}\right)$
Neper: $\frac{P}{P_0}(Np) = \frac{1}{2} \cdot ln(\frac{P}{P_0})$
$1Np = 10log(e^2) = 8.686dB$ $\hookrightarrow \frac{V}{V_0}(Np) = ln(\frac{V}{V_0})$

2.4 Power

Only real power is dissipated $p(t) = u(t) \cdot i(t)$
$P_{peak} = Re(VI^*) = VI^*cos(\theta)$
$P_{avg} = \frac{1}{2}Re(VI^*) = \frac{1}{2}VI^*cos(\theta)$
Maximum Power Transfer: $Z_{Load} = Z^*_{Generator}$

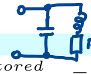
2.5 Loss in AC circuits

Insertion Loss: IL $= 10\log(\frac{P_{L1}}{P_{L2}})$ dB
$= \frac{Power-without-2-port}{Power-with-2-port}$ dB $\rightarrow P_A$: max. power available at load if you do $Z_L = Z_G^*$
Transducer Loss TL $= 10\log(\frac{P_A}{P_{L2}})$ dB
$= \frac{Max.-Power-available}{Power-with-2-port}$ dB
Series: TL $= |1 + \frac{Z}{2Z_0}|^2$ Parallel: TL $= |1 + \frac{Y}{2Y_0}|^2$

$\rightarrow$ see appendix for more TL-stuff!

2.6 Resonators

Resonance Frequency: Admittance equal zero.
A circuit with several L and C has several resonance freq.
normal: $\omega_0 = \frac{1}{\sqrt{LC}}$

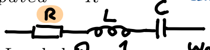


this would be a series resonator, since R is in series with a reactive element

2.7 Quality Factor

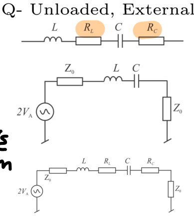
$Q = \frac{avergaeenergystored}{averageenergydissipated} = \frac{X}{R}$

Series Resonator



$Q = \frac{1}{\omega_0 C R} = \frac{\omega_0 L}{R}$

Q- Unloaded, External and Loaded



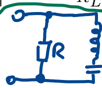
$Q_U = \frac{\omega_0 L}{R_L + R_C} = \frac{1}{\omega_0 C(R_L + R_C)}$

$Q_E = \frac{\omega_0 L}{2Z_0} = \frac{1}{\omega_0 C(2Z_0)}$

$\uparrow$ series

$Q_L = \frac{\omega_0 L}{R_L + R_C + 2Z_0}$

$\frac{1}{Q_L} = \frac{1}{Q_U} + \frac{1}{Q_E}$



this would be a parallel resonator, since R is in parallel with a reactive element

2.7.1 Parallel Resonator

$Q = \frac{1}{\omega_0 LG} = \frac{\omega_0 C}{G}$ Parallel: see appendix

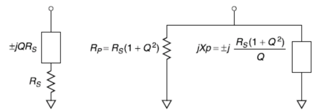
2.7.2 Bandwidth $\rightarrow$ see appendix

$Q = \frac{f_0}{\Delta f} = \frac{\omega_0}{\Delta\omega}$; $B = \frac{1}{Q}$ for high Q only $(B \cdot Q = 1)$!
higher Q means narrower Bandwidth! $\hookrightarrow$ valid for $Q > 10$
$B = \frac{1}{Q}$: fractional BW (no units!) $\rightarrow BW = f_0 B = \frac{f_0}{Q}$
$\hookrightarrow [BW] = Hz$ $\hookrightarrow$ if $Q > 10$

2.7.3 Fano's Limit

Matching narrowband, only works for the designed frequency. Fano's limit calculates the minimum obtainable Γ_{MIN} over the selected bandwith:
$\omega_0 = \sqrt{\omega_1\omega_2}$, $f_0 = \frac{\omega_0}{2\pi}$, $\Delta f = \frac{\omega_2 - \omega_1}{2\pi}$
$\Gamma_{MIN} \geq e^{-(\frac{\pi}{Q})(\frac{f_0}{\Delta f})}$
$\hookrightarrow$ target Q

2.8 Series to parallel equivalent circuit



$R_P = R_S(1 + Q^2)$; $X_P = \frac{R_S(1+Q^2)}{Q} = \frac{R_P}{Q}$

$Z = R_S + jX = R_S(1 + jQ)$
$Y = \frac{1 - jQ}{R_S(1+Q^2)} = G - jB$

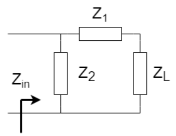
3 Q-Matching

To increase bandwith use multiple stages with lower Q use n sections: $1 + Q^2 = \sqrt[n]{R} \rightarrow$ do the transformation n times
$\hookrightarrow$ see appendix

3.1 Low to high Resistance if $Re(Z_L) < R_{goal}$

R_P = target Resistance; $R_S = R_L$

1. Find target Q ($R_P = R_S(1 + Q^2)$); $Q = \sqrt{\frac{R_P}{R_S} - 1}$
2. Add L or C (X_{LC}) in series (X_S is the target reactance and we want to archieve it with the help of the series reactance)
$\rightarrow X_{LC} + X_L \stackrel{!}{=} \pm X_S = \pm Q \cdot R_S$
If $X_S = \pm X_L$ no L or C element has to be added.
3. Convert to parallel($X_P = \frac{R_s(1+Q^2)}{Q}$, keep sign)
4. Add C or L in parallel to cancel out (resonate) the reactive part (X_P)
5. Check input impedance!

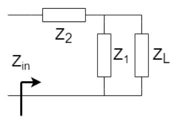


3.2 High to low Resistance if $Re(Z_L) > R_{goal}$

$Q_L = \left|\frac{X_L}{R_L}\right|$; R_S = target Resistance $\quad sgn(X_{PL}) = sgn(Im(Z_L))$

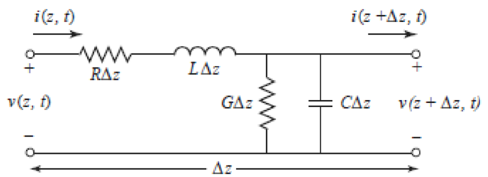
1. Convert to parallel representation
$R_{PL} = R_L(1 + Q_L^2)$; $X_{PL} = \frac{R_L(1+Q_L^2)}{Q_L}$ Sign of imag. part stays the same!
2. Find target Q: $Q = \sqrt{\frac{R_{PL}}{R_S} - 1}$
3. Add L or C (X_{LC}) in parallel,
$X_P = \pm\frac{R_{PL}}{Q}$ $\rightarrow$ $X_P \stackrel{!}{=} \frac{X_{PL}X_{LC}}{X_{PL}+X_{LC}}$
$\rightarrow X_{LC} = \frac{X_P X_{PL}}{X_{PL} - X_P}$
4. Convert to series representation $X_S = \pm Q \cdot R_S$ (keep the sign)
5. Add L or C in series to cancel out (resonate) the reactive part (X_S)
6. Check input impedance!

$Z_0 \in \mathbb{R}$ iff $\frac{R}{G} = \frac{L}{C}$ or $R = G = 0$ or $L = C = 0$



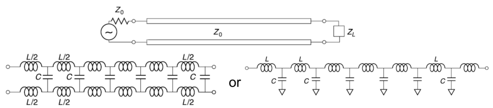
4 Transmission Line

- Device/circuit dimensions are in the same order as λ $(l > \frac{\lambda}{10})$
- Lumped Elements are not used at HF because of parasitic behaviour
- One piece of transmission line with length Δz can be modeled as a lumped element circuit $\rightarrow$ TL = periodic circuit of infinitely many of those elements



Characteristics:

1. Uniform Crosssection
2. Separation between the two conductors $<< \lambda$
3. Characteristic Impedance Z_0 (= voltage to current ratio traveling in a direction)
4. distributed behaviour



4.1 Telegraphers equations & some formulas

Voltage and Current are not constant, it is a superposition of a forward $\rightarrow$ and reflected $\leftarrow$ wave. We get a standing wave pattern. For $\Delta z \rightarrow 0$: $\frac{\delta V}{\delta z} = -(R + j\omega L) \cdot I$;
$\frac{\delta I}{\delta z} = -(G + j\omega C) \cdot V$
Current: $I = I_I e^{-\gamma z} - I_R e^{\gamma z} = \frac{1}{Z_0}(V_I e^{-\gamma z} - V_R e^{\gamma z})$
Voltage: $V = V_I e^{-\gamma z} + V_R e^{\gamma z} = V_I e^{-\gamma z}(1 + \Gamma_L e^{2\gamma z})$
propagation const: $\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$
Phase constant: $\beta = \frac{rad}{m}$; $\lambda = \frac{2*\pi}{\beta}$; $\beta z = \omega t$
attenuation constant: $\alpha = \frac{Np}{m}$ low loss $\rightarrow \alpha \approx \frac{R}{2Z_0}$
The attenuation const. α defines the loss of the TL often given in dB.
$Z_0 = \frac{V_I}{I_I} = \frac{V_R}{I_R} = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$ complex!
Wavelength: $\lambda = \frac{2\pi}{\beta}$

$\boxed{V_R = \Gamma_L V_I \; ; I_R = \Gamma_L I_I}$

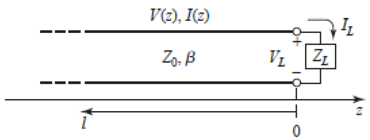
4.2 Lossless Line

$\beta = \omega\sqrt{LC}$, $\alpha = 0$, $Z_0 = \sqrt{\frac{L}{C}}$
Incident + Reflected wave:
Current: $I = I_I e^{-j\beta z} - I_R e^{j\beta z} = I_I e^{-j\beta z}[1 - \Gamma_L e^{2j\beta z}]$
Voltage: $V = V_I e^{-j\beta z} + V_R e^{j\beta z} = V_I e^{-j\beta z}[1 + \Gamma_L e^{2j\beta z}]$
Phase caused by a TL: $\theta = \beta l = \frac{2\pi}{\lambda}l = \frac{2\pi}{\nu_p}l = \frac{\omega l}{\nu_p}$
if lossy, replace β with γ

4.3 Phase & Group Velocities

phase velocity: $\nu_p = \frac{\omega}{\beta} = \lambda f = \frac{c_0}{\sqrt{\epsilon_r}} = \frac{1}{\sqrt{LC}}$
$e^{j(\omega t - \beta z)} = e^{j\omega(t - z/\nu_p)}$
group velocity: $\nu_G = \frac{d\omega}{d\beta} = \frac{1}{1 - (\frac{\omega}{\nu_p})(\frac{d\nu_p}{d\omega})}$

4.4 Reflection Coefficient



$\Gamma = |\Gamma|\angle\phi = |\Gamma|e^{j\phi}$
Derivation at z=0 (load):
$V(0) = V_I(0) + V_R(0) = I(0) \cdot Z_L$
$I(0) = I_I(0) - I_R(0) = \frac{1}{Z_0}(V_I(0) - V_R(0))$
$V_I(0) + V_R(0) = \frac{1}{Z_0}(V_I(0) - V_R(0)) \cdot Z_L$
$\Rightarrow V_R(0) = \frac{Z_L - Z_0}{Z_L + Z_0}V_I(0)$
$\Gamma(z) = \frac{V_R(z)}{V_I(z)} = \frac{I_R(z)}{I_I(z)} = \Gamma(0)e^{2j\beta z}$ (lossless) $\quad \Gamma(0) = \Gamma_L$
$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$
$\Gamma_{in} = \frac{Z_{in} - Z_G}{Z_{in} + Z_G} = \Gamma_L e^{-2\gamma l} == \Gamma_L e^{-2l(\alpha + j\beta)}$
No reflection $\rightarrow \Gamma = 0$
$\Gamma_g = \frac{Z_g - Z_0}{Z_g + Z_0}$

$V(z) = V_I(z)(1+\Gamma(z))$
$I(z) = I_I(z)(1-\Gamma(z))$

$P_{avg} = \frac{1}{2}\frac{|V_{I0}|^2}{Z_0}(1-|\Gamma(0)|^2)$ sign depending on convention
Return Loss RL $= \pm 10\log(|\Gamma|^2)$ dB , $|\Gamma| = 10^{\pm\frac{RL}{20}}$
(the fraction of power reflected from the load)
Mismatch loss $= \pm 10\log(1-|\Gamma|^2)$ dB
(the fraction of power absorbed at the load)

$|V(z)| = |V_I(z)+V_R(z)| = |V_{I0}||e^{-j\beta z}+\Gamma(0)e^{j\beta z}| = |V_{I0}||e^{-j\beta z}||1+\Gamma(0)e^{2j\beta z+\theta_L}|$

on smithchart : VSWR = z_{Max} → proof in appendix

4.5 Voltage Standing Wave Ratio (VSWR)

Max. Voltage of standing wave: $V_{max} = |V_I|(1+|\Gamma|)$
Min. Voltage of standing wave: $V_{min} = |V_I|(1-|\Gamma|)$
VSWR $= \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$ maxima $\Leftrightarrow e^{2j\beta z+\theta_L} = 1$
$\Gamma = \frac{VSWR-1}{VSWR+1} = 10^{\frac{-RL(dB)}{20}}$ minima $\Leftrightarrow e^{2j\beta z+\theta_L} = -1$

- Distance between maxima (minima) is $\frac{\lambda}{2}$
- Distance between maxima and minima is $\frac{\lambda}{4}$

Just for lossless:

- Because voltage and current on the line aren't constant, impedance looking into the transmission line will vary with position x: $Z(x) = \frac{V(x)}{I(x)} = Z_0\frac{1+\Gamma(x)}{1-\Gamma(x)}$
- Since $\Gamma(x) = \frac{Z(x)-Z_0}{Z(x)+Z0}$ and $\Gamma_L = \frac{Z_L-Z_0}{Z_L+Z0}$ we get $Z(x) = Z_0\frac{Z_L+jZ_0tan\beta x}{Z_0+jZ_Ltan\beta x}$

4.6 Input Impedance

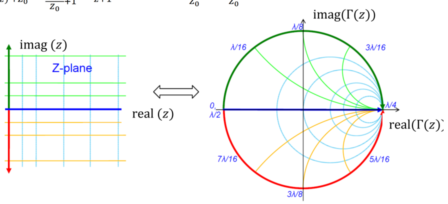
- At input z = -1 we get: $\boxed{Z_{in} = Z_0\frac{Z_L+jZ_0tan\theta}{Z_0+jZ_Ltan\theta}}$ $\theta = \beta l = \frac{2\pi}{\lambda}l$
- with loss: $Z_{in} = Z_0\frac{Z_L+Z_0tanh\gamma l}{Z_0+Z_Ltanh\gamma l}$

Special Cases (lossless):

- impedance matched: $Z_L = Z_0 \rightarrow Z_{in} = Z_0$
- $\boxed{\text{lambda half: } l = n\frac{\lambda}{2} \rightarrow Z_{in} = Z_L}$
- $\boxed{\text{lambda quarter: } l = \frac{\lambda}{4} \rightarrow Z_{in} = \frac{Z_0^2}{Z_L}}$ → see Quarter wave transformer
- shorted TL: $Z_{in} = jZ_0tan\theta \rightarrow$ inductive for $\theta < 90$, capacitive for $90 < \theta < 180$
- open TL: $Z_{in} = -jZ_0cot\theta \rightarrow$ capacitive for $\theta < 90$, inductive for $90 < \theta < 180$

5 Smith Chart

$\Gamma(z) = \frac{Z(z)-Z_0}{Z(z)+Z_0} = \frac{\frac{Z(z)}{Z_0}-1}{\frac{Z(z)}{Z_0}+1} = \frac{z-1}{z+1}$ where $z = \frac{Z(z)}{Z_0} = \frac{R+jX}{Z_0} = r + x$



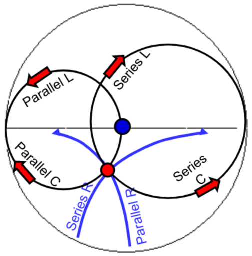
Addmitance Smith chart: $\Gamma = \frac{1-y}{1+y}$

- At a fixed frequency, movement from the load toward the generator results in a clockwise rotation on the Smith Chart.
- As frequency increases, reflection coefficients always rotate clockwise

5.1 Matching

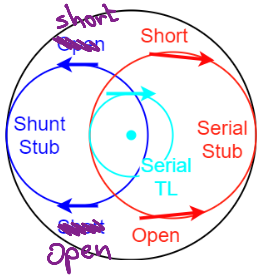
- There are two types:
  1. impedance matching (no reflection)
  2. complex conjugate matching (max power is delivered to the load)
- Matching can be done with:
  - Lumped LC elements
  - Sections of Transmission line (stub with open or short circuit, series TL)
  - Mix of both
- Matching narrowband, only works for the designed frequency. Fano's limit calculates the minimum obtainable Γ_{MIN} over the selected bandwidth: $\Gamma_{MIN} \geq e^{-(\frac{\pi}{Q})(\frac{f_0}{\Delta f})}$

5.2 LC Matching



High to Low: first Parallel (stub) then Serial
Low to High: first Serial then Parallel (stub)

5.3 Transmission Line (TL) Matching



5.3.1 Serial TL

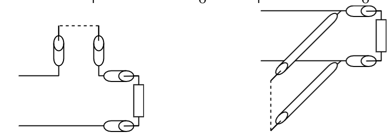
- Making TL to the load longer → rotate around center point in clockwise direction on const. Γ (radius) → lossless.
- If Loss is respected the line spirals towards center (matched point) of the smith chart.
- $Z_0 = \sqrt{Z_{max}Z_{min}}$

Match any complex load to any complex generator:
We want: $Z_{in} = R_G - jX_G \stackrel{!}{=} Z_0\frac{Z_L+jZ_0tan\theta}{Z_0+jZ_Ltan\theta} \rightarrow$ solve for complex Z_0; often $\theta = \beta l$ (lossless)

5.3.2 Stub TL

theoratically both (open or short) can be used as cap/induc. but the TL would have to be $> \pi/2$ and therefor it takes more area
$\beta = \frac{2\pi}{\lambda}$

	open circuit	short ciruit
Impedance	$Z = -jZ_0cot\beta l$	$Z = jZ_0tan\beta l$
Admittance	$Y = \frac{jtan\beta l}{Z_0}$	$Y = \frac{-jcot(\beta l)}{Z_0}$

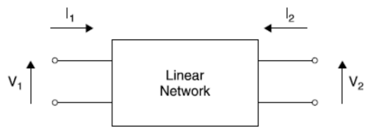


Serial Stub (impedance) Shunt Stub (admittance)

5.4 Quarter wave transformer see appendix

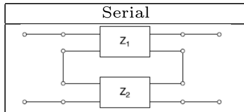
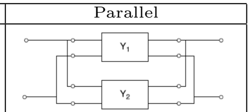
TL with a length of $\frac{\lambda}{4}$ and a characteristic impedance which has to be defined.
deriviation:
$Z_{in} = Z_0\frac{Z_L+jZ_0tan\beta l}{Z_0+jZ_Ltan\beta l} = Z_0\frac{Z_0}{Z_L}$ for $l = \frac{\lambda}{4}$
$\rightarrow Z_{in} = \frac{Z_0^2}{Z_L} \Rightarrow Z_0 = \sqrt{Z_{in}Z_L}$
to make the transformation more wideband by doing the transformation in $n\frac{\lambda}{4}$ steps, each providing a $\sqrt[n]{\frac{Z_L}{Z_{in}}}$ transformation.

6 Matrix Network Analysis



$V_1 = Z_{11}I_1 + Z_{12}I_2$
$V_2 = Z_{21}I_1 + Z_{22}I_2$
Impedance: $\begin{pmatrix}V_1\\V_2\end{pmatrix} = \begin{pmatrix}Z_{11} & Z_{12}\\Z_{12} & Z_{22}\end{pmatrix}\begin{pmatrix}I_1\\I_2\end{pmatrix}$
$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$ $Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$
$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$ $Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$
Admittance: $\begin{pmatrix}I_1\\I_2\end{pmatrix} = \begin{pmatrix}Y_{11} & Y_{12}\\Y_{12} & Y_{22}\end{pmatrix}\begin{pmatrix}V_1\\V_2\end{pmatrix}$
$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$ $Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$
$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$ $Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$

6.1 Serial and Parallel

Serial	Parallel
	
$Z_T = Z_1 + Z_2$	$Y_T = Y_1 + Y_2$

6.2 Special Networks

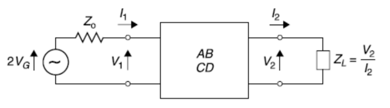
6.2.1 Reciprocal Network

Its elements are bilateral and linear (not containing any active devices, ferrites, plasma..) $Z_{12} = Z_{21}$; $Y_{12} = Y_{21}$

6.2.2 Symmetric Network

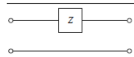
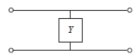
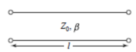
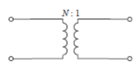
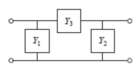
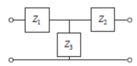
$Z_{11} = Z_{22}$; $Z_{12} = Z_{21}$

6.3 ABCD (transmission) Matrix



$$\boxed{A = \frac{V_1}{V_2}\Big|_{I_2=0} \quad B = \frac{V_1}{I_2}\Big|_{V_2=0} \quad C = \frac{I_1}{V_2}\Big|_{I_2=0} \quad D = \frac{I_1}{I_2}\Big|_{V_2=0}}$$

ABCD matric of the cascade connection of several two port network can be found just by multiplying the ABCD matrices of individual two ports:
$\begin{pmatrix}A_T & B_T\\C_T & D_T\end{pmatrix} = \begin{pmatrix}A_1 & B_1\\C_1 & D_1\end{pmatrix}\cdot\begin{pmatrix}A_2 & B_2\\C_2 & D_2\end{pmatrix} = \begin{pmatrix}(A_1A_2+B_1C_2) & (A_1B_2+B_1D_2)\\(C_1A_2+D_1C_2) & (C_1B_2+D_1D_2)\end{pmatrix}$
Reciprocal: $AD - BC = 1$
Symmetrical: $A = D$
Lossless: $|S_{11}|^2 + |S_{21}|^2 = 1$ and $|S_{21}|^2 + |S_{22}|^2 = 1$

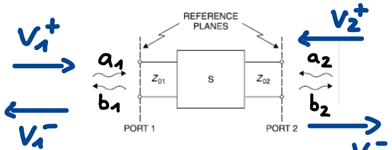
Circuit	*ABCD* Parameters	
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos\beta\ell$ $C = jY_0\sin\beta\ell$	$B = jZ_0\sin\beta\ell$ $D = \cos\beta\ell$
	$A = N$ $C = 0$	$B = 0$ $D = \frac{1}{N}$
	$A = 1 + \frac{Y_2}{Y_3}$ $C = Y_1 + Y_2 + \frac{Y_1Y_2}{Y_3}$	$B = \frac{1}{Y_3}$ $D = 1 + \frac{Y_1}{Y_3}$
	$A = 1 + \frac{Z_1}{Z_3}$ $C = \frac{1}{Z_3}$	$B = Z_1 + Z_2 + \frac{Z_1Z_2}{Z_3}$ $D = 1 + \frac{Z_2}{Z_3}$

Input Impedance:
$V_1 = AV_2 + BI_2$; $I_1 = CV_2 + DI_2$
$Z_{in} = \frac{AV_2+BI_2}{CV_2+DI_2} = \frac{A(V_2/I_2)+B}{C(V_2/I_2)+D} = \frac{AZ_L+B}{CZ_L+D}$
Insertion Loss: $IL = \frac{1}{4}|A + \frac{B}{Z_0} + CZ_0 + D|^2$ ($Z_L = Z_0 = real$)

6.4 Scattering Matrix (S-Parameters)

S_{ii} is the reflection coefficient seen looking into port i when all other ports are terminated in matched loads
S_{ij} is the transmission coefficient from port j to port i when all other ports are matched loads
Instead of measuring voltages and currents at the ports (for Z,Y, ABCD), S parameters are obtained by measuring intensities of the incident and reflected waves (at certain conditions: matching is achieved!)

Reciprocal: $[S] = [S]^T \rightarrow$ symmetric
Lossless: $|S_{11}|^2 + |S_{21}|^2 = 1$,
transmitted power: $|S_{12}|^2 = |S_{21}|^2$
dissipated power = 1 - transmitted power



b_i: reflected wave, a_i: incident wave

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$a_1 = \frac{V_{I1}}{\sqrt{Z_{01}}}$ $b_1 = \frac{V_{R1}}{\sqrt{Z_{01}}}$

$a_2 = \frac{V_{I2}}{\sqrt{Z_{02}}}$ $b_2 = \frac{V_{R2}}{\sqrt{Z_{02}}}$

Reflection:

$$S_{11} = \frac{b_1}{a_1}\Big|_{a_2=0} = \frac{V_{R1}}{V_{I1}}\Big|_{V_{I2}=0} = \Gamma_1\Big|_{V_{I2}=0}$$

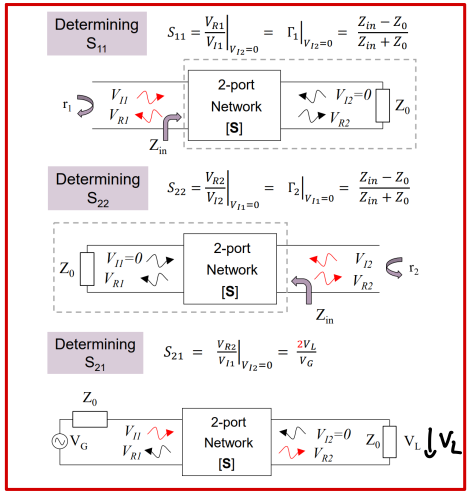
$$S_{22} = \frac{b_2}{a_2}\Big|_{a_1=0} = \frac{V_{R2}}{V_{I2}}\Big|_{V_{I1}=0} = \Gamma_2\Big|_{V_{I1}=0}$$

Transmission:

$$S_{21} = \frac{b_2}{a_1}\Big|_{a_2=0} = \frac{V_{R2}}{V_{I1}}\Big|_{V_{I2}=0}$$

$$S_{12} = \frac{b_1}{a_2}\Big|_{a_1=0} = \frac{V_{R1}}{V_{I2}}\Big|_{V_{I1}=0}$$

Even though Z, Y, and ABCD parameters cannot be practically measured at RF frequencies they can be obtained by conversion from the measured S parameters



S_{12} is similar to S_{21}

$$S_{21} = \frac{b_2}{a_1}\Big|_{a_2=0} = \frac{V_2^-}{V_1^+}\Big|_{Z_0 at port 2}$$

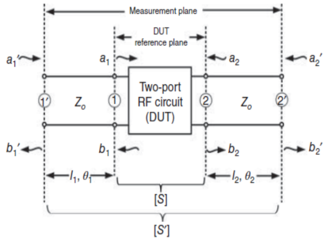
often: $V_2^- = V_2$ when the load is matched; $V_1^+ = V_1 - S_{11}V_1^+ \rightarrow$ solve for V_1^+

6.4.1 S parameters of a TL

$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} 0 & e^{-\gamma l} \\ e^{-\gamma l} & 0 \end{pmatrix}$$

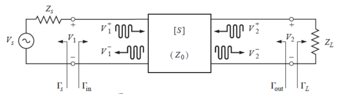
6.4.2 Problems when determining S parameters

- Measurements of individual circuits, components or devices often cannot be done directly at individual ports
- As a result, the actual measurement at reference planes are different from those of the interested RF device under test (DUT)
- However, S parameters of a DUT can be obtained from the measured S parameters with a simple calculation: shifting of the reference planes



$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} S'_{11}e^{j2\theta_1} & S'_{12}e^{j(\theta_1+\theta_2)} \\ S'_{21}e^{j(\theta_1+\theta_2)} & S'_{22}e^{j2\theta_2} \end{pmatrix}$$

mismatched load



$$\Gamma_L = \frac{a_2}{b_2} = \frac{V^+_2}{V^-_2} = \frac{Z_L - Z_0}{Z_L + Z_0} \qquad \Gamma_{out} = \frac{b_2}{a_2} = \frac{V^-_2}{V^+_2} = \frac{1}{\Gamma_L} = \frac{Z_{out} - Z_0}{Z_{out} + Z_0}$$

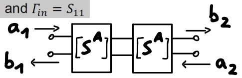
$$\Gamma_S = \frac{a_1}{b_1} = \frac{V^+_1}{V^-_1} = \frac{Z_S - Z_0}{Z_S + Z_0} \qquad \Gamma_{in} = \frac{b_1}{a_1} = \frac{V^-_1}{V^+_1} = \frac{1}{\Gamma_S} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$\begin{cases} V_1^- = S_{11}V_1^+ + S_{12}V_2^+ = S_{11}V_1^+ + S_{12}\Gamma_L V_2^-, \\ V_2^- = S_{21}V_1^+ + S_{22}V_2^+ = S_{21}V_1^+ + S_{22}\Gamma_L V_2^-. \end{cases} \rightarrow \begin{cases} \Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} \\ \Gamma_{out} = \frac{V_2^-}{V_2^+} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S} \end{cases}$$

If $Z_L = Z_0$ and $Z_S = Z_0 \longrightarrow \begin{cases} \Gamma_L = \Gamma_S = 0 \\ \Gamma_{out} = S_{22} \text{ and } \Gamma_{in} = S_{11} \end{cases}$

cascaded Networks

↳ ex. 10+ - Task 1



$$\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = \begin{pmatrix} S_{11}{}^A + S_{11}{}^B \cdot \frac{S_{12}{}^A S_{21}{}^A}{1 - S_{22}{}^A S_{11}{}^B} & S_{12}{}^A \cdot \frac{S_{12}{}^B}{1 - S_{22}{}^A S_{11}{}^B} \\ S_{21}{}^A \cdot \frac{S_{21}{}^B}{1 - S_{22}{}^A S_{11}{}^B} & S_{22}{}^B + S_{22}{}^A \cdot \frac{S_{12}{}^B S_{21}{}^B}{1 - S_{11}{}^B S_{22}{}^A} \end{pmatrix}$$

7 Amplifier Design

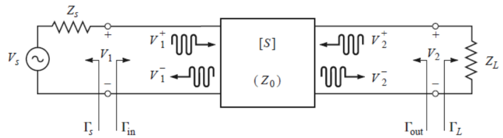
7.1 Stability

$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$; $\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}$ Oscillation is possible if either the input or output port reflection coefficient has a magnitude larger than 1

7.1.1 Types of stability

unconditional stability:
The network is unconditionally stable if $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ for all passive source and load impedances ($|\Gamma_s < 1|$ and $|\Gamma_L| < 1$)
conditional stability:
The network is conditionally stable if $|\Gamma_{in}| < 1$ and $|\Gamma_{out}| < 1$ only for a certain range of passive source and load impedances. This case is also referred to as potentially unstable

7.1.2 Determining Stability



$$|\Gamma_{in}| = |S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}| \overset{!}{<} 1$$

$$|\Gamma_{out}| = |S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}| \overset{!}{<} 1$$

7.1.3 Stability Circles

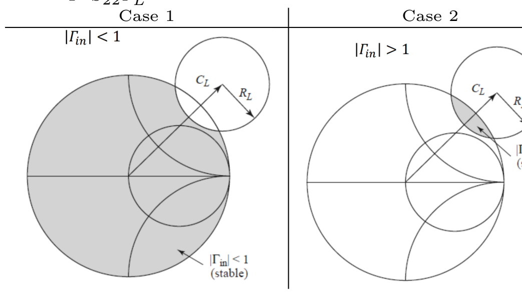
Output Stability circle: ($\Gamma_{out} = 1$)
Center: $C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2}$
Radius: $R_L = \left|\frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2}\right|$
Input Stability circle: ($\Gamma_{in} = 1$)
Center: $C_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2}$
Radius: $R_S = \left|\frac{S_{12}S_{21}}{|S_{11}|^2 - |\Delta|^2}\right|$

Analyzing stability circles:

- We find the output stability circle
- The center of the Smith Chart is Z_0
- Consider the load to be $Z_L = Z_0 \rightarrow \Gamma_L = 0$

Case 1: if $|S_{11}| < 1$ then from $\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$; $|\Gamma_{in}| < 1$

Case 2: if $|S_{11}| > 1$ then from $\Gamma_{in} = \frac{V_1^-}{V_1^+} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$; $|\Gamma_{in}| > 1$



7.1.4 Test for unconditional Stability (K-Δ Test)

③

$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|} > 1$
$|\Delta| = |S_{11}S_{22} - S_{12}S_{21}| < 1$
$\Longrightarrow$ unconditionally stable

both have to be satisfied simultaneously, if the condition is not satisfied, stability circles should be constructed for the designed frequency and input and output matching network should be designed away from the unstable regions

7.2 Gain

For a two-port network characterized by its S parameter matrix and source and load impedances Z_S and Z_L we can define three types of gain:

- **Voltage Gain**: $A_v = \frac{S_{21}\Gamma_L + S_{21}}{1 - S_{22}\Gamma_L + S_{11}(1 - S_{22}\Gamma_L) + S_{21}\Gamma_L S_{12}}$
- **Power gain** is the ratio of power dissipated in the load to the power delivered to the two-port network $G_P = \frac{P_L}{P_{in}} = \frac{|S_{21}|^2(1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)|1-S_{22}\Gamma_L|^2}$
- **Available power gain** is the ratio of the power available from the two-port

$$G_A = \frac{P_{avn}}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_S|^2)}{(1-|\Gamma_{out}|^2)|1-S_{11}\Gamma_S|^2}$$

7.2.1 Transducer Power Gain

G_T is the ratio of the power delivered to the load to the power available from the source

$G_T = \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_{in}\Gamma_S|^2|1-S_{22}\Gamma_L|^2}$ general

$G_{TU} = |S_{21}|^2 \frac{(1-|\Gamma_S|^2)}{|(1-S_{11}\Gamma_S)|^2} \frac{(1-|\Gamma_L|^2)}{|(1-S_{22}\Gamma_L)|^2}$ unilateral

- **Input matching gain**: $G_S = \frac{(1-|\Gamma_S|^2)}{|1-\Gamma_{in}\Gamma_S|^2}$, if transistor is unilateral then $S_{12} = 0$ and

$$G_S = \frac{(1-|\Gamma_S|^2)}{|1-S_{11}\Gamma_S|^2}$$

- **Transistor gain**: $G_0 = |S_{21}|^2$
- **Output matching gain**: $G_L = \frac{(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2}$

For unilateral transistor:
$G_{Smax} = \frac{1}{1-|S_{11}|^2}$ ($\Gamma_S = S_{11}^*$), $g_S = \frac{G_S}{G_{Smax}}$
$G_{Lmax} = \frac{1}{1-|S_{22}|^2}$ ($\Gamma_L = S_{22}^*$), $g_L = \frac{G_L}{G_{Lmax}}$
Constant input gain circle: $C_S = \frac{g_S S_{11}^*}{1-(1-g_S)|S_{11}|^2}$,
$R_S = \frac{\sqrt{1-g_S}(1-|S_{11}|^2)}{1-(1-g_S)|S_{11}|^2}$
Constant output gain circle: $C_L = \frac{g_L S_{22}^*}{1-(1-g_L)|S_{22}|^2}$,
$R_L = \frac{\sqrt{1-g_L}(1-|S_{22}|^2)}{1-(1-g_L)|S_{22}|^2}$

7.3 Noise

- Unbiased resistor generates a random thermal noise voltage $P_n = (\frac{V_n}{2R})^2; R = \frac{V_n^2}{4R} = kTB$; $V_n = \sqrt{4kTBR}$
- The measure of signal degradation in the signal-to-noise ration between input and output is called the noise figure: $F = \frac{S_I/N_I}{S_O/N_O}$ „F = noise"
- Noise figure of the cascaded system is determined by the Friis formula: $F = F_1 + \frac{F_2-1}{G_1} + \frac{F_3-1}{G_1G_2}$...

- The noise figure of a two port amplifier can be expressed as: $\boxed{F = F_{min} + \frac{R_N}{G_S}|Y_S - Y_{S,opt}|}$
  - source admittance: $Y_s = G_S + jB_S$
  - source admittance that results in the minimum noise figure $Y_{s,opt}$
  - minimum noise figure when admittance is $Y_{s,opt}$: F_{min}
  - equivalent noise resistance: R_N
- instead of $Y_s, Y_{s,opt}$ we can use $\Gamma_S, \Gamma_{S,opt}$: $\boxed{F = F_{min} + \frac{4R_N}{Z_0}\frac{|\Gamma_S - \Gamma_{s,opt}|^2}{(1-|\Gamma_s|^2)|1+\Gamma_{s,opt}|^2}}$
- We can define a noise figure parameter N: $\boxed{N = \frac{|\Gamma_S - \Gamma_{s,opt}|^2}{1-|\Gamma_s|^2} = \frac{F - F_{min}}{4R_N/Z_0}|1+\Gamma_{s,opt}|^2}$
- circle solution: $\boxed{C_F = \frac{\Gamma_{opt}}{N+1},\ R_F = \frac{\sqrt{N(N+1-|\Gamma_{opt}|^2)}}{N+1}}$

7.4 Amplifier Design Techniques

→ see next page

7.4.1 Design for specific gain and best possible noise

1. Draw noise circles for several noise figures F close to F_{min}
2. Find the sum of the gains for the input and output matching networks
3. Take into account that maximum gain you can get from the output is $10log\frac{1}{1-|S_{22}|^2}$
4. Draw several G_S and choose which input matching intersects with the smallest noise circle
5. For a fixed G_S now you can calculate G_L and then draw G_L circle
6. Choose Γ_S and Γ_L where G_S and G_L circles intersect the least noise figure circle and are close as possible to center of the Smith chart

7.4.2 Design for specific noise and best possible gain

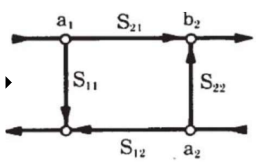
1. $\Gamma_L = \Gamma_{out}^*$ we can maximize it because noise figure does not depend on output. If unilateral : $\Gamma_L = S_{22}^*$
2. Draw the noise circle for desired F
3. Find maximum input matching circuit gain G_S for which the noise circle and input gain circle have 1 common point. That point will be the desired Γ_S
4. Amplifier gain is now:
$G_T(dB) = 10log|S_{21}|^2 + 10log\frac{1-|\Gamma_S|^2}{|1-\Gamma_{in}\Gamma_S|^2} + 10log\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$
if unilateral:
$G_{TU}(dB) = 10log|S_{21}|^2 + 10log\frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} + 10log\frac{1}{1-|S_{22}|^2}$

7.4.3 Design for minimum noise and best possible gain

1. $\Gamma_L = \Gamma_{out}^*$ if unilateral: $\Gamma_L = S_{22}^*$
2. Minimum noise occurs for $\Gamma_S = \Gamma_{s,opt}$
3. Amplifier gain is now:
$G_T(dB) = 10log|S_{21}|^2 + 10log\frac{1-|\Gamma_S|^2}{|1-\Gamma_{in}\Gamma_S|^2} + 10log\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$
if unliteral:
$G_{TU}(dB) = 10log|S_{21}|^2 + 10log\frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2} + 10log\frac{1}{1-|S_{22}|^2}$

7.5 Signal Flow Graphs (SFG)

- A SFG is a directed graph in which nodes represent system variables, and branches (edges, arcs, or arrows) represent functional connections between pairs of nodes.
- SFGs are used to represent the signal flow in electronic networks.
- The SFG here is used in association with our S-parameters!
- each variable (a_1, a_2, b_1, b_2) is marked at a node
- S-Params are marked as branches



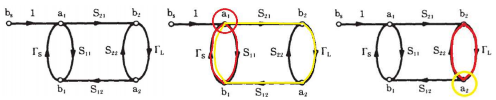
for ex: $b_2 = a_1S_{21} + a_2S_{22}$; $b_1 = a_1S_{11} + a_2S_{12}$

Mason's Rule states:

$$\frac{b_1}{b_s} = T = \frac{(P_1[1-\sum L(1)^{(1)}+\sum L(2)^{(1)}-\sum L(3)^{(1)}...]+P_2[1-\sum L(1)^{(2)}+\cdots]+\cdots)}{1-\sum L(1)+\sum L(2)-\sum L(3)+\cdots}$$

Where:

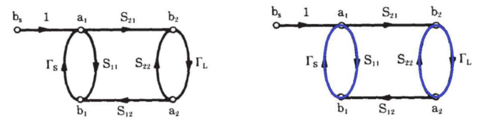
- $\sum L(1)^{(1)}$ is the sum of all **first order loops** that do not touch the first path between the variables
- $\sum L(2)^{(1)}$ is the sum of all **second order loops** that do not touch the first path between the variables
- $\sum L(1)^{(2)}$ is the sum of all **first order loops** that do not touch the second path between the variables
- The denominator is 1 – the sum of all first order loops, plus the sum of all second order loops, minus the third order loops, and so on…



- First Order Loop: is defined as the product of the branches encountered in a journey **starting from a node** and moving in the direction of the arrows **back to that original node.**

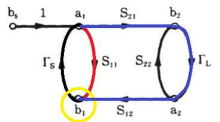
Example: Starting at node a_1: $S_{11}\Gamma_s$ $S_{21}\Gamma_L S_{12}\Gamma_s$
Starting at node a_2: $S_{22}\Gamma_L$

Any of the other loops we encounter includes one of these three first order loops!



Second Order Loop: is defined as the product of any **two non-touching first order loops.** Of the three first order loops just found only $S_{11}\Gamma_s$ and $S_{22}\Gamma_L$ do not touch.

Third Order Loop: is the product of any three non-touching first order loops. (no third order loop for this example)



Let's find the value of b_1. Since b_s is the only independent variable, its value will determine the value of all other variables in the network.

To determine b_1, we first have to identify the paths leading to b_1 from b_S:

- S_{11}
- $S_{21}\Gamma_L S_{12}$

Let's identify the non-touching loops with respect to the paths just found:

- $\boldsymbol{S_{11}}$ → First order loop $S_{22}\Gamma_L$ have no nodes or branches in common
- $\boldsymbol{S_{21}\Gamma_L S_{12}}$ touches all of the network's first order loop → no non-touching loops

It is now time to use **Mason's Rule** to determine the ratio of the variables b_1 and b_S.

Let's apply this rule to our case:

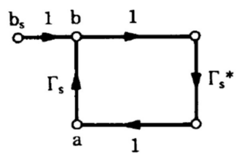
$$\frac{b_1}{b_s} = T = \frac{(S_{11}[1-S_{22}\Gamma_L]+S_{21}\Gamma_L S_{12}[1])}{1-(S_{11}\Gamma_S+S_{22}\Gamma_L+S_{21}\Gamma_L S_{12}\Gamma_S)+S_{11}\Gamma_S S_{22}\Gamma_L}$$

Although the example exploits a rather simple network, the expressions can become complicate.

Mason`s Rule provides a systematic approach to determine various transfer functions (usable for far more complex systems, as the error correction model for a VNA seen during the lecture).

Power Available from the Source

Power available from the source is the power delivered to a conjugate-matched load ($\Gamma_L = \Gamma_s^*$).



$P_{avs} = |b|^2 - |a|^2$

Applying **Mason's Rule**:

$b = \frac{b_s}{1-\Gamma_S\Gamma_S^*}$ $\quad a = \frac{b_s\Gamma_S^*}{1-\Gamma_S\Gamma_S^*}$

So:

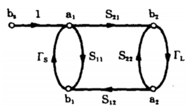
$$P_{avs} = \frac{|b_s|^2(1-|\Gamma_s|^2)}{(1-|\Gamma_s|^2)^2} = \frac{|b_s|^2}{1-|\Gamma_s|^2}$$

Voltage Gain

For a two-port network:

$$A_V = \frac{V_{out}}{V_{in}}$$

Recalling the total voltage on a Tline is $V = V^+ + V^-$:



$$\frac{a_1}{b_s} = \frac{1[1-S_{22}\Gamma_L]}{1-(S_{11}\Gamma_s+S_{22}\Gamma_L+S_{21}\Gamma_L S_{12}\Gamma_s)+S_{11}\Gamma_s S_{22}\Gamma_L}$$
$$\frac{b_1}{b_s} = \frac{S_{11}[1-S_{22}\Gamma_L]+S_{21}\Gamma_L S_{12}[1]}{1-(S_{11}\Gamma_s+S_{22}\Gamma_L+S_{21}\Gamma_L S_{12}\Gamma_s)+S_{11}\Gamma_s S_{22}\Gamma_L}$$
$$\frac{a_2}{b_s} = \frac{S_{21}\Gamma_L[1]}{1-(S_{11}\Gamma_s+S_{22}\Gamma_L+S_{21}\Gamma_L S_{12}\Gamma_s)+S_{11}\Gamma_s S_{22}\Gamma_L}$$
$$\frac{b_2}{b_s} = \frac{S_{21}[1]}{1-(S_{11}\Gamma_s+S_{22}\Gamma_L+S_{21}\Gamma_L S_{12}\Gamma_s)+S_{11}\Gamma_s S_{22}\Gamma_L}$$

$$A_V = \frac{a_2+b_2}{a_1+b_1} \Longrightarrow A_V = \frac{\frac{a_2}{b_s}+\frac{b_2}{b_s}}{\frac{a_1}{b_s}+\frac{b_1}{b_s}} = \frac{S_{21}+S_{21}\Gamma_L}{1(1-S_{22}\Gamma_L)+S_{11}(1-S_{22}\Gamma_L)+S_{21}\Gamma_L S_{12}}$$

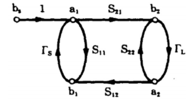
4

Transducer Power Gain

The **Transducer power gain** is defined as the power delivered to a load divided by the power available from a source.

$$G_T = \frac{P_{del}}{P_{avs}} = \frac{\boxed{|b_2|^2(1-|\Gamma_L|^2)}}{|b_s|^2/(1-|\Gamma_s|^2)}$$

$$\boxed{\begin{aligned}P_{del} &= P_{inc} - P_{refl}\\ &= |a|^2 - |b|^2\end{aligned}}$$



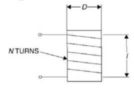
$$\frac{b_2}{b_s} = \frac{S_{21}[1]}{1-(S_{11}\Gamma_s+S_{22}\Gamma_L+S_{21}\Gamma_L S_{12}\Gamma_s)+S_{11}\Gamma_s S_{22}\Gamma_L}$$

$$G_T = \frac{|S_{21}|^2(1-|\Gamma_s|^2)(1-|\Gamma_L|^2)}{|(1-S_{11}\Gamma_s)(1-S_{22}\Gamma_L)-S_{21}\Gamma_L S_{12}\Gamma_s|^2} \xrightarrow{S_{12}\cong 0} G_{TU} = |S_{21}|^2\frac{(1-|\Gamma_s|^2)}{|(1-S_{11}\Gamma_s)|^2}\frac{(1-|\Gamma_L|^2)}{|(1-S_{22}\Gamma_L)|^2}$$

8 Apendix

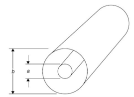
8.1 Designing Components

Spiral Coil:



$L = \frac{n^2r^2}{9r+10l}[\mu H] \rightarrow L = \frac{n^2r}{29}$, if $l = 2r$, for higher Q

Wire Inductance:



$L = \frac{\mu_0\mu_r}{2\pi}ln\frac{b}{a}[\frac{H}{m}] \rightarrow$ set $\frac{b}{a} = 50$ = distant ground plane

Plate Capacitor:
$C = \frac{\epsilon_0\epsilon_r A}{d}[F]$

8.2 Slotted T-line

1. A known load e.g. Short is connected:
   a) $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$, here: $Z_L = 0$
   b) $SWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \infty$
   c) Distance between 2 minima $= \frac{\lambda}{2}$
2. The unknown load is connected:
   a) $SWR = \frac{1+|\Gamma|}{1-|\Gamma|} \rightarrow |\Gamma|$
   b) $e^{j(\theta_L - 2\beta l)} = -1$; $l = l_{minshort} - l_{minload} > 0$
   c) $\Gamma_L = |\Gamma|e^{j\theta_L}$
   d) $Z_L = Z_0(\frac{1+\Gamma_L}{1-\Gamma_L})$

You can also solve from 2.a) with the help of the smith chart: just move "l" towards load.

8.3 Critical Length for Digital Interconnects

$l_c = \frac{t_r \cdot v}{2}$, t_r = rise time, v = wave speed
also used: $l_c = \frac{t_r \cdot v}{1.5}$

8.4 Transmission matrix

The T-matrices are multiplied in the same way as the ABCD parameters when networks are cascaded.

$$\begin{pmatrix}b_1\\a_1\end{pmatrix} = \begin{pmatrix}T_{11} & T_{12}\\T_{21} & T_{22}\end{pmatrix}\cdot\begin{pmatrix}a_2\\b_2\end{pmatrix}$$

$$\begin{pmatrix}T_{11} & T_{12}\\T_{21} & T_{22}\end{pmatrix} = \begin{pmatrix}\frac{S_{12}S_{21}-S_{11}S_{22}}{S_{21}} & \frac{S_{11}}{S_{21}}\\ \frac{-S_{22}}{S_{21}} & \frac{1}{S_{21}}\end{pmatrix}$$

reciprocal: $S_{12} = S_{21} \rightarrow T_{11}T_{22} - T_{12}T_{21} = 1$

Amplifier Design

given: • $\begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22}\end{pmatrix}$

• goal gain and/or noise

• sometimes Γ_{opt}, F_{min}, R_N

goal: find Γ_s, Γ_L and/or G_T and/or design matching networks

$$G_T = \frac{P_L}{P_{avs}} = \frac{|S_{21}|^2(1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_{in}\Gamma_S|^2|1-S_{22}\Gamma_L|^2}$$ general

$$G_{TU} = |S_{21}|^2 \frac{(1-|\Gamma_s|^2)}{|(1-S_{11}\Gamma_s)|^2}\frac{(1-|\Gamma_L|^2)}{|(1-S_{22}\Gamma_L)|^2}$$ unilateral

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}; \Gamma_{out} = S_{22}+\frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}$$

See page 3&4 for more formulas

maximum gain (unilateral) → $S_{12} \approx 0$

1. $\Gamma_s = S_{11}^*$ $\Gamma_L = S_{22}^*$
2. $G_{TU,max} = \frac{|S_{21}|^2}{(1-|S_{11}|^2)(1-|S_{22}|^2)}$
3. $G_{TU,max}(dB) = 10\log\frac{1}{1-|S_{11}|^2} + 10\log|S_{21}|^2 + 10\log\frac{1}{(1-|S_{22}|^2)}$
4. Input matching network should match S_{11} to Z_S
5. Output matching network should match S_{22} to Z_L

maximum gain (general)

1. $\Gamma_s = \Gamma_{in}^*$ $\Gamma_L = \Gamma_{out}^*$
2. $G_{TU,max}(dB) = 10\log\frac{1}{1-|\Gamma_S|^2} + 10\log|S_{21}|^2 + 10\log\frac{(1-|\Gamma_L|^2)}{|(1-S_{22}\Gamma_L)|^2}$
3. Input matching network should match $\Gamma_s{}^*$ to Z_S
4. Output matching network should match $\Gamma_L{}^*$ to Z_L

while $\begin{cases} \Gamma_s^* \overset{!}{=} S_{11} + \dfrac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} \\ \Gamma_L^* \overset{!}{=} S_{22} + \dfrac{S_{12}S_{21}\Gamma_s}{1-S_{11}\Gamma_s} \end{cases}$

specific gain, don't care about noise

1. Find the sum of the gains for the input and output matching networks
2. Distribute the sum in a reasonable way between G_S and G_L and draw the G_S and G_L circles
3. Choose Γ_S and Γ_L so they are as close as possible to center of the Smith chart
4. Input matching network should match $\Gamma_S{}^*$ to Z_S
5. Output matching network should match $\Gamma_L{}^*$ to Z_L

specific gain, best possible noise

1. Draw noise circles for several noise figures F close to F_{min}
2. Find the sum of the gains for the input and output matching networks (G_S+G_L)
3. Take into account that maximum gain G_{Lmax} you can get from the output is $10\log\frac{1}{(1-|S_{22}|^2)}$!

→ $\begin{cases} \text{Case A: } G_S + G_L < G_{Lmax} \\ \text{Case B: } G_S + G_L > G_{Lmax} \end{cases}$

Case A: $G_S + G_L < G_{Lmax}$

- Distribute the sum in a reasonable way between G_S and G_L and draw the G_S and G_L circles (try several options depending on intersections with the noise circles)
- Find the smallest noise circle which intersects with G_S circle
- Choose Γ_S where G_S circle intersect the least noise figure circle and Γ_L on G_L circle as close as possible to center of the Smith chart (or take Γ_L to be as close as possible to Γ_S because od symmetry)
- Input matching network should match $\Gamma_S{}^*$ to Z_S
- Output matching network should match $\Gamma_L{}^*$ to Z_L

Case B: $G_S + G_L > G_{Lmax}$

- $\Gamma_L = S_{22}^*$ (we can maximize it because noise figure does not depend on output) if unilateral, otherwise $\Gamma_L = \Gamma_{out}^*$
- Draw G_S circle (for $G_S = (G_S + G_L) - G_{Lmax}$ and choose the smallest noise circle which intersects with G_S circle
- Choose Γ_S where G_S circle intersect the least noise figure circle
- matching network should match $\Gamma_S{}^*$ to Z_S
- Output matching network should match S_{22} to Z_L if unilateral, otherwise match $\Gamma_L{}^*$ to Z_L

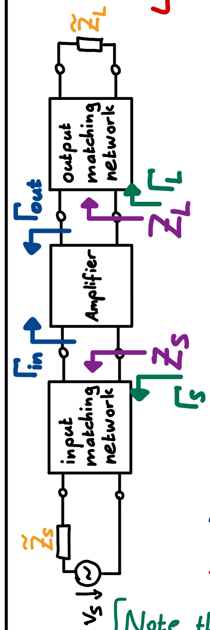
specific noise, best possible gain

1. $\Gamma_L = S_{22}^*$ (we can maximize it because noise figure does not depend on output) if unilateral, otherwise $\Gamma_L = \Gamma_{out}^*$
2. Draw the noise circle for desired noise figure F
3. Find maximum input matching circuit gain G_S for which the noise circle and input gain circle have 1 common intersection point
4. That point will be the desired Γ_S
5. Input matching network should match $\Gamma_S{}^*$ to Z_S
6. Output matching network should match S_{22} to Z_L if unilateral, otherwise match $\Gamma_L{}^*$ to Z_L

minimum noise, best possible gain

1. $\Gamma_L = S_{22}^*$ if unilateral, otherwise $\Gamma_L = \Gamma_{out}^*$
2. Minimum noise occurs for $\Gamma_s = \Gamma_{s,opt}$
3. Input matching network should match $\Gamma_s{}^*$ to Z_S
4. Output matching network should match S_{22} to Z_L if unilateral otherwise match $\Gamma_L{}^*$ to Z_L

matching networks



$\tilde{Z}_S$: actual source impedance

$\tilde{Z}_L$: actual Load impedance

Z_S: impedance that the Amp. should see at the input

Z_L: impedance that the Amp. should see at the output

$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} \Rightarrow Z_S = \frac{1+\Gamma_S}{1-\Gamma_S}$

$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow Z_L = \frac{1+\Gamma_L}{1-\Gamma_L}$

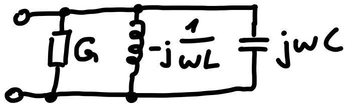
now we want to match $\tilde{Z}_S \to Z_S$ and $\tilde{Z}_L \to Z_L$

but this is difficult, so we do a trick and match $Z_S^* \to \tilde{Z}_S$ and $Z_L^* \to \tilde{Z}_L$

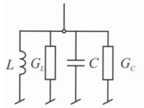
[Note that $Z_A \to Z_B \Leftrightarrow Z_B^* \to Z_A$]

Appendix

parallel resonator circuit

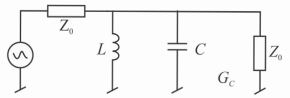


- unloaded



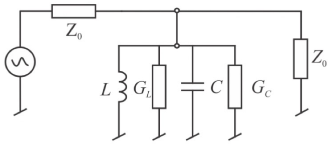
$$Q_U = \frac{\omega_0 C}{G_L + G_C} = \frac{1}{\omega_0 L (G_L + G_C)}$$

- external



$$Q_E = \frac{\omega_0 C}{2Y_0} = \frac{1}{\omega_0 L (2Y_0)}$$

- loaded



$$Q_L = \frac{\omega_0 C}{G_L + G_C + 2Y_0} = \frac{1}{\omega_0 L (G_L + G_C + 2Y_0)}$$

$$\frac{1}{Q_L} = \frac{1}{Q_E} + \frac{1}{Q_U}$$

Trigonometry

$$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j} \;;\; \sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2} \;;\; \cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)} \;;\; \tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

$$\cot(x) = \frac{1}{\tan(x)} \;;\; \coth(x) = \frac{1}{\tanh(x)}$$

$$\sec(x) = \frac{1}{\cos(x)} \;;\; \mathrm{sech}(x) = \frac{1}{\cosh(x)}$$

$$\csc(x) = \frac{1}{\sin(x)} \;;\; \mathrm{csch}(x) = \frac{1}{\sinh(x)}$$

$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$$
$$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$$
$$\sin(x)\sin(y) = \frac{1}{2}(\cos(x-y) - \cos(x+y))$$
$$\cos(x)\cos(y) = \frac{1}{2}(\cos(x-y) + \cos(x+y))$$
$$\sin(x)\cos(y) = \frac{1}{2}(\sin(x-y) + \sin(x+y))$$
$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$
$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$
$$\sin^3(x) = \frac{1}{4}(3\sin(x) - \sin(3x))$$
$$\cos^3(x) = \frac{1}{4}(3\cos(x) + \cos(3x))$$
$$\sin^4(x) = \frac{1}{8}(3 - 4\cos(2x) + \cos(4x))$$
$$\cos^4(x) = \frac{1}{8}(3 + 4\cos(2x) + \cos(4x))$$
$$\sin(x) + \sin(y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\sin(x) - \sin(y) = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$
$$\cos(x) + \cos(y) = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\cos(x) - \cos(y) = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Taylorseries

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k$$

$$f(\vec{x})\big|_{\vec{x}=\vec{a}} = \sum_{k=0}^{\infty} \frac{1}{k!}\left(\Delta x_1 \frac{\partial}{\partial x_1} + \dots + \Delta x_n \frac{\partial}{\partial x_n}\right)^k \Bigg|_{\vec{x}=\vec{a}}$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \mathcal{O}(x^4) \quad \text{um } x_0 = 0$$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$\sin(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

$$\cos(x) = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}$$

$$\sinh(x) = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}$$

$$\cosh(x) = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}$$

Q-matching with n-stages

low to high-example

match $Z_L = (8 - j12)\Omega$ to 50Ω with $n=2$; $f = 10\,GHz$

$Re(Z_L) = 8\Omega < 50\Omega \Rightarrow$ low to high

$1 + Q^2 \overset{!}{=} \sqrt[n]{R}$ with $R = \frac{R_{goal}}{Re(Z_L)} = \frac{50\Omega}{8\Omega} = 6{,}25$

$\Rightarrow Q = \sqrt{R^{1/n} - 1} = 1{,}22$

now the 1st is „normal" but it will only match

$Z_L \rightarrow \underbrace{Re(Z_L)}_{R_{S,1} = R_L} \cdot \sqrt[n]{R} = 20\Omega$

- first stage

$R_{S,1} = R_L = Re(Z_L) = 8\Omega$

$Q = 1{,}22$ (see above)

$X_{new,1} + Im(Z_L) \overset{!}{=} \pm Q R_{S,1}$ (let's choose +)

$\Rightarrow X_{new,1} = Q R_{S,1} - Im(Z_L) = 21{,}8\Omega$

$X_{new,1} > 0 \Rightarrow$ series L_1 with $j\omega L_1 = j21{,}8\Omega$

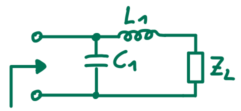
$\Rightarrow L_1 = \frac{21{,}8\Omega}{\omega} = \underline{346{,}9\,pH}$

convert to parallel: $X_p = \frac{R_{S,1}(1+Q^2)}{Q} = 16{,}3\Omega$

$\Rightarrow$ we need a shunt capacitor to cancel X_p

$\frac{1}{j\omega C_1} = -j\frac{1}{\omega C_1} \overset{!}{=} -j16{,}3\Omega \Rightarrow C_1 = \frac{1}{16{,}3\Omega \cdot \omega} = \underline{\underline{974{,}6\,fF}}$

so far we have this



$R_{S,2} = Z_{IN,1} = \underbrace{R_{S,1}}_{Re(Z_L)} \underbrace{(1+Q^2)}_{\sqrt[n]{R}} = Re(Z_L) \cdot \sqrt{R} = 20\Omega$

- second stage

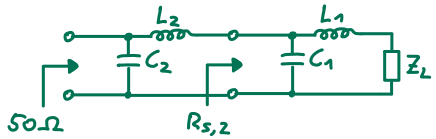
Q & R stay across all stages

$R_{S,2} = R_{S,1}\sqrt{R} = 20\Omega$; $Q \overset{!}{=} 1{,}22$

„Z_L & 1st-stage produce a $Z_{L,2} = R_{S,2} + j \cdot 0$"

$X_{new,2} + \underbrace{X_{L,2}}_{0} \overset{!}{=} \pm Q R_{S,2} \xrightarrow[\text{choose +}]{} L_2 = \frac{Q R_{S,2}}{\omega} = \dots$

$X_p = \frac{R_{S,2}(1+Q^2)}{Q} = 40{,}8\Omega \rightarrow$ shunt cap $C_2 = \frac{1}{\omega \cdot 40{,}8\Omega} = \dots$



high to low-example

6

match $Z_L = (250 - j200)\Omega$ to 50Ω, $n=2$, $f = 1\,GHz$

$Q_L = \left|\frac{X_L}{R_L}\right| = \left|\frac{Im(Z_L)}{Re(Z_L)}\right| = 0{,}8$

$R_{PL,1} = R_L(1 + Q_L^2) = 410\Omega$

$X_{PL,1} = -\frac{R_L(1+Q_L^2)}{Q_L} = -512{,}5\Omega$

$Im(Z_L) < 0 \Rightarrow X_{PL,1} \overset{!}{<} 0$

$R = \frac{R_{PL,1}}{R_{goal}} = \frac{410\Omega}{50\Omega} = 8{,}2$

$\Rightarrow$ target Q: $Q = \sqrt{R^{1/n} - 1} = 1{,}365$

$X_{P,1} = \pm \frac{R_{PL}}{Q} = 300{,}3\Omega$ (chose +)

can choose

$X_{P,1} \overset{!}{=} \frac{X_{PL,1} X_{new,1}}{X_{PL,1} + X_{new,1}} \Rightarrow X_{new,1} = \frac{X_{P,1} X_{PL,1}}{X_{PL,1} - X_{P,1}} = 189{,}4\Omega$

$X_{new,1} > 0 \Rightarrow$ shunt inductor with $j\omega L_1 \overset{!}{=} j189{,}4\Omega$

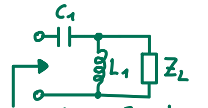
$\Rightarrow L_1 = \frac{189{,}4\Omega}{\omega} = \underline{30{,}14\,nH}$

$R_{S,1} = \frac{R_{PL,1}}{1+Q^2} = 143{,}18\Omega$; $X_{S,1} = Q R_{S,1} = 195{,}46\Omega$

$\Rightarrow$ we need a series capacitor to cancel $X_{S,1}$

$\Rightarrow \frac{1}{j\omega C_1} = -j\frac{1}{\omega C_1} \overset{!}{=} -j195{,}46\Omega \Rightarrow C_1 = \underline{814{,}3\,fF}$

so far we have:



$Z_{IN,1} = \frac{1}{j\omega C_1} + \frac{Z_L j\omega L_1}{Z_L + j\omega L_1} = 143{,}18\Omega = R_{S,1}$ (!)

- second stage

„match $R_{S,1}$ to 50Ω; $Q = 1{,}365$"

! $R_{PL,2} = R_{S,1} = 143{,}18\Omega$!

$X_{PL,2} = \pm \frac{R_{PL,2}}{Q} = 104{,}9\Omega$

I think we can choose but do + to be safe

$X_{PL,2} > 0 \Rightarrow$ we need to add* a shunt L with

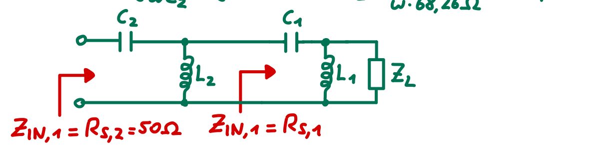
$j\omega L_2 \overset{!}{=} j104{,}9\Omega \Rightarrow L_2 = \frac{104{,}9\Omega}{\omega} = \underline{16{,}7\,nH}$

*here we „match real to real" → we need to add L_2 to „conserve Q"

$R_{S,2} = \frac{R_{PL,2}}{1+Q^2} = 50\Omega$ (as expected) ; $X_{S,2} = \pm Q R_{S,2} = 68{,}26\Omega$

$X_{S,2} > 0 \Rightarrow$ need series C to cancel $X_{S,2}$

$\Rightarrow -j\frac{1}{\omega C_2} \overset{!}{=} -j68{,}26\Omega \Rightarrow C_2 = \frac{1}{\omega \cdot 68{,}26\Omega} = 2{,}33\,pF$



Quarter wave transformer

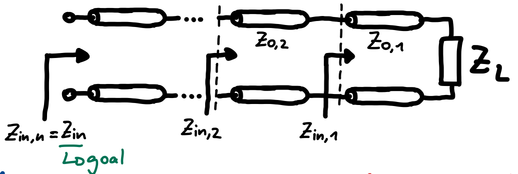
$R = \sqrt[n]{\frac{Z_L}{Z_{in}}}$ ← ratio

[Z_{in} : goal input imp. → often $Z_{in} \stackrel{!}{=} 50\Omega$ etc.]

$Z_{in,1} = \frac{Z_L}{R}$; $Z_{0,1} = \sqrt{Z_{in,1} Z_L}$

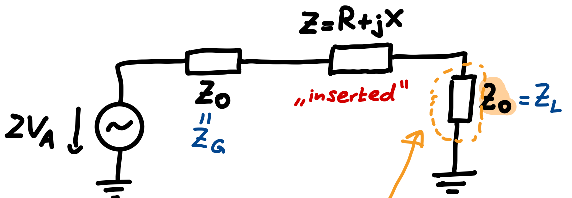
$Z_{in,2} = \frac{Z_{in,1}}{R}$; $Z_{0,2} = \sqrt{Z_{in,2} Z_{in,1}}$

this is like the „new load" seen by $Z_{0,2}$



Loss in AC Circuits (continued)

Series:



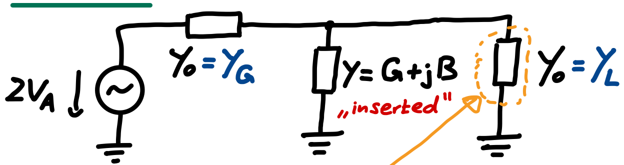
Transducer Loss

$$TL = \frac{P_A}{P_L} = \frac{\frac{V_A^2}{Z_0}}{\left(\frac{2V_A}{|2Z_0+Z|}\right)^2 Z_0} = \frac{|2Z_0+Z|^2}{4Z_0^2} = \left|1+\frac{Z}{Z_0}+\frac{Z^2}{4Z_0^2}\right| = \left|1+\frac{Z}{2Z_0}\right|^2$$

$$TL = \left|1+\frac{z}{2}\right|^2 = \left|\frac{2+r+jx}{2}\right|^2 = \left(1+\frac{r}{2}\right)^2 + \frac{x^2}{4} = 1 + r + \frac{r^2}{4} + \frac{|x|^2}{4}$$

for normalised $z = \frac{Z}{Z_0} = r + jx$

Parallel:



Transducer Loss

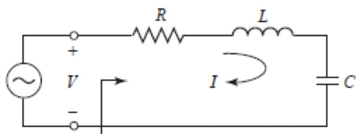
$$TL = \frac{P_A}{P_L} = \frac{V_A^2 Y_0}{\left(\frac{2V_A \frac{1}{Y+Y_0}}{\left|\frac{1}{Y_0}+\frac{1}{Y+Y_0}\right|}\right)^2 Y_0} = \frac{|2Y_0+Y|^2}{4Y_0^2} = \left|1+\frac{Y}{Y_0}+\frac{Y^2}{4Y_0^2}\right| = \left|1+\frac{Y}{2Y_0}\right|^2$$

$$TL = \left|1+\frac{y}{2}\right|^2 = \left|\frac{2+g+jb}{2}\right|^2 = \left(1+\frac{g}{2}\right)^2 + \frac{b^2}{4} = 1 + g + \frac{g^2}{4} + \frac{|b|^2}{4}$$

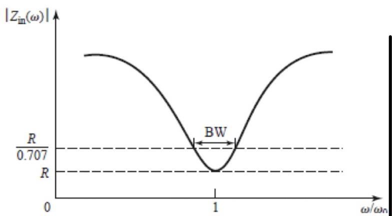
for normalised $y = \frac{Y}{Y_0} = g + jb$

Bandwith (continued)

Series Resonator:



$Z_{in} = R + j\omega L + \frac{1}{j\omega C}$



$$Z_{in} = R + j\omega L\left(1 - \frac{1}{\omega^2 LC}\right) = R + j\omega L\left(\frac{\omega^2-\omega_0^2}{\omega^2}\right)$$

If $\omega = \omega_0 + \Delta\omega$ then $Z_{in} \simeq R + j2L\Delta\omega \simeq R + j\frac{2RQ_0\Delta\omega}{\omega_0}$

↳ $\Delta\omega \ll \omega_0$

Half-power (3dB) fractional Bandwith BW:

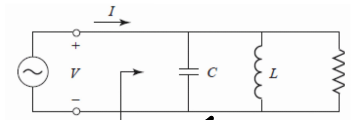
check where $|Z_{in}|^2 = 2R^2$

$\Rightarrow |R + jRQ_0(BW)|^2 \stackrel{!}{=} 2R^2$

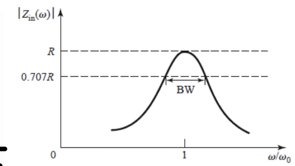
$\Rightarrow \underbrace{BW}_{\text{fractional}} = \frac{1}{Q_0}$ for $Q > 10$ only!

$\Rightarrow$ „absolute" Bandwith $BW_{absolute} = f_0 \underbrace{BW}_{\text{fractional}} \stackrel{!}{=} \frac{f_0}{Q_0}$ ($Q_0 \stackrel{!}{>} 10$)

Parallel Resonator:



$Z_{in} = \frac{1}{\frac{1}{R} + \frac{1}{j\omega L} + j\omega C}$



Half-power (3dB) fractional Bandwith BW:

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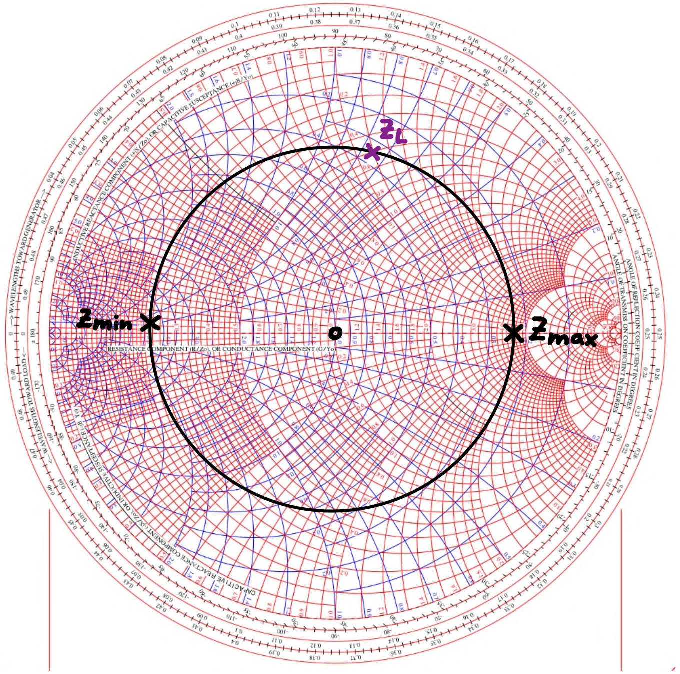
$$\boxed{\text{„Q is the number of ringing cycles"}}$$

→ count peaks until envelope dies down to $e^{-\pi} \approx 4,3\%$ of it's max. Amplitude

Smithchart Tricks

- if you go along a TL, the maximal (minimal) impedance is at the right (left) intersection of the const-$|\Gamma|$-circle and the real axis



- VSWR = z_{max}

Proof:

at z_{max}: Γ is real and positive (since it has phase 0°)

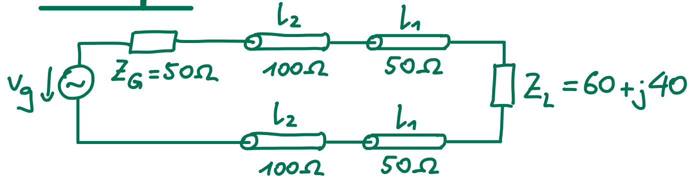
$\hookrightarrow |\Gamma| = \Gamma = \frac{Z_{max} - Z_0}{Z_{max} + Z_0} = \frac{z_{max} - 1}{z_{max} + 1}$

$$\Rightarrow VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+\Gamma}{1-\Gamma} = \frac{1 + \frac{z_{max}-1}{z_{max}+1}}{1 - \frac{z_{max}-1}{z_{max}+1}}$$

$$= \frac{z_{max}+1+z_{max}-1}{z_{max}+1-(z_{max}-1)} = \frac{2z_{max}}{2} = \underline{\underline{z_{max}}} \quad \square$$

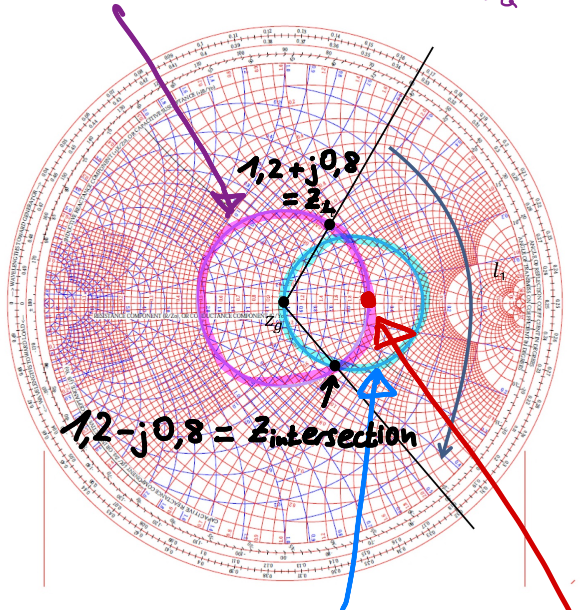
- On a smitchard, that's normalised to $Z_{0,original}$ a TLine with impedance Z_{weird} will make you move along a circle whose center is at $z = \frac{Z_{weird}}{Z_{0,original}}$ in the „$Z_{0,original}$-Chart".

Example (see Ex. 8, Problem 2)



this circle is for the 50Ω-TLine

the center is at $\frac{\text{characteristic impedance}}{\text{normalisation of chart}} = \frac{50\Omega}{Z_G} = 1+j0$

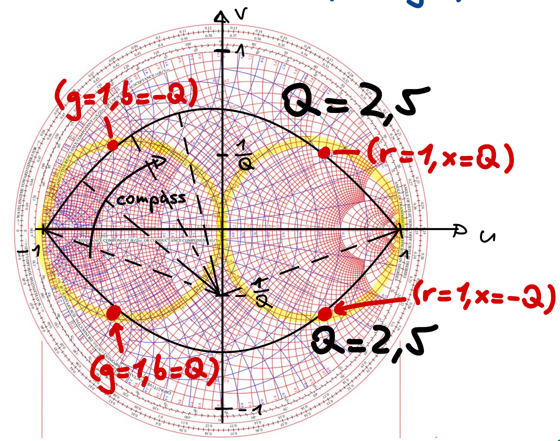


this circle is for the 100Ω-TLine

the center is at $\frac{\text{characteristic impedance}}{\text{normalisation of chart}} = \frac{100\Omega}{Z_G} = \underline{2+j0}$

Drawing Q-Circles

1. put center of compass in $v = -\frac{1}{Q}$ with $v = Im(\Gamma)$
2. Draw a „segment" from $u=-1$ to $u=1$ with $u = Re(\Gamma)$
3. put center of compass in $v = \frac{1}{Q}$
4. Draw a „segment" from $u=-1$ to $u=1$
5. Double check that the upper segment intersects $(r=1, x=Q)$ & $(g=1, b=-Q)$
6. Double check that the lower segment intersects $(r=1, x=-Q)$ & $(g=1, b=Q)$



Note that here it's pretty ugly but it works