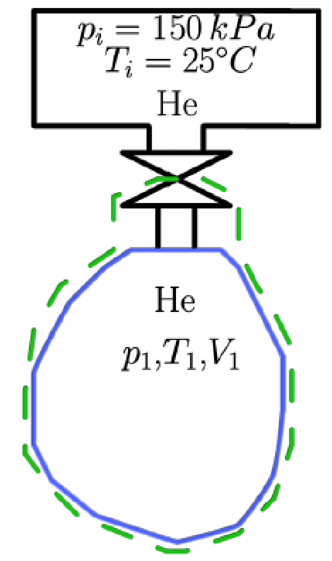
Aufgabe 4

Annahmen:

- Helium ist ein ideales Gas,
- der Tank ist unendlich gross, daraus folgt: $p_2 = p_i$
- $c_p = 5.1926 \frac{kJ}{kg\,K}$
- $p_1 = 100\,kPa$
- $T_1 = 22°C$
- $V_1 = 65\,m^3$



$$R_{He} = \frac{\bar{R}}{M_{He}} = \frac{8.314\frac{J}{mol\,K}}{4.003 \cdot 10^{-3}\frac{kg}{mol}} = \underline{\underline{2.0769\,kJ/kg\,K}}$$

$$c_v = c_p - R_{He} = 5.1926\frac{kJ}{kg\,K} - 2.0769\frac{kJ}{kg\,K} = \underline{\underline{3.1157\frac{kJ}{kg\,K}}}$$

$$m_1 = \frac{p_1 V_1}{R_{He} T_1} = \frac{100\,kPa\,65\,m^3}{2.0769\,kJ/kg\,K\;295\,K} = \underline{\underline{10.609\,kg}}$$

1.HS für offene Systeme mit einem Einlass: (KE=0, PE=0)

$$\Delta E = \Delta U = \overbrace{Q}^{=0} - W + m_{in} \cdot h_{in}$$

Prozess 1–2:

$$p \sim V \Rightarrow \frac{p_1}{p_2} = \frac{V_1}{V_2} \Rightarrow V_2 = V_1 \frac{p_2}{p_1} = 65\,m^3\,\frac{150\,kPa}{100\,kPa} = \underline{\underline{97.5\,m^3}}$$

Arbeit vom Gas:

$$W_{12} = \int_1^2 p \cdot dV \text{ mit } p \sim V \Rightarrow p = \frac{p_1}{V_1} \cdot V$$
$$W_{12} = \frac{p_1}{2 \cdot V_1} \cdot (V_2^2 - V_1^2) = \frac{100 * 10^3\,Pa}{2 \cdot 65\,m^3} \cdot (97.5^2 - 65^2)\ m^3 = \underline{\underline{4.0625\,MJ}}$$

somit 1.HS:

$$\begin{aligned}
\Delta U = U_2 - U_1 = m_2 \cdot u_2 - m_1 \cdot u_1 &= -W_{12} + (m_2 - m_1) \cdot h_{in} \\
(m_2 - m_1) \cdot u_2 + m_1 \cdot c_v \cdot (T_2 - T_1) &= -W_{12} + (m_2 - m_1) \cdot h_{in} \\
(m_2 - m_1)[u_2 - h_{in}] &= -W_{12} - m_1 \cdot c_v \cdot (T_2 - T_1) \\
(m_2 - m_1)[u_2 - \underbrace{(u_i + p_i \cdot v_i)}_{h=u+pv}] &= \\
(m_2 - m_1)[u_2 - (u_i + \underbrace{R_{He} \cdot T_i}_{He=ig:\,pv=RT})] &= \\
(m_2 - m_1)[(u_2 - u_i) - R_{He} T_i] &= \\
(m_2 - m_1)[c_v \cdot (T_2 - T_i) - R_{He} T_i] &= \\
(m_2 - m_1)[c_v \cdot T_2 - \underbrace{c_p \cdot T_i}_{c_p = R + c_v}] &= -W_{12} - m_1 \cdot c_v \cdot (T_2 - T_1)
\end{aligned}$$

$$m_2 \cdot c_v \cdot T_2 - m_1 \cdot c_v \cdot T_1 - (m_2 - m_1) \cdot c_p \cdot T_i = -W_{12}$$

$$\overbrace{\frac{p_2 V_2}{R_{He} T_2}}^{m_2} \cdot c_v \cdot T_2 - m_1 \cdot c_v \cdot T_1 - (\overbrace{\frac{p_2 V_2}{R_{He} T_2}}^{m_2} - m_1) \cdot c_p \cdot T_i = -W_{12}$$

$$\Rightarrow T_2 = \frac{p_2 \cdot V_2}{R_{He}\left[\frac{W_{12} + \frac{p_2 V_2 c_v}{R_{He}} - m_1 c_v T_1}{c_p T_i} + m_1\right]} = \underline{\underline{333.55\,K}} = 60.55°C$$