Informatik - Exercise Session
Numerical representation

1. Perform the following steps:
   1.1 Convert the integer numbers $a = 4$ and $b = 7$ into their binary representation.
   1.2 Add the binary representations.
   1.3 Convert the result into decimal.
2. Evaluate the following expressions:
   2.1 `5 < 4 < 1`
   2.2 `true > false`

Ob . . .

Solutions

1.
   1.1 $4_{10} = 100_2$ and $7_{10} = 111_2$
   1.2 $100_2 + 111_2 = 1011_2$
   1.3 $1011_2 = 11_{10}$
2.
   2.1

```
5 < 4 < 1
(5 < 4) < 1
false < 1
0 < 1
true
```

   2.2

```
true > false
true > 0
1 > 0
true
```

Binary Representation: floating point numbers

dec.

$$\ldots \underline{\quad}_{10^2}\ \underline{\quad}_{10^1}\ \underline{\quad}_{10^0}\ .\ \underline{\quad}_{10^{-1}}\ \underline{\quad}_{10^{-2}}\ \underline{\quad}_{10^{-3}}$$

0.1

bin

$$\underline{\quad}_{2^2}\ \underline{\quad}_{2^1}\ \underline{\quad}_{2^0}\ .\ \underline{\quad}_{2^{-1}}\ \underline{\quad}_{2^{-2}}\ \underline{\quad}_{2^{-3}}$$

$$\frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8}$$

$1.9 = 1 + 0.9$

Converting decimal FPs to binary FPs

x	b_i	$x - b_i$	$2(x - b_i)$
0.9	0	0.9	1.8
1.8	1	0.8	1.6
1.6	1	0.6	1.2
1.2	1	0.2	0.4
0.4	0	0.4	0.8
0.8	0	0.8	1.6
1.6	1	0.6	1.2
1.2	1	0.2	0.4

$1 + 0.11100110 0\ldots$

$1.1\overline{1100}$

Compute the binary expansions of the following decimal numbers.

1. 0.25
2. 11.1

Solutions:

1. $0.25_{10} = 0.01_2$

x	b_i	$x - b_i$	$2 \cdot (x - b_i)$
0.25	0	0.25	0.5
0.5	0	0.5	1
1	1	0	0

2. $11.1_{10} = 1011.0\overline{0011}_2$
   11.1_{10} is first split into $11_{10} + 0.1_{10}$. The binary representation of 0.1_{10} is derived as follows:

x	b_i	$x - b_i$	$2 \cdot (x - b_i)$
0.1	0	0.1	0.2
0.2	0	0.2	0.4
0.4	0	0.4	0.8
0.8	0	0.8	1.6
1.6	1	0.6	1.2
1.2	1	0.2	0.4
0.4	0	0.4	0.8

   Hence, 11.1_{10} evaluates to $1011_2 + 0.0\overline{0011}_2$.

Normalized FP-systems

$$F^{*}(\underbrace{b}_{\text{Basis}},\ \underbrace{p}_{\#\text{Stellen}},\ \underbrace{e_{min},\ e_{max}}_{\text{Exponenten}})$$

$e \in \{e_{min}, \dots, e_{max}\}$

$$\pm \underbrace{b_0 . b_1 b_2 b_3 \dots b_{p-1}}_{p \text{ Stellen}} \cdot b^e$$

$b_0 \neq 0$
(weil normalis.)

$1.010 \cdot 2^{2}$

$0.1 \cdot 10^2$

Consider: $F^*(2, 4, \underbrace{-2, 2}_{\text{exp.}}) \longrightarrow \{-2, -1, 0, 1, 2\}$

b #stellen

↳ d.h. 3 stellen nach "."

Do the following numbers belong to this set and why?

1. $0.000 \cdot 2^1$
2. $1.000 \cdot 2^1$
3. $1.001 \cdot 2^{-1}$
4. $1.0001 \cdot 2^{-1}$
5. $1.111 \cdot 2^1$
6. $1.111 \cdot 2^5$

Consider: $F^*(2, 4, -2, 2)$

Do the following numbers belong to this set and why?

1. $0.000 \cdot 2^1$
2. $1.000 \cdot 2^1$
3. $1.001 \cdot 2^{-1}$
4. $1.0001 \cdot 2^{-1}$
5. $1.111 \cdot 2^1$
6. $1.111 \cdot 2^5$

$$- 1.\underbrace{\_\ \_\ \_}\cdot 2^{\bigcirc}$$

State the following numbers in $F^*(2, 4, -2, 2)$:

1. the largest number;
2. the smallest number;
3. the smallest non-negative number.

Compute how many numbers are in the set $F^*(2, 4, -2, 2)$.

Solutions:

1. The largest number is $1.111 \cdot 2^2$ which is 7.5 in decimal.
2. The smallest number is $-1.111 \cdot 2^2$ which is -7.5 in decimal.
3. The smallest non-negative number is $1.000 \cdot 2^{-2}$ which is 0.25 in decimal.

The set has 80 numbers in it. This can be seen as follows. For a fixed exponent there are three digits we can vary freely, and for each number also the negative number is in the set, thus resulting in $2 \cdot 2^3 = 16$ numbers per exponent. On the other hand, there are 5 possible exponents, thus resulting in $5 \cdot 16 = 80$ numbers. Notice that in normalized number systems we cannot "count some numbers twice" as we've seen in the lecture that the representation of a number is unique.

$2^3 \longrightarrow$ 3 bits

2

5

$5(2 \cdot 2^3) = 80$

Adding within a FP-system:

1. Bring numbers to same exponent.
2. Add the two numbers, remember the decimal point and exponent for result.
3. Re-normalize the result.
4. Round the result.

Add $1.001 \cdot 2^{-1}$ (i.e. 0.5625) and $1.111 \cdot 2^{-2}$ (i.e. 0.46875) in $F^*(2, 4, -2, 2)$.

Solution:

The two numbers $1.001 \cdot 2^{-1}$ (i.e. 0.5625) and $1.111 \cdot 2^{-2}$ (i.e. 0.46875) are added as follows:

1. Bring both to say exponent -1: $\quad 1.001 \cdot 2^{-1}$ and $0.1111 \cdot 2^{-1}$
2. Add as binary numbers:

$$\begin{array}{rr} & 1.001 \cdot 2^{-1} \\ + & 0.1111 \cdot 2^{-1} \\ \hline & 10.0001 \cdot 2^{-1} \end{array}$$

1.

3. Re-normalize: $1.00001 \cdot 2^{0}$
4. Round: $1.000 \cdot 2^{0}$

Notice that this does not coincide with their exact sum 1.03125 as the precision 4 is not high enough to represent this number.

$1.1 \cdot 2^{-1}$