ETH Zürich
Prof. Dr. H. M. Soner

Winter 2017

Exam Analysis III
D–MAVT, D–MATL

Please fill!

Surname:	
First Name:	
Student Card Nr.:	

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Exercise	Value	Points	Control
1	10		
2	10		
3	10		
4	10		
5	10		
Total	50		

Please do not fill!

Completeness	

Please turn!

Important: Before the exam starts, please

- Turn off your mobile phone and place it inside your Briefcase/Backpack, underneath the table.
- Place your Student Card (Legi) on the desk.
- Fill in the front page of the exam with your generalities.

During the exam, please

- Start every exercise on a new piece of paper.
- Put your name on the top right corner of every page.
- You are expected to motivate your answers. Write down every step of your calculation and, where sensible, write down results of partial answers.
- The general solutions of the PDE's which were obtained **throughout the lecture** can be used without further motivation. This **does not apply** to solutions obtained in the exercise sheets.
- Provide at most **one** solution to each exercise.
- **Do not** write with **pencils**. Please avoid using **red** or **green** ink pens.

Allowed :

- 20 pages (=10 sheets) DIN A4 handwritten or typed personal summary.
- an English dictionary
- **no** further aids are allowed, such as formula collections (Papula or similar), Mobile phones or Pocket calculators.

Good Luck!

Laplacetransformations:

	$f(t)$	$\mathcal{L}(f)$		$f(t)$	$\mathcal{L}(f)$		$f(t)$	$\mathcal{L}(f)$
1	1	$\frac{1}{s}$	5	$t^a,\ a>0$	$\frac{\Gamma(a+1)}{s^{a+1}}$	9	$\cosh(at)$	$\frac{s}{s^2-a^2}$
2	t	$\frac{1}{s^2}$	6	e^{at}	$\frac{1}{s-a}$	10	$\sinh(at)$	$\frac{a}{s^2-a^2}$
3	t^2	$\frac{2!}{s^3}$	7	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$	11	$u(t-a)$	$\frac{e^{-as}}{s}$
4	$t^n,\ n\in\mathbb{N}_0$	$\frac{n!}{s^{n+1}}$	8	$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$	12	$\delta(t-a)$	e^{-as}

(Γ=Gamma function, u=Heaviside function, δ=Delta function)

1. (*10 Points*) Solve the initial value problem

$$\begin{cases} y''' + y'' = \delta(t-6), & t \geq 0 \\ y(0) = 0 = y'(0) \\ y''(0) = 1 \end{cases}$$

Hint: You may use the identity

$$\frac{1}{s^2(s+1)} = \frac{1}{s+1} - \frac{1}{s} + \frac{1}{s^2}$$

2. (*10 Points*) Given the 2π periodic function

$$f(x) = \begin{cases} -1 & -\pi < x < 0, \\ 1 & 0 < x < \pi, \end{cases}$$

calculate the Fourier Series of $f(x)$. (Please show the details)

3. (*6+4 Points*) Let $u(x,t)$ be the solution of the one dimensional wave Equation.

$$\begin{cases} u_{tt} = u_{xx}, & x \in \mathbb{R},\ t > 0 \\ u(x,0) = f(x) = \begin{cases} \cos(x), & |x| \leq 2\pi \\ 0, & |x| > 2\pi \end{cases}, & x \in \mathbb{R} \\ u_t(x,0) = g(x) = \begin{cases} 0, & |x| \leq 2\pi \\ e^{-x}, & |x| > 2\pi \end{cases}, & x \in \mathbb{R} \end{cases}$$

Please turn!

a) Find $u(0, \pi)$. You may use the D'Alambert solution.

b) Find $\lim_{a \to \infty} u(a, a)$.

4. (*10 Points*)
Consider the string of length $L = \pi$ and $c^2 = 1$ with zero initial velocity, initial deflection $u(x, 0) = x(x - \pi)$ and fixed endpoints. The deflection $u(x, t)$ is a solution of the following PDE

$$\begin{cases} u_{tt} = u_{xx}, & t > 0,\ x \in (0, \pi) \\ u(0, t) = 0 = u(\pi, t), & t \geq 0 \\ u(x, 0) = x(x - \pi), & 0 \leq x \leq \pi \\ u_t(x, 0) = 0, & 0 \leq x \leq \pi \end{cases}$$

Find $u(x, t)$.

5. (*6+4 Points*) Consider the system

$$\begin{cases} u_{xx} + u_{yy} = 0, & 0 < x, y < \pi \\ u(0, y) = 0 = u(\pi, y), & 0 < y < \pi, \\ u(x, 0) = 0, & 0 \leq x \leq \pi, \\ u(x, \pi) = 3\sin(2x), & 0 \leq x \leq \pi, \end{cases}$$

a) Find the general solution $u(x, t)$ of

$$\begin{cases} u_{xx} + u_{yy} = 0, & 0 < x, y < \pi \\ u(0, y) = 0 = u(\pi, y), & 0 < y < \pi, \\ u(x, 0) = 0, & 0 \leq x \leq \pi, \end{cases}$$

via a separation of variables argument (please show the details).

b) Find the solution of

$$\begin{cases} u_{xx} + u_{yy} = 0, & 0 < x, y < \pi \\ u(0, y) = 0 = u(\pi, y), & 0 < y < \pi, \\ u(x, 0) = 0, & 0 \leq x \leq \pi, \\ u(x, \pi) = 3\sin(2x), & 0 \leq x \leq \pi, \end{cases}$$