D-MAVT
D-MATL

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Analysis III

Serie 7

Topics: Discret Fourier transform DFT, Fast Fourier transform FFT.

1. Discrete Fourier transform (DFT)

Let $N = 4$ and f be a function whose the following values,

$$f(0) = 2, \quad f\left(\frac{2\pi}{N}\right) = 0, \quad f\left(2\frac{2\pi}{N}\right) = 6, \quad f\left(3\frac{2\pi}{N}\right) = 3.$$

Find the discrete Fourier transform (DFT) of the function f with the numerical values given above. And write down the finite trigonometric representation of the function f with the coefficients that you found.

Steps:

1) Find the value of w_4.
2) Compute the entries of the matrix $\mathbf{M}_4^{-1}$ using the formula
$$\mathbf{M}^{-1} = \frac{1}{N}[w^{-jk}].$$
3) Use the formula
$$\mathbf{C} = \mathbf{M}^{-1}\mathbf{F},$$
where $\mathbf{F} = \begin{bmatrix} 2 & 0 & 6 & 3 \end{bmatrix}^\top$ to find $\mathbf{C} = \begin{bmatrix} c_0 & c_1 & c_2 & c_3 \end{bmatrix}^\top$.
4) Use Euler's formula to pass from the finite complex Fourier series
$$f(t) = c_0 + c_1 e^{it} + c_2 e^{2it} + c_3 e^{3it}$$
to the finite trigonometric representation.

2. Write out the matrix $\mathbf{M}_8$ in terms of w_8, expressing each entry as the lowest possible positive power of w_8. You do not need to write w_8 explicitly.

Do the same for the inverse matrix $\mathbf{M}_8^{-1}$.

3. Fast Fourier Transform (FFT)

Compute the Fast Fourier Transform (FFT) of the same function given in exercise 1. Check that you get the same result.

Steps:

1) Find the value of w_M, where $M = \frac{N}{2}$.
2) Compute the even and odd coefficients $\mathbf{C}^{(o)}$ and $\mathbf{C}^{(e)}$ using the formula

$$\mathbf{C}^{(o)} = \begin{bmatrix} c_0^{(o)} \\ c_2^{(o)} \end{bmatrix} = \mathbf{M}_2^{-1}\mathbf{f}^{(o)}, \qquad \text{and} \qquad \mathbf{C}^{(e)} = \begin{bmatrix} c_0^{(e)} \\ c_2^{(e)} \end{bmatrix} = \mathbf{M}_2^{-1}\mathbf{f}^{(e)}$$

3) Find the value of w_N.
4) Compute the coefficient c_k using the formulas for $k < M$

$$c_k = \frac{1}{2}\left(c_k^{(o)} + w_N^k c_k^{(e)}\right).$$

And for the coefficient c_k with $k \geq M$,

$$c_k = \frac{1}{2}\left(c_k^{(o)} - w_N^k c_k^{(e)}\right).$$

4. Let $\mathbf{C} = \begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix}^\top$.

- Find $\mathbf{F}$ using $\mathbf{F} = \mathbf{M}_4\mathbf{C}$.
- Find $\mathbf{F}$ using the fast Fourier transform.

5. Fast Fourier Transform (FFT)

Let $N = 4$ and f be a function whose the following values,

$$f(0) = 0, \quad f\left(\frac{2\pi}{N}\right) = 1, \quad f\left(2\frac{2\pi}{N}\right) = 2, \quad f\left(3\frac{2\pi}{N}\right) = 3.$$

Compute the Fast Fourier Transform (FFT) of the function f with the numerical values given above. And write down the finite trigonometric representation of the function f with the coefficients that you found.

<u>**Hand in by:**</u> Wednesday 6 November 2024.