Impacts of Transmission Switching in Zonal Electricity Markets - Part I

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***Abstract*—In this paper, we present a two-stage model of zonal electricity markets with day-ahead market clearing and real-time re-dispatch and balancing that accounts for transmission line switching at both stages. We show how the day-ahead problem with switching can be formulated as an adaptive robust optimization problem with mixed integer recourse and present a new algorithm for solving the adversarial max-min problem that obeys the structure of an interdiction game. We apply the model on a realistic instance of the Central Western European system and comment on the impacts of both proactive and reactive transmission switching on the operating costs of the system.**

Part I presents day-ahead models of a short-term zonal electricity market with switching, and describes our algorithmic approach for solving these models efficiently.

Part II describes variants of the real-time model, and presents the results of our case study on the Central Western European market.

***Index Terms*—Transmission switching, Zonal electricity market, Robust optimization**

I. Introduction

The European day-ahead electricity market is organized as a zonal market. In a zonal design, the nodes of the network are aggregated into a set of zones and the market clears with a unique price for each zone. The market clearing model does not account explicitly for the network constraints within a zone. Instead, it constrains the net positions (i.e. exports - imports) of each zone or the inter-zonal exchanges of power.

There is a long history of papers about zonal pricing, discussing its merits and drawbacks and describing its practical implementations. The development of this literature was triggered by the liberalization of the electric power sector that revealed the challenges of transmission pricing. These challenges were debated in the early stages of US market deregulation [1], [2]. In Europe, an early trigger for the literature on zonal pricing was the deregulation of the Nordic system, see Bjørndal and Jørnsten [3]. This early research focused on the challenge of defining zones and proposed practical computational approaches for improving the method that was in place at the time in the Norwegian market. Transmission capacity allocation became a central topic in subsequent EU market design discussions, we refer the reader to Ehrenmann and Smeers [4]. In [4], the authors highlight certain key challenges that are inherent to zonal market design proposals. Since then, the debates on the relative advantages and disadvantages of zonal pricing have remained vigorous, and new contributions have been inspired by the successive reforms of the European zonal market. The interested reader is referred to [5] for a comprehensive literature review on zonal pricing, that would be outside the scope of this paper.

One recurrent observation that previous work on zonal pricing highlights is that there is no unique or obvious way of organizing a zonal aggregation. Consequently, different methods have been proposed. The original market coupling model that was used for European day-ahead market clearing was the so-called Available-Transfer-Capacity Market Coupling (ATCMC) model. This is effectively a transportation network model; it amounts to defining a set of inter-connectors between each pair of adjacent zones and setting a limit on the maximum electricity exchange that can take place on these inter-connectors. In other words, the model enforces box constraints on the amount of power exchange between zones.

In recent years, the European day-ahead zonal model has been revisited in an effort to improve the efficiency of the zonal design. As an alternative to the ATC model, the so-called Flow-Based Market Coupling (FBMC) model has been adopted. The idea of the flow-based model is to define polyhedral constraints on the zonal net injections of the market clearing model. The goal in adopting FBMC, which is a more general network model than ATCMC, is to increase operational efficiency by increasing the set of feasible trades that can be concluded in the day-ahead market. Flow-based market coupling went live in the Central Wastern Europe (CWE) area in May 2015. Recent research by Aravena [6] raises questions about whether the introduction of FBMC indeed increases short-term operational efficiency relative to ATCMC. Aravena analyzes the implication of both designs on both the day-ahead commitment of thermal resources, as well as real-time operations, when the physical constraints of the network need to be satisfied.

The assessment of short-term operational efficiency hinges on the degree of operational flexibility that is

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afforded to the system operator. A recurrent argument in favor of the zonal design is that its efficiency can be significantly improved by considering the possibility to actively configure the network using active network management measures, such as transmission switching[1]. Our interest in this paper is to provide a quantitative framework for substantiating this argument.

Our two-part paper presents a modeling and algorithmic framework for analyzing zonal markets with switching, and also develops a policy analysis using a simulation model of the Central Western European market. Although there is a strong link between the two parts, we have organized it so that each part is self-contained. Part I presents the model that we have employed for analyzing transmission switching in a zonal market, as well as the algorithmic procedures that we have developed for tackling the problem. Part II focuses on numerical results and policy insights.

Part I is organized as follows: Section II reviews the literature concerning the modeling of short-term electricity markets and the algorithmic approaches for solving the class of problems that emerge from zonal modeling. Section III presents the two-stage models of zonal electricity markets which are considered in the present analysis. In section IV, we present the algorithm that we have developed for solving the day-ahead market-clearing model with proactive switching. First, we show how this problem can be modeled as an adaptive robust optimization problem with mixed integer recourse (AROMIP). Then, we develop a new approach for solving the adversarial max-min subproblem corresponding to robustness to N-1 contingencies, and we show how this algorithm can be inserted into a column-and-constraint generation procedure for solving the AROMIP. Finally, section V discusses the conclusions of the analysis.

II. Literature review

A. Transmission switching

The co-optimization of topology along with generation dispatch in the Optimal Power Flow (OPF) problem as a means of reducing operating cost has received considerable attention by the research community. O'Neill et al. formulated the problem of employing transmission switching in order to improve operational efficiency as a mathematical optimization problem in [7], and dubbed the term *optimal transmission switching*. Fisher et al. [8] demonstrate how this problem can be formulated as an MILP if we consider the DC approximation of power flow. Hedman *et al.* [9] focus on quantifying these gains when the N-1 security criterion is accounted for. This early literature on transmission switching considered a *dispatch* model. Therefore, they only focused on the benefits of switching as an option to be considered in *dispatch*, where commitment decisions are assumed to be fixed. These studies are not directly relevant in the context of our paper which is focused on the interaction of switching with unit commitment.

It is only in subsequent work [10] that the interactions of switching with day-ahead unit commitment were considered in the literature. However, these models are also not directly applicable to our context, since they represent a nodal transmission model. Instead, the novelty of our work is in considering a zonal transmission model, which is the predominant network model that is used in European day-ahead market clearing. The introduction of the zonal network representation, and its interaction with unit commitment, introduces a host of modeling and computational challenges, that are the focus of the present paper.

B. Modeling of the European electricity market

In addition to the different representation of network constraints in day-ahead, the European design differs with respect to several other aspects from the US design. Accounting for these differences requires a different modeling set-up. This paper builds off of recent research on developing a precise model of the day-ahead and real-time operation of the European electricity market. Aravena and Papavasiliou [11] develop a hierarchy of European electricity market models that are targeted at accounting for unit commitment and the separation between energy and reserves. Based on this work, Han and Papavasiliou [12] develop a model of the European market that accounts for transmission switching. Although their model is highly simplified (the representation of day-ahead flow scheduling is inaccurate, security criteria are not accounted for directly, the simultaneous optimization of unit commitment and switching is treated heuristically), this first analysis demonstrates encouraging results by demonstrating that transmission switching can lead to significant cost savings in a zonal market when re-dispatch and balancing are perfectly coordinated in real time.

C. Robust optimization

Accounting for the N-1 security criterion introduces significant complexity to the problem at hand. We review here some previous work that relates to our problem either from a modeling or from an algorithmic point of view. The goal is not to be exhaustive. Sun and Lorca [13] provide an in-depth review of modeling and algorithmic approaches for robust optimization in power systems. A common attribute of the papers cited below is that they tackle a certain class of robust optimization problems. We distinguish models and algorithms for adaptive and reactive robust optimization. In both cases, the recourse problem can be continuous or include integer restrictions. This structure is represented in Figure 1.

[1]It is worth noting that transmission switching is employed extensively in Europe, much more so than in US system operations.

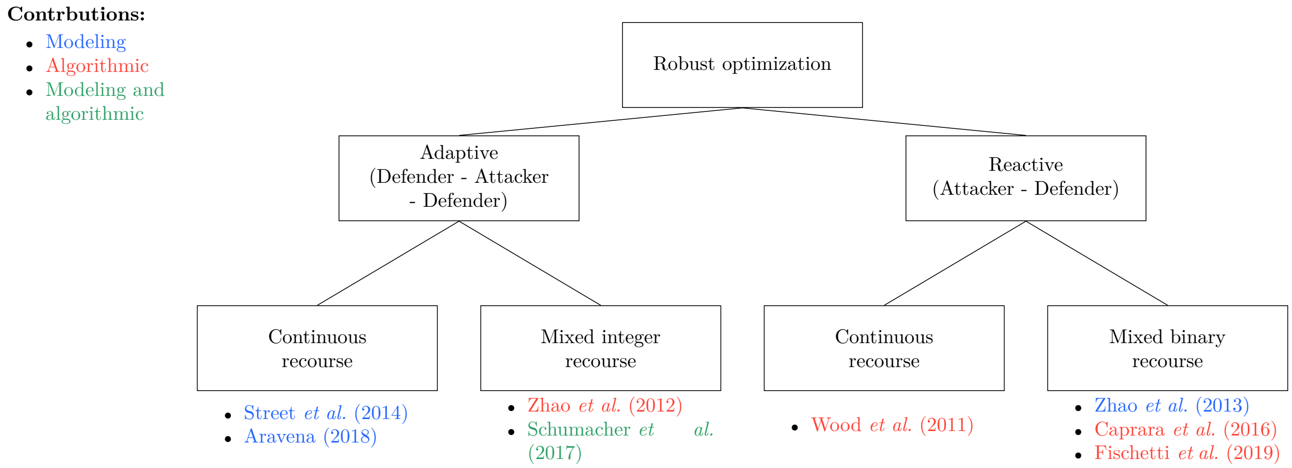

Figure 1: Decomposition of robust optimization into the different subproblems and the papers associated to each case. We also use a color code to distinguish the modeling and algorithmic contributions.

Street et al. [14] propose a formulation of the OPF problem with N-k robustness as an Adaptive Robust Optimization (ARO) problem and present a cutting plane algorithm for solving it. The algorithm is inspired by dual methods for ARO that rely on Benders decomposition. Similarly, Aravena [6] proposes a cutting plane approach for solving the day-ahead market clearing problem of a zonal market clearing model, which respects the N-1 security criterion and the European rules for setting day-ahead prices. A new layer of complexity is added to these two models when we consider transmission switching as a recourse action. In this setting, the aforementioned approaches that rely on Benders decomposition cannot be used due to the presence of integer variables in the inner problem. An algorithm for attacking this class of problems has been proposed recently by Zhao and Zeng [15]. Their idea is to consider only a subset of the possible integer values in the inner problem, which results in an LP formulation that can be dualized. Promising candidate integer values are identified and added in a sequential manner. This approach has been applied to the optimal power flow problem with line interdiction [16]. In a similar spirit, Schumacher et al. [17] also generate switching variables as needed, in order to solve the N-2 security-constrained unit commitment problem with transmission switching.

In parallel to this work, new approaches have been developed in the literature on interdiction games for solving robust optimization problems with binary uncertainty and mixed integer recourse. In a survey paper on the subject, Wood presents a cutting plane algorithm for solving interdiction games with continuous linear recourse [18]. This approach has been employed in by Caprara et al. [19] for solving the bi-level knapsack with interdiction constraints, which is a particular instance of an interdiction game with binary recourse. This algorithm has been extended in by Fischetti et al. [20] to a particular class of interdiction games with mixed integer recourse, i.e. those with the property of monotonicity. Most network interdiction games, however, do not satisfy the property of monotonicity. The line and generator interdiction game with transmission switching, which is the problem that we are interested in, is one of them. To the best of our knowledge, no efficient algorithm based on this approach has been proposed for solving the problem.

Our paper combines the modeling and computational literature cited above by formulating the problem of day-ahead N-1 market clearing as an ARO with mixed integer recourse. We use the column-and-constraint generation algorithm of Zeng and Zhao for solving the outer loop and present a new approach based on the cutting plane formulation used in interdiction games for solving the max-min adversarial subproblem.

D. Contributions

Our present work provides the following contributions to the literature:

1) We present a model for the organization of a zonal market that accounts for transmission switching at both the day-ahead and the real-time stages.
2) We present a new approach for solving the adversarial subproblem (i.e. the max-min stage) of an adaptive robust optimization problem, which is a min-max-min problem in its entirety, when it has the structure of an interdiction game. The adversarial subproblem is a mixed integer recourse problem with binary uncertainty. We integrate our approach for the resolution of the max-min adversarial subproblem into a known column-and-constraint generation algorithm for resolving AROMIP. In this way, we obtain a tractable procedure for solving the day-ahead market clearing problem with proactive switching.

III. Models of transmission switching in zonal markets

In a zonal market, the nodes of the network are aggregated into a set of zones. Network constraints inside a zone are ignored, and there is a unique price for each zone. Market clearing takes place in two stages. The zonal market is cleared in the day-ahead stage and results in the commitment of slow units. Then, as network constraints have not been represented exactly, and as the state of the grid evolves between the day ahead and real time, the system operator conducts re-dispatch (also referred to as congestion management) and balancing close to real time, while respecting the commitment of units determined in the day-ahead stage. This process ensures the feasibility of the dispatch with respect to the actual state of the grid at the time of delivery, as well as the balancing of supply and demand.

In this work, we follow this two-settlement organization of the market by presenting a two-stage model with a zonal market clearing in the day ahead and a re-dispatch and balancing process in real time. Transmission switching can be used in both the first stage (in which case we refer to proactive switching) and the second stage (in which case we refer to reactive switching). The two-stage structure of our model with the different inputs and outputs of each module is represented graphically in Fig. 2.

A. Day-ahead market clearing with proactive switching

We define the net position of a node (resp. zone) as the difference between the power produced and consumed within that node (resp. zone). Whereas a nodal market clearing model defines constraints on the net position of each node, a zonal market clearing model constrains the *zonal* net positions. The set of feasible zonal net positions depends on the chosen topology of the grid. We index the chosen topology by t in the sequel.

Extending the formulation in [6], we can write a zonal market clearing model with transmission switching in its simplest form as follows:

$$\min_{v\in[0,1],p,t} \sum_{g\in G} P_g Q_g v_g \tag{1}$$

$$\text{s.t.} \sum_{g\in G(z)} Q_g v_g - p_z = \sum_{n\in N(z)} Q_n \qquad \forall z \in Z \tag{2}$$

$$p \in \mathcal{P}_t \tag{3}$$

where Q_g and P_g correspond to the quantity and price bid by generator $g \in G$, $G(n)$ is the set of generators at node $n \in N$, Q_n is the forecast demand at node n, p_z corresponds to the net positions of zone $z \in Z$, v_g is the acceptance/rejection decision for the bid placed by generator g, and $G(z), N(z)$ correspond to the set of generators and nodes within a zone z. The set $\mathcal{P}_t$ corresponds to the feasible set of net positions for a particular topology t. In practice, the day-ahead model (1)-(3) results also in the commitment of slow units that are allowed to submit block bids. We do not represent explicitly the binary commitment variables in this model for simplicity of the exposition.

Note that, in formulation (1)-(3), the control space is limited to a $|Z|$-dimensional space, which is much smaller than an $|N|$-dimensional space (the control space in nodal pricing). For instance, in the CWE instance that we use in our case study, there are 632 buses but only 5 zones.

We adopt the following assumption in the sequel:

Assumption 1. We assume that $\mathcal{P}_t$ can be described as a set of linear inequalities which implicate a binary vector $t \in \{0,1\}^{|L|}$, where L is the set of lines in the network.

This set $\mathcal{P}_t$, that imposes constraints on the net positions in the day-ahead market, has a central role in the zonal market model. It is the basic building block that must be defined in order to characterize the market and that should be modified in order to account for new features, such as transmission switching or the N-1 security criterion. In practice, the set $\mathcal{P}_t$ is defined by PTDF-like constraints as follows [6]:

$$\mathcal{P}_t^{TSO} = \Bigg\{ p \in \mathbb{R}^{|Z|} \;\Bigg|\; \sum_{z\in Z} p_z = 0, \\ \sum_{z\in Z} PTDF_{cb,z} \cdot p_z \le RAM_{cb} \;\forall cb \in CB \Bigg\} \tag{4}$$

Here, cb corresponds to a critical branch, i.e. a branch that is significantly impacted by cross-zonal exchanges. The parameter $PTDF_{cb,z}$ is the zone-to-line PTDF from zone z to line cb, which indicates the change in power flow on line cb resulting from a unit increase in the net position of zone z. The parameter RAM_{cb} is the Remaining Available Margin, a parameter that evaluates the capacity of line cb that can be used for cross-border exchanges. These parameters, and the FBMC method in general, are described in further detail in the literature [21], as well as in documentation published by the European transmission system operators (TSOs) [22], [23].

The following challenges emerge when modeling flow-based market coupling for the purpose of policy analysis:

1) The flow-based polytope presented above is characterized by parameters (e.g. CB, $PTDF_{cb,z}$, RAM, ...), the definition of which differs among TSOs. This makes it difficult to represent exactly the current TSO practice, particularly since the market clearing outcome is sensitive to these parameters [24].
2) The method used in practice faces a circularity problem: the parameters that are used for defining the flow-based domain are computed from a forecast

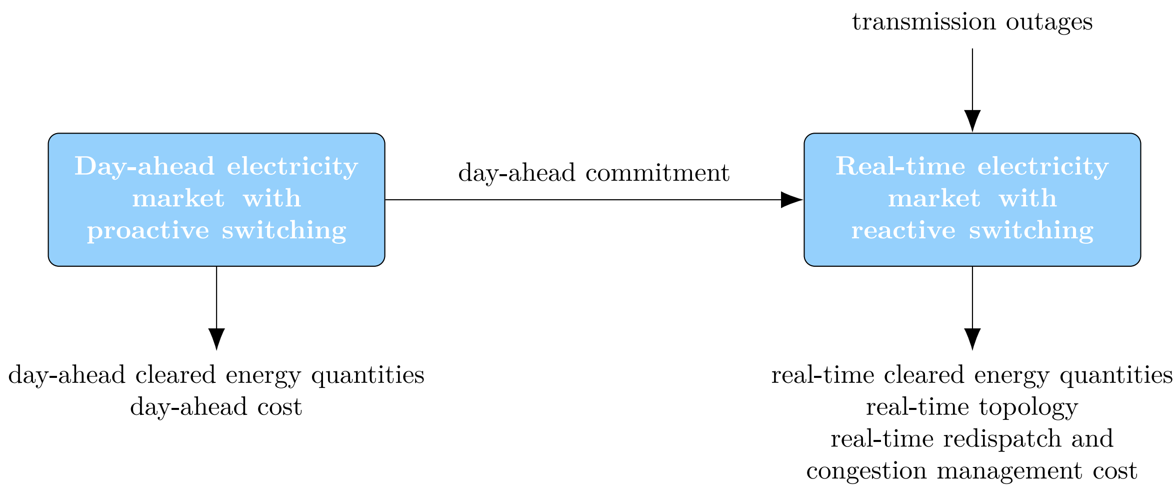

Figure 2: Block diagram of the two-settlement system used in this work.

of the outcome of the market, which depends itself on the the value of these parameters.

Instead of building a model that attempts to replicate the current practice, Aravena [6] proposes a definition of the flow-based domain that does not depend on arbitrary parameters. We extend this definition in order to account for the possibility to use transmission switching while defining the acceptable set of net positions. This set can be written as:

$$
\begin{aligned}
\mathcal{P}_t = \Big\{ & p \in \mathbb{R}^{|Z|} : \\
& \exists(\bar{v}, f, \theta, t) \in [0,1]^{|G|} \times \mathbb{R}^{|L|} \times \mathbb{R}^{|N|} \times \{0,1\}^{|L|} : \\
& \sum_{g \in G(z)} Q_g \bar{v}_g - p_z = \sum_{n \in N(z)} Q_n, \quad \forall z \in Z \qquad \text{(5a)} \\
& \sum_{g \in G(n)} Q_g \bar{v}_g - \sum_{l \in L(n,\cdot)} f_l + \sum_{l \in L(\cdot,n)} f_l = Q_n, \qquad \text{(5b)} \\
& \qquad \forall n \in N \\
& - t_l F_l \le f_l \le t_l F_l, \quad \forall l \in L \qquad \text{(5c)} \\
& f_l \le B_l(\theta_{m(l)} - \theta_{n(l)}) + M(1 - t_l), \quad \forall l \in L \qquad \text{(5d)} \\
& f_l \ge B_l(\theta_{m(l)} - \theta_{n(l)}) - M(1 - t_l), \quad \forall l \in L \Big\} \qquad \text{(5e)}
\end{aligned}
$$

Here, B_l is the susceptance of line $l \in L$, F_l is the thermal limit, and $m(l)$ and $n(l)$ are the adjacent nodes of line l (in the outgoing and incoming directions respectively), $L(m, n)$ is the set of lines directed from node m to node n, f_l is the flow through line l, θ_n is the voltage angle at node n, and M is a sufficiently large constant.

The interpretation of set $\mathcal{P}_t$ is as follows: for every acceptable vector of net positions p, there should exist a generator dispatch $\bar{v}$ that aggregates to this vector of net positions (Eq. (5a)) while respecting the nodal grid constraints (Eq. (5b)-(5e)) under transmission switching. Switching off a line is modeled here by binary variables t_l. If $t_l = 0$, then line l is disconnected. This implies that the flow on the line must be zero (Eq. (5c)) and the voltage angles at the two ends of the line must be independent ((5d) and (5e) become trivially satisfied if M is large enough).

In this way, the definition of $\mathcal{P}_t$ that we use circumvents the problem of circular definitions and discretionary values for the flow-based polytope. Instead, it is inspired by the following principles laid out in European legislation [25], [26]:

1) All feasible trade should be allowed to be cleared (Regulation (EC) 2009/714, Annex I, Art. 1.7 [25], Annex I, Art. 1.1, 1.2 and 1.6; Regulation (EU) 2019/943, Art. 7.2.(c) [27]).
2) Cross-zonal capacity should be firm, which implies that no trade that provably leads to infeasible zonal net positions should be cleared (Regulation (EU) 2015/1222, Art. 69 [26]).

The domain $\mathcal{P}_t$ represents exactly the set of net positions that respects both principles. The reader is referred to Aravena [6] for a more detailed description of $\mathcal{P}_t$, as well as its implications on zonal market clearing.

B. Day-ahead market clearing with proactive switching and security criterion

In practice, the clearing of the European day-ahead market needs to respect the N-1 security criterion. We define N-1 robustness as the ability of a system to serve demand under any outage of a single transmission line in the system. The zonal market clearing model, as presented in (1) - (3), does not respect N-1 security. The modifications that are required in order to introduce N-1 security depend on whether the remedial actions (i.e. the actions that the TSOs resort to in response to a contingency) are preventive (i.e. applied before the realization of a contingency) or curative (i.e. applied in reaction to a contingency). In theory, RAs can be preventive or curative [23]. In practice, however, re-dispatch is always used in a preventive way as curative re-dispatch is not considered to be safe enough by TSOs[2]. Topology measures, in contrast, can be applied both preventively and curatively. In what follows, we modify model (1) - (3) in order to account for the N-1 security criterion with purely preventive dispatch (subsection III-B1), purely curative re-dispatch (subsection III-B2) and a hybrid preventive-curative re-dispatch (subsection III-B3).

Let $u \in \{0,1\}^{|L|}$ be the vector of contingencies. When one element of vector u is equal to 1, it means that the corresponding transmission line is out of service.

1) Preventive re-dispatch: The constraint on the acceptable net positions with preventive dispatch can be written as:

$$p \in \mathcal{P}_t^{\text{prev}}, \tag{6}$$

with

$$\begin{aligned}\mathcal{P}_t^{\text{prev}} = \Big\{ & p \in \mathbb{R}^{|Z|} : \exists\ \bar{v} \in [0,1]^{|G|} : \\ & \sum_{g \in G(z)} Q_g \bar{v}_g - p_z = \sum_{n \in N(z)} Q_n, \quad \forall z \in Z \\ & \bar{v} \in \bigcap_{\|u\|_1 \le 1} \mathcal{V}_t(u) \Big\}\end{aligned}$$

and

$$\begin{aligned}\mathcal{V}_t(u) = \Big\{ & v \in [0,1]^{|G|} : \\ & \exists (f, \theta, t) \in \mathbb{R}^{|L|} \times \mathbb{R}^{|N|} \times \{0,1\}^{|L|} : \\ & \sum_{g \in G(n)} Q_g v_g - \sum_{l \in L(n,\cdot)} f_l + \sum_{l \in L(\cdot,n)} f_l = Q_n, \quad \forall n \in N \\ & - t_l F_l \le f_l \le t_l F_l, \quad \forall l \in L \\ & f_l \le (1-u_l) B_l (\theta_{m(l)} - \theta_{n(l)}) + M(1-t_l), \quad \forall l \in L \\ & f_l \ge (1-u_l) B_l (\theta_{m(l)} - \theta_{n(l)}) - M(1-t_l), \quad \forall l \in L \Big\}\end{aligned}$$

The set $\mathcal{V}_t(u)$ corresponds to all dispatch decisions that respect power flow constraints and line limits under contingency u, when transmission switching is allowed. The interpretation of the set $\mathcal{P}_t^{\text{prev}}$ is thus the following: to every acceptable vector of net positions p, there should exist a generator dispatch $\bar{v}$ that aggregates to this vector of net positions, and that respects grid constraints for every contingency u.

The constraints of the market clearing model under the N-1 security criterion with preventive dispatch can be represented through Eqs. (1), (2) and (6).

2) Curative re-dispatch: The flow-based domain with curative re-dispatch can be described as follows:

$$p \in \bigcap_{\|u\|_1 \le 1} \mathcal{P}_t^{\text{cur}}(u) \tag{7}$$

[2]This has been initially pointed out by an anonymous reviewer and confirmed to us during personal communications with G. Maes (ENGIE)

with

$$\begin{aligned}\mathcal{P}_t^{\text{cur}}(u) = \Big\{ & p \in \mathbb{R}^{|Z|} : \\ & \exists (\bar{v}, f, \theta, t) \in [0,1]^{|G|} \times \mathbb{R}^{|L|} \times \mathbb{R}^{|N|} \times \{0,1\}^{|L|} : \\ & \sum_{g \in G(z)} Q_g \bar{v}_g - p_z = \sum_{n \in N(z)} Q_n, \quad \forall z \in Z \\ & \sum_{g \in G(n)} Q_g \bar{v}_g - \sum_{l \in L(n,\cdot)} f_l + \sum_{l \in L(\cdot,n)} f_l = Q_n, \quad \forall n \in N \\ & - t_l F_l \le f_l \le t_l F_l, \quad \forall l \in L \\ & f_l \le (1-u_l) B_l (\theta_{m(l)} - \theta_{n(l)}) + M(1-t_l), \quad \forall l \in L \\ & f_l \ge (1-u_l) B_l (\theta_{m(l)} - \theta_{n(l)}) - M(1-t_l), \quad \forall l \in L \Big\}\end{aligned}$$

The constraints of the market clearing model under the N-1 security criterion with curative re-dispatch can be represented through Eqs. (1), (2) and (7).

The interpretation of Eq. (7) is as follows: a vector of net positions is acceptable if, for every contingency, there exists a dispatch that respects all grid constraints. Note the *fundamental difference* with the case of preventive re-dispatch: in *curative* re-dispatch, the dispatch can be *different* for every contingency, while in the case of *preventive* re-dispatch, the dispatch must be *the same* for every contingency. The reader is referred to Appendix A for an illustrative example of the difference between a day-ahead model with preventive and curative re-dispatch.

Mathematically, the main difference between the flow-based domain described by Eq. (6) and the domain described by Eq. (7) is that, in Eq. (6), the intersection over all contingencies is over a set of dimension $|G|$; instead, in Eq. (7), the intersection is over a set of dimension $|Z|$, which is much smaller. This has computational implications, as we discuss later.

3) Hybrid preventive-curative re-dispatch: Combining the two previous models, we can easily extend them to the case of a re-dispatch that is neither purely preventive nor purely curative: the *hybrid* preventive-curative model. A hybrid model corresponds to a dispatch that is preventive for a subset of contingencies U_{prev}, and curative for all other contingencies $U_{\text{cur}} = U \backslash U_{\text{prev}}$. The flow-based domain for this hybrid model can be written as follows:

$$\begin{aligned}\mathcal{P}_{\text{hyb}}^{\text{FB}} = \Big\{ & p \in \mathbb{R}^{|Z|} : \exists\ \bar{v} \in [0,1]^{|G|} : \\ & \sum_{g \in G(z)} Q_g \bar{v}_g - p_z = \sum_{n \in N(z)} Q_n, \quad \forall z \in Z && \text{(8a)} \\ & \bar{v} \in \mathcal{V}_t(u) \quad \forall u \in U_{\text{prev}}, && \text{(8b)} \\ & p \in \mathcal{P}_t^{\text{cur}}(u) \quad \forall u \in U_{\text{cur}} \Big\} && \text{(8c)}\end{aligned}$$

Model (8) is a direct extension of the preventive and the curative cases. Eqs. (8a)-(8b) express the fact that the acceptable net positions should disaggregate in a dispatch that is feasible for all contingencies belonging to set U_{prev}, at the same time. Eq. (8c) express the fact that for any contingency belonging to U_{cur}, the net positions disaggregate in a dispatch that is feasible for the grid, after the realization of that contingency.

C. Real-time re-dispatch and balancing

The goal of the re-dispatch and balancing process is to modify the day-ahead dispatch so as to balance the system at minimum cost in real time, while respecting network constraints. We assume that there is no uncertainty in demand and renewable production. Uncertainty can only materialize in a transmission line outage. Thus, we focus on the re-dispatch and balancing actions that are required (i) in case the zonal day-ahead auction violates any inter or intra-zonal transmission constraints and (ii) in case the operator is required to react to a transmission contingency that may occur between the day ahead and real time.

We consider various real-time models in part 2, depending on the objective of the TSO and the degree of coordination among TSOs. The ideal standard in terms of coordination and cost minimization is the perfectly coordinated re-dispatch and balancing model which aims at minimizing real-time cost. We introduce reactive transmission switching to this model, and formulate it as an optimization problem as follows:

$$\begin{aligned}
\min_{\substack{v\in[0,1]\\ f,\theta,t}} \; & \sum_{g\in G} P_g Q_g v_g \\
\text{s.t.} \; & \sum_{g\in G(n)} Q_g v_g - \sum_{l\in L(n,\cdot)} f_l + \sum_{l\in L(\cdot,n)} f_l = Q_n, \\
& \hspace{18em} \forall n \in N \\
& -F_l t_l \le f_l \le F_l t_l, \quad \forall l \in L \\
& f_l \le B_l(\theta_{m(l)} - \theta_{n(l)}) + M(1-t_l), \quad \forall l \in L \\
& f_l \ge B_l(\theta_{m(l)} - \theta_{n(l)}) - M(1-t_l), \quad \forall l \in L
\end{aligned}$$

We stress the fact that, although this second-stage real-time problem is nodal, the day-ahead zonal market clearing drives the real-time behavior because of the fixed unit commitment decisions that must be respected. We do not represent here the dependence of real-time dispatch on day-ahead unit commitment decisions, in order to simplify the exposition, but enforce this requirement for the simulations presented in part II.

IV. An algorithm for proactive transmission switching

In this section, we present an algorithm for solving the zonal day-ahead market clearing model under N-1 robustness with proactive transmission switching and a purely curative re-dispatch. We then discuss how the algorithm can be adapted to the case of a preventive re-dispatch. The zonal day-ahead market clearing problem can be written as:

$$\min_{\substack{v\in[0,1],p,\\ t\in\{0,1\}}} \sum_{g\in G} P_g Q_g v_g \tag{9}$$

$$\text{s.t.} \sum_{g\in G(z)} Q_g v_g - p_z = \sum_{n\in N(z)} Q_n, \qquad \forall z \in Z \tag{10}$$

$$p \in \bigcap_{\|u\|_1\le 1} \mathcal{P}_t(u) \tag{11}$$

The difficulty in solving this problem lies in the fact that its equivalent monolithic formulation is too large to be solved directly, while the non-convexity of the set of feasible net positions prevents the use of a cutting plane approach similar to the one proposed by [6].

Our idea for solving this problem is to rewrite it as an adaptive robust optimization problem with mixed integer recourse (AROMIP), and to use a known column-and-constraint generation (C&CG) algorithm for this class of problems. The general AROMIP that we consider can be described as follows:

$$\min_{\mathbf{x}\in\mathbb{X}} \mathbf{cx} + \max_{\mathbf{u}\in\mathbb{U}} \min_{\mathbf{z},\mathbf{y}\in\mathbb{F}(\mathbf{u},\mathbf{x})} \mathbf{dy} + \mathbf{gz} \tag{12}$$

where $\mathbb{X} = \{\mathbf{x} \in \mathbb{R}^m_+ \times \mathbb{Z}^m_+ : A\mathbf{x} \ge b\}, \mathbb{F}(\mathbf{u},\mathbf{x}) = \{(\mathbf{z},\mathbf{y}) \in \mathbb{Z}^n_+ \times \mathbb{R}^p_+ : E(\mathbf{u})\mathbf{y} + G(\mathbf{u})\mathbf{z} \ge f(\mathbf{u}) - R\mathbf{u} - D(\mathbf{u})\mathbf{x}\}$, and the uncertainty set $\mathbb{U}$ is a bounded binary set in the form of $\mathbb{U} = \{u \in \mathbb{B}^q_+ : H\mathbf{u} \le a\}$. This formulation is similar to that of [15]. The only difference is that we restrict ourselves to a pure binary uncertainty set $\mathbb{U}$, which corresponds in our case to the set of line contingencies. We also consider a more general form of $\mathbb{F}(\mathbf{u},\mathbf{x})$, where every parameter (E, G, f, D) can depend on the realization of uncertainty.

Let us now show how we can reformulate our problem in the general form (12). First, notice that in problem (9) - (11) it is equivalent to replace constraint (11) by

$$d(p, \bigcap_{\|u\|_1\le 1} \mathcal{P}_t(u)) = 0 \tag{13}$$

where $d(p, \bigcap_{\|u\|_1\le 1} \mathcal{P}_t(u))$ is the L_1 distance of injection p to the set of net positions.

We then penalize this function in the objective and show in Proposition 1 that there exists a penalizing factor λ^* such that problem (9) - (10), (13) is equivalent to

$$\min_{v\in[0,1],p,t} \sum_{g\in G} P_g Q_g v_g + \lambda^* \Big(d(p, \bigcap_{\|u\|_1\le 1} \mathcal{P}_t(u)) \Big) \tag{14}$$

$$\text{s.t.} \sum_{g\in G(z)} Q_g v_g - p_z = \sum_{n\in N(z)} Q_n, \qquad \forall z \in Z \tag{15}$$

Proposition 1. *Consider the following two optimization problems:*

$$\begin{aligned}
(P1) : \min_{p,t} \; & cp \\
\text{s.t.} \; & Ap \le b \\
& d(p, \mathcal{P}_t) = 0
\end{aligned}$$

and

$$(P2): \min_{p,t} \; cp + \lambda^* \, d(p, \mathcal{P}_t)$$
$$\text{s.t. } Ap \leq b$$

where $\mathcal{P}_t$ is a general polyhedron described by a set of linear inequalities implicating an integer vector of variables t, and where $d(\cdot,\cdot)$ is the L1 distance function of a vector to a polyhedron. There exists a scalar λ^* such that (P1) and (P2) are equivalent.

Proof. The proof is inspired by [28]. Suppose, without loss of generality, that $\mathcal{P}_t = \{p \in \mathbb{R}^P : V_t p \leq W_t\}$. Then,

$$\begin{aligned}
(P1) \Leftrightarrow \min_t \; & \min_p \; cp \\
& \text{s.t. } Ap \leq b \\
& \quad\;\; V_t p \leq W_t \\
\Leftrightarrow \min_t \; & \min_{p,\tilde{p},s_1,s_2} cp \\
& \text{s.t. } Ap \leq b \\
& \quad\;\; \mathbb{1}^\top s_1 + \mathbb{1}^\top s_2 \leq 0 \quad [\lambda_t] \\
& \quad\;\; s_{1i} \geq p_i - \tilde{p}_i \quad \forall i = \{1, ..., Z\} \\
& \quad\;\; s_{2i} \geq \tilde{p}_i - p_i \quad \forall i = \{1, ..., Z\} \\
& \quad\;\; V_t \tilde{p} \leq W_t \\
& \quad\;\; s_1, s_2 \geq 0
\end{aligned}$$

where the scalar λ_t is the inner-problem dual variable of the constraint on the distance between p and $\tilde{p}$, that depends on t. Let $\overline{\lambda}$ be an upper bound of λ_t for each t. By using Lemma 1 of [28], we have that

$$\begin{aligned}
(P1) \Leftrightarrow \min_t \; & \min_{p,\tilde{p},s_1,s_2} cp + \overline{\lambda}(\mathbb{1}^\top s_1 + \mathbb{1}^\top s_2)^+ \\
& \text{s.t. } Ap \leq b \\
& \quad\;\; s_{1i} \geq p_i - \tilde{p}_i \quad \forall i = \{1, ..., Z\} \\
& \quad\;\; s_{2i} \geq \tilde{p}_i - p_i \quad \forall i = \{1, ..., Z\} \\
& \quad\;\; V_t \tilde{p} \leq W_t \\
& \quad\;\; s_1, s_2 \geq 0
\end{aligned}$$

By the non-negativity of s_1 and s_2, $(\mathbb{1}^\top s_1 + \mathbb{1}^\top s_2)^+ = \mathbb{1}^\top s_1 + \mathbb{1}^\top s_2$. Therefore, we conclude, by the definition of the L1 distance, that

$$(P1) \Leftrightarrow \begin{bmatrix} \min_{p,t} \; cp + \overline{\lambda} \, d(p, \mathcal{P}_t) \\ \text{s.t. } Ap \leq b \end{bmatrix} \Leftrightarrow (P2)$$

□

We now define explicitly the distance function as the following max-min problem:

$$d(p, \bigcap_{\|u\|_1 \leq 1} \mathcal{P}_t(u)) = \max_{u \in \mathbb{U}} \; \min_{\tilde{p},t} \; \|p - \tilde{p}\|_1 \tag{16}$$
$$\text{s.t. } \tilde{p} \in \mathcal{P}_t(u) \tag{17}$$

With the formulation (14) - (15) which is justified by the result of Proposition 1, we are now in the framework of adaptive robust optimization with mixed integer recourse. The correspondence in notation between the generic AROMIP and our specific application is the following: $\mathbf{x} = (v, p)$, $\mathbf{z} = t$, $\mathbf{y} = (s_1, s_2, \tilde{p})$, $\mathbf{dy} + \mathbf{gz} = \sum_{g \in G} s_{1,g} + s_{2,g}$, $\mathbb{Y} = \{v, p : \sum_{g \in G(z)} Q_g v_g - p_z = \sum_{n \in N(z)} Q_n \; \forall z \in Z\}$, $\mathbb{U}$ is the set of all possible contingencies such that $\|u\|_1 \leq 1$, and

$$\begin{aligned}
\mathbb{F}(\mathbf{u}, \mathbf{x}) = \Big\{ \tilde{p} \; : & \tilde{p} \in \mathcal{P}_t(u) \\
& s_{1,g} \geq p_g - \tilde{p}_g, \;\; \forall g \in G \\
& s_{2,g} \geq \tilde{p}_g - p_g, \;\; \forall g \in G \Big\}
\end{aligned}$$

which can be written as a mixed integer linear feasibility set if Assumption 1 holds.

A. Outer-level column-and-constraint generation algorithm

Two different classes of methods have been proposed in the literature for solving two-stage robust optimization problems [29]. *Benders dual methods*, as in Benders decomposition, use the dual information of the second-stage problem to sequentially approximate the first-stage value function. *Column-and-constraint generation methods* gradually include the variables and constraints of the monolithic formulation. In [6], the first approach has been used, leading to a cutting plane based algorithm for solving the market clearing problem. However, as mentioned previously, the presence of binary variables in the second-stage problem, which correspond to transmission switching decisions, prevents the use of Benders dual methods in the context of our problem. In contrast, [15] describes how a column-and-constraint generation method can be used for solving adaptive robust optimization problems with integer variables in the recourse problem, provided we can solve exactly the second-stage max-min problem for a given choice of first-stage decisions.

Let us therefore assume that we can solve this second-stage problem (which corresponds, in our case, to computing the distance of a net position vector to the set of net positions) and let us apply the column-and-constraint generation algorithm of [15] to our problem. We will then explain how we can solve the second-stage problem in section IV-B.

Algorithm 1 : Column-And-Constraint Generation Algorithm

1) Set $LB = +\infty, UB = -\infty$ and $k = 0$
2) Solve the following master problem:

$$\begin{aligned}
\textbf{MP:} \min_{\substack{v \in [0,1] \\ p,t,\eta}} & \sum_g Q_g P_g v_g + \lambda^* \eta \\
\text{s.t.} & \sum_{g \in G(z)} Q_g v_g - p_z = \sum_{n \in N(z)} Q_n \\
& \eta \geq |p^i - p|, \qquad \forall i \in \{1, ..., k\} \\
& p^i \in \mathcal{P}_{t^i}(u^i), \qquad \forall i \in \{1, ..., k\}
\end{aligned}$$

Update $LB = \sum_g Q_g P_g v_g^* + \lambda^* \eta^*$. If $UB - LB < \epsilon$, terminate.

3) Call the oracle to solve subproblem $d(p^*, \underset{\|u\|_1 \le 1}{\cap} \mathcal{P}_t(u))$ and update UB as

$$\min \left(UB, \sum_g Q_g P_g v_g^* + d(p^*, \underset{\|u\|_1 \le 1}{\cap} \mathcal{P}_t(u)) \right)$$

If $UB - LB < \epsilon$, terminate.

4) Create variable p^i and add the following constraints

$$\eta \ge |p^i - p|$$
$$p^i \in \mathcal{P}_{t^i}(u_i^*)$$

where u^* is the optimal value of variable u in the subproblem.

□

This algorithm can be further simplified by noticing that, by definition of λ^*, $\eta^* = 0$ at each iteration of the algorithm. This renders the algorithm totally independent of λ^*, for λ^* sufficiently large. This also implies that $LB = \sum_g Q_g P_g v_g^*$ at each iteration, and thus that the algorithm will terminate when

$$d(p^*, \underset{\|u\|_1 \le 1}{\cap} \mathcal{P}_t(u_i)) < \epsilon \quad \forall i \in \{1, ..., k\},$$

i.e. when the optimal net position obtained with only a subset of the possible contingencies is actually robust to all contingencies.

B. Inner level max-min problem

So far, we have assumed that we were able to solve exactly the second-stage problem. To have a complete algorithm, it remains to show how we can solve the second-stage problem. Recall that the second-stage problem can be written as follows:

$$\begin{aligned} d(p, \underset{\|u\|_1 \le 1}{\cap} \mathcal{P}_t(u)) = \max_{u \in \mathbb{U}} \ &\min_{\tilde{p}, t} \ |p - \tilde{p}| \\ &\text{s.t. } \tilde{p} \in \mathcal{P}_t(u) \end{aligned} \tag{18}$$

Zhao and Zeng [15] propose solving this inner problem by using a column-and-constraint generation algorithm, exactly in the same fashion as the outer problem. With this method, the master problem first considers only a subset of topologies. The master problem clears with the best net position that corresponds to this subset of topologies. Then, the subproblem identifies the best topology than can react to the vector of net positions identified and this topology is added to the master problem. This procedure repeats until no more new topologies are added to the master. The drawback of this method is that, for each topology under considerarion, the other variables (v, f, θ in our case) must be duplicated. This quickly introduces a bottleneck in terms of efficiency in the master problem, which increases the run time of the algorithm.

In what follows, we propose a new approach for the inner max-min problem that avoids the bottleneck of the nested C&CG method. Our approach builds on the observation that this problem falls in the class of interdiction games, i.e. two player Stackelberg games where the decision variables of the leader are binary. When set to 1, these variables force the corresponding follower variables to be 0, thereby interdicting the follower from choosing certain actions. In our context, the leader is Nature and is looking for the line to place out of service so as to maximize the disruption to the system operator. The system operator can react with switching.

Based on [30], we will first show how a cutting plane formulation of our problem can be obtained. Let $\mathcal{Q}$ be the set $\mathcal{P}_t(\mathbf{0})$ in the space of p and t, i.e. the feasible set of net positions and switching variables under no contingency. Then, problem (18) can be formulated equivalently as

$$\begin{aligned} \max_{u \in \mathbb{U}} \ \min_{\tilde{p}, t} \ & |p - \tilde{p}| \\ \text{s.t. } & (\tilde{p}, t) \in \mathcal{Q} \\ & t_l u_l = 0 \quad \forall l \in L \end{aligned} \tag{19}$$

We also have the following result.

Proposition 2. *Let us consider the following problem for a fixed vector u^*:*

$$\min_{\tilde{p}, t} \ |p - \tilde{p}| \tag{20a}$$
$$s.t. \ (\tilde{p}, t) \in \mathcal{Q} \tag{20b}$$
$$t_l u_l^* = 0 \quad \forall l \in L \tag{20c}$$

If λ_l is an optimal Lagrangian dual multiplier of constraint (20c)*, then problem* (20) *is equivalent to*

$$\begin{aligned} \min_{\tilde{p}, t} \ & |p - \tilde{p}| + \sum_{l \in L} \lambda_l t_l u_l^* \\ s.t. \ & (\tilde{p}, t) \in \mathcal{Q}, \end{aligned} \tag{21}$$

i.e. there is no Lagrangian duality gap for constraint (20c).

Proof. First notice that, because constraint (20c) is an equality constraint, if the solution of problem (21) satisfies $t_l u_l^* = 0 \ \forall l \in L$, then the duality gap is zero. We define the following two quantities:

$$\begin{aligned} \alpha = \min_{\tilde{p}, t} \ & |p - \tilde{p}| \\ \text{s.t. } & (\tilde{p}, t) \in \mathcal{Q} \end{aligned} \quad \text{and} \quad \begin{aligned} \beta_l = \min_{\tilde{p}, t} \ & |p - \tilde{p}| \\ \text{s.t. } & (\tilde{p}, t) \in \mathcal{Q} \\ & t_l u_l^* = 0 \end{aligned}$$

If u^* is the zero vector, every real value is a dual optimal multiplier, the constraint $t_l u_l^* = 0$ is naturally satisfied $\forall l \in L$, and the duality gap is zero. Else, u^* is a vector of zeros with one entry set to one. Let m be the index of that entry. Note that the optimal objective value of problem (20) is β_m by definition. Let t_m^* be the optimal value of problem (21) with $\lambda_l = \beta_l - \alpha \ \forall l \in L$. If $t_m^* = 1$, then the optimal objective value of problem (21) is $\alpha + \beta_m - \alpha = \beta_m$. If $t_m^* = 0$, then its optimal objective is also β_m. We conclude that the duality gap is zero and that $\beta_l - \alpha$ is a dual optimal Lagrange multiplier. □

Using Proposition 2, we deduce that if λ_l is a dual optimal multiplier, problem (19) is equivalent to

$$\begin{aligned} \max_{u\in\mathbb{U}} \ \min_{\tilde{p},t} \ & |p-\tilde{p}| + \sum_{l\in L} \lambda_l t_l u_l \\ \text{s.t. } & (\tilde{p},t) \in \mathcal{Q} \end{aligned} \tag{22}$$

Note that the feasible set of the inner level of problem (22) does not depend on u anymore. Moreover, as its objective function is linear, at least one extreme point of $\text{conv}(\mathcal{Q})$ is optimal, where conv denotes the convex hull. Let us denote by $\text{ext}(\mathcal{Q})$ the set of extreme points of $\mathcal{Q}$. Then, problem (22) is consequently also equivalent to

$$\max_{u\in\mathbb{U}} \ \min \left\{ |p-\tilde{p}| + \sum_{l\in L} \lambda_l t_l u_l : (\tilde{p},t) \in \text{ext}(\mathcal{Q}) \right\}$$

and to

$$\begin{aligned} \max_{u\in\mathbb{U}} \ & \eta \\ \text{s.t. } & \eta \le |p-\tilde{p}| + \sum_{l\in L} \lambda_l t_l u_l, \ \ \forall (\tilde{p},t) \in \text{ext}(\mathcal{Q}) \end{aligned} \tag{23}$$

where our problem has been rewritten in the form of a cutting plane formulation. Using this formulation, a cutting plane algorithm can be obtained by noticing the two following facts: (i) if problem (23) is solved with a subset of $\text{ext}(\mathcal{Q})$, then we obtain an upper bound as well as an interdiction plan u; (ii) solving the inner level of problem (22) for a fixed u gives a lower bound as well as a new extreme point for the set $\text{conv}(\mathcal{Q})$. The algorithm that is suggested above can be presented formally as follows.

Algorithm 2 : Inner Level

1) Set $LB = +\infty, UB = -\infty$, $k = 0$ and $\text{ext}(\mathcal{Q})^0 = \emptyset$
2) Solve the following Worst Uncertainty Oracle :

$$\begin{aligned} \max_{u\in\mathbb{U}} \ & \eta \\ \text{s.t. } & \eta \le |p-\tilde{p}| + \sum_{l\in L} \lambda_l t_l u_l, \ \ \forall (\tilde{p},t) \in \text{ext}(\mathcal{Q})^k \end{aligned}$$

   Denote by u^* the optimal value of variable u and update UB to the optimal objective value.
3) Solve the following Best Reaction Oracle :

$$\begin{aligned} \min_{\tilde{p},t} \ & |p-\tilde{p}| + \sum_{l\in L} \lambda_l t_l u_l^* \\ \text{s.t. } & (\tilde{p},t) \in \mathcal{Q} \end{aligned}$$

   Denote by $\tilde{p}^*$ and t^* the optimal values of variables $\tilde{p}$ and t respectively.
   Let $\text{ext}(\mathcal{Q})^{k+1} \leftarrow \text{ext}(\mathcal{Q})^k \cup (\tilde{p}^*, t^*)$.
   Let $LB \leftarrow \max\{LB, |p-\tilde{p}^*| + \sum_{l\in L} \lambda_l t_l^* u_l\}$.
4) If $UB - LB < \epsilon$, terminate. Else, let $k \leftarrow k+1$ and go back to step 2.

□

If λ_l is a dual optimal multiplier, algorithm 2 is guaranteed to converge in a finite number of iterations. Note, however, that if λ_l is a dual optimal multiplier, then every $\tilde{\lambda}_l$ such that $\tilde{\lambda}_l > \lambda_l$ is also a dual optimal multiplier. This method also results in the decomposition of the problem into two subproblems, as in the case of Zhao and Zeng [15]. The difference is that the Worst Uncertainty Oracle subproblem is solved much more efficiently than the master problem of [15]. Since the master problem is the bottleneck of [15], our approach achieves a material improvement over Zhao and Zeng.

The speed of convergence is largely determined by the value of the dual multiplier. Notice that the smaller λ_l is, the tighter the formulation (23) is. Thus, the goal is to find the smallest possible dual optimal multiplier of constraint (20c). If λ_l is set to a large trivial value, the algorithm will have to generate almost all possible values of u before converging. In contrast, if the value chosen is close to its optimal value, the convergence can be much faster. In what follows, we present our idea for generating values for λ_l that yield fast convergence in the case of our problem.

We first mention that the proof of Proposition 2 highlights how we can obtain the best value of λ_l, which we denote by λ_l^*. Indeed, let

$$\begin{aligned} \gamma_l = \max_{u\in\mathbb{U}} \ \min_{\tilde{p},t} \ & |p-\tilde{p}| \\ \text{s.t. } & (\tilde{p},t) \in \mathcal{Q} \\ & t_l u_l = 0 \end{aligned}$$

Then, $\lambda_l^* = \gamma_l - \alpha$. This value can be interpreted as *the price of robustness*, i.e. the price to pay for being robust to the contingency of line l. It turns out that it can be easily upper bounded as follows: Let δ_l be defined as the objective value of the inner problem when we are robust to the contingency of line l without the possibility for switching as a recourse action:

$$\begin{aligned} \delta_l = \min_{\substack{v\in[0,1] \\ \tilde{p},f,\theta}} \ & |p-\tilde{p}| \\ \text{s.t. } & \sum_{g\in G(z)} Q_g v_g - \tilde{p}_z = \sum_{n\in N(z)} Q_n, \ \ \forall z \in Z \\ & \sum_{g\in G(n)} Q_g v_g - \sum_{j\in L(n,\cdot)} f_j + \sum_{j\in L(\cdot,n)} f_j = Q_n, \\ & \qquad \forall n \in N \\ & -F_j \le f_j \le F_j, \ \ \forall j \in L \\ & f_j = B_j(\theta_{m(j)} - \theta_{n(j)}), \ \ \forall j \in L \backslash \{l\} \\ & f_l = 0 \end{aligned} \tag{24}$$

The value δ_l can be obtained much more efficiently than the value γ_l as it is a simple monolhitic LP. Then, the following inequalities naturally hold:

$$\alpha \le \gamma_l \le \delta_l.$$

It follows that λ_l^* is bounded from above by $\delta_l - \alpha$. Algorithm 2 with $\lambda_l = \delta_l - \alpha$ is the approach that we use for solving (18).

C. Extension to preventive dispatch

It is straightforward to adapt the algorithm to the case of a purely preventive dispatch. In the master problem, instead of adding constraints

$$p \in \mathcal{P}_{t^i}(u^i), \qquad \forall i \in \{1, ..., k\}$$

we will add the following set of constraints, which describe the fact that the dispatch should be the same for every contingency:

$$\sum_{g \in G(z)} Q_g \bar{v}_g - p_z = \sum_{n \in N(z)} Q_n, \qquad \forall z \in Z$$
$$\bar{v} \in \mathcal{V}_{t^i}(u^i), \qquad \forall i \in \{1, ..., k\}$$

The inner problem, that identifies the next contingency to add in the master problem, now reads as follows:

$$d(\bar{v}, \bigcap_{\|u\|_1 \le 1} \mathcal{V}_t(u)) = \max_{u \in \mathbb{U}} \min_{\tilde{v}, t} \; |\bar{v} - \tilde{v}| \quad \text{s.t. } \tilde{v} \in \mathcal{V}_t(u) \tag{25}$$

The structure of problem (25) is exactly the same as that of problem (18). The same cutting plane algorithm can thus be used to solve the inner problem. Note, however, that although the algorithm can be adapted in a straightforward way, the problem with preventive re-dispatch and the problem with curative re-dispatch vary in terms of solution difficulty. This is due to the fact that, in the case of a preventive re-dispatch, the inner problem computes the distance to a set that is significantly higher dimensional than in the case of curative dispatch. Whereas the set $\mathcal{P}_t(u)$ has dimension $|Z|$, the number of zones in the system, the set $\mathcal{V}_t(u)$ is of dimension $|G|$, which corresponds to the number of generators. For instance, in our case study on a realistic instance of the Central Western European (CWE) system, there are 5 zones, whereas the number of generators is almost 2000. This translates to additional computational burden for achieving preventive N-1 robustness, compared to curative robustness.

D. Extension to hybrid dispatch

A simple combination of the algorithm for the preventive case and for the curative case can be used to clear the market with a hybrid N-1 robust dispatch. We note, however, that the important question of how to determine the set U_{prev} (i.e. the set of contingencies to be considered in a preventive way) remains. Our column-and-constraint generation algorithm for solving the day-ahead model suggests a way of defining this set. In this algorithm, each iteration generates a severe contingency that is added to the master problem, until convergence is reached. We propose that U_{prev} should consist of the n first contingencies produced by the algorithm. By construction, these contingencies are selected among the most severe contingencies that the system operator is called to react to.

Note that this hybrid model, being a combination of the preventive and curative case, has a computational complexity that is intermediate between that of the curative and that of the preventive model. This complexity is increasing with n.

V. Conclusion

In the first part of this two-part paper, we have proposed a two-stage model of a zonal electricity market with transmission switching at both the day-ahead and real-time stage. We have cast our problem as an AROMIP, and we have described a new algorithm for solving this class of challenging mixed integer optimization problems when the inner level has the structure of an interdiction game.

In the second part, we propose different variations of models for the re-dispatch and balancing phase. We use these models and the resolution method presented in this first part for analyzing the relative performance of various market design options that include transmission switching. We test these models on a realistic case study of the Central Western European market. The second part of the paper also validates the efficiency of the algorithm proposed here, by showing that it achieves a considerable reduction in computation time compared to the technique proposed in [16].

Appendix

A. Illustration of the difference between preventive and curative re-dispatch

In this appendix, we describe an illustrative example that highlights the difference between a day-ahead model with preventive versus curative re-dispatch. We consider a three-node two-zone network, as shown in Fig. 3. Zone A consists of two buses, An and As. Zone B consists of a single bus, bus B. Buses An and As are connected by two lines, each with a reactance of 0.01 per unit (p.u.). Bus An is connected to bus B by two lines, each with a reactance of 0.001 p.u. Bus As is connected to bus B by two lines, each with a reactance of 0.001 p.u. All lines obey a capacity limit of 1 GW. In order to simplify the exposition, we consider only contingencies that involve cross-zonal lines.

In the case of purely **curative re-dispatch**, the net position of zone A could reach up to 3 GW. Indeed, if a contingency occurs on a line between bus An and bus B, the transmission limits on the remaining elements can still be respected by a net injection of 1 GW in bus An and a net injection of 2 GW in bus As. If a contingency occurs on a line between bus As and bus B, the transmission limits on the remaining elements can still be respected by a net injection of 2 GW in bus An and a net injection of 1 GW in bus As.

However, in the case of purely **preventive re-dispatch**, the N-1 security constraints on all cross-zonal transmission elements must be met simultaneously. In

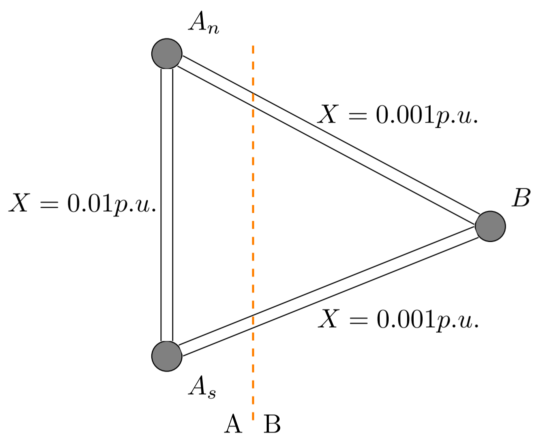

Figure 3: Network data of the three-node two-zone example that is presented in the appendix.

that case, it is not possible to inject more than 1083 MW in bus An and more than 1083 MW in bus As. In the day-ahead market clearing problem, the maximum net position of zone A is thus limited to 2.17 GW (preventive), and not to 3 GW (curative).

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