Distributed Derandomization via Network Decomposition

Longer talk

Mohsen Ghaffari (ETH), Vasek Rozhon (ETH)
Plan

1. More on **LOCAL** and **CONGEST** model

2. A deterministic algorithm for **network decomposition**.
   - a. Sequential algorithm
   - b. Distributed algorithm

3. Applications
   - a. Derandomization and a bigger picture of the **LOCAL** model
   - b. $\Delta+1$ coloring, MIS, Lovász local lemma
   - c. **CONGEST** model and open problems
Plan

1. More on **LOCAL** and **CONGEST** model
The **LOCAL** model of distributed graph algorithms

- Undirected graph on $n$ nodes
- One computer in each node
- Synchronous message passing rounds
- Unbounded message size and computation
- Initially, nodes know only (upper bound on) $n$ and their unique $O(\log n)$ bit identifier
- In the end, each node should know its part of output
- Time complexity: number of rounds

*LOCAL model [Linial FOCS’87]*
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[LOCAL model](Linial FOCS'87)
The **LOCAL** model of distributed graph algorithms

“unbounded message size and computation”:

**CONGEST** model: message size bounded to $O(\log n)$.

deterministic **LOCAL** algorithm is a function mapping neighbourhoods to labels.
The **LOCAL** model of distributed graph algorithms

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The **LOCAL** model of distributed graph algorithms

Why unique $O(\log n)$ bit identifier?

Otherwise, not much to be done with deterministic algorithms, especially on vertex-transitive graphs!
Plan

1. More on **LOCAL** and **CONGEST** model
2. A deterministic algorithm for *network decomposition*.
   a. Sequential algorithm
Network decomposition

Network decomposition with $C$ colors and diameter $D$:

Coloring of the vertices with $C$ colors, such that each component induced by a particular color has diameter at most $D$. 

(2,6) decomposition
Weak-diameter network decomposition

Weak-diameter network decomposition with $C$ colors and weak-diameter $D$:

Coloring of the vertices with $C$ colors, such any two vertices in a component are of distance $\leq D$ in the original graph

$$(3,4)$$ decomposition
Network decomposition

But is there such a thing (with reasonable parameters)?

Yes, we let’s see a sequential algorithm for \((O(\log n), O(\log n))\) network decomposition.

(Sequential) ball carving

1. clusters at least \(\frac{1}{2}\) fraction of vertices
2. such that each cluster has diameter \(O(\log n)\)
3. clusters are non-adjacent

\(\Rightarrow (O(\log n), O(\log n))\) network decomposition by repeated application
Sequential ball carving
Sequential ball carving

we let a cluster grow, while its size increases by a factor of 2
Sequential ball carving
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1. clusters at least $\frac{1}{2}$ fraction of vertices

   Each cluster $C$ is responsible for deleting $< |C|$ vertices
   $\Rightarrow < \frac{1}{2}$ fraction of vertices deleted.

2. each cluster has diameter $O(\log n)$

   After $1+\log n$ steps, a cluster would contain the whole graph, as $2^{1+\log n} > n$.

3. clusters are non-adjacent

   By construction.
Plan

1. More on **LOCAL** and **CONGEST** model

2. A deterministic algorithm for **network decomposition**.
   a. Sequential algorithm
   b. Distributed algorithm
Distributed ball carving

We follow the sequential strategy and show a deterministic $\text{poly}(\log n)$-round algorithm that

1. clusters at least $\frac{1}{2}$ fraction of vertices
2. such that each cluster has weak-diameter $O(\log^3 n)$ and
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Distributed ball carving

The identifiers have $B = O(\log n)$ bits.

The algorithm has $B$ phases.

In the $i$-th phase we deal with “bad edges” between clusters whose identifiers differ in the $i$-th bit.
Distributed ball carving

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blue clusters grow,
red vertices are recolored/deleted
Distributed ball carving
Distributed ball carving

Red vertices propose to join an arbitrary neighbouring blue cluster.

A blue cluster $C$ accepts all proposals if at least $|C|/(2B)$ vertices are proposing.

Otherwise, it denies all of them, therefore deleting proposing red nodes permanently.
Distributed ball carving

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We let this process run for $4B \ln n$ steps.
Distributed ball carving

In the second phase (and other phases) we do the same, based on the $i$-th rightmost bit.

Note that the coloring here has different meaning than in the first phase.

**Red vertices** propose, not whole clusters.
Distributed ball carving

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1. clusters at least \( \frac{1}{2} \) fraction of vertices
2. such that each cluster has weak-diameter \( O(\log^3 n) \)
3. clusters are non-adjacent

Property 2:

The weak-diameter grows additively by \( \leq 2 \) in each step.
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Hence, the weak diameter is $O(B^2 \log n) = O(\log^3 n)$. 
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*Observation:* If a blue cluster does not grow in some step, it does not have red neighbours in any future steps.

Hence, it does not grow any more during the phase.
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Property 1:

Each blue cluster \( C \) is at the end of a phase responsible for \( |C|/(2B) \) deleted red vertices.
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**Property 3:**
Distributed ball carving

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Property 3:

At the end of each phase, there are no edges between red and blue nodes.
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**Property 3:**

At the end of each phase, there are no edges between red and blue nodes.

Otherwise there is a blue cluster of size $> (1+1/(2B))^{4B \ln n} > n$. 
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After \( i \)-th phase, clusters in each connected component agree on their \( i \)-th bit, and this stays so during next phases.
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The running time of the whole algorithm is

\[ O(\log^7 n) = \]

\[ O(\log n) \] \# of colors of decomposition

\[ \cdot O(\log n) \] \# of phases

\[ \cdot O(\log^2 n) \] steps per phase

\[ \cdot O(\log^3 n) \] complexity of one step
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The whole algorithm works in the CONGEST model (see Section 2.2 in our paper).
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1. More on **LOCAL** and **CONGEST** model

   a. Sequential algorithm
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3. Applications
   a. Derandomization and a bigger picture of the **LOCAL** model
General derandomization theorem

Theorem: [Ghaffari, Harris, Kuhn FOCS’18 + R., Ghaffari STOC’20]

\[ \text{P-LOCAL} = \text{P-RLOCAL}. \]

**P-LOCAL:** problems* solvable by a deterministic \( \text{poly}(\log n) \)-round algorithm in the \text{LOCAL} model

**P-RLOCAL:** problems* solvable by a randomized \( \text{poly}(\log n) \)-round algorithm in the \text{LOCAL} model

*problems needs to be locally checkable = if a proposed solution is not correct, at least one node recognises that after looking at its \( \text{poly}(\log n) \)-hop neighbourhood
General derandomization theorem

Theorem: [Ghaffari, Kuhn, Maus STOC’17 + Ghaffari, Harris, Kuhn FOCS’18 + R., Ghaffari STOC’20]

\[ \text{P-LOCAL} = \text{P-RLOCAL} = \text{P-SLOCAL} = \text{P-RSLOCAL}. \]

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Sequential $\Delta+1$ coloring algorithm

Iterate over nodes in arbitrary order.

Decide their color based only on their 1-hop neighbourhood.
Sequential $\Delta + 1$ coloring algorithm

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**SLOCAL** - sequential variant of the **LOCAL** model

Iterate over nodes in adversarial order.

Decide their label based only on their $r$-hop neighbourhood.

Write the label to the node so that other nodes can see it.

**P-SLOCAL:**  \( r = \text{poly}(\log n) \), locally checkable

**P-RSLOCAL:**  \( r = \text{poly}(\log n) \), locally checkable, can use randomness
P-SLOCAL
“deterministic sequential”

P-LOCAL
“deterministic distributed”

P-RSLOCAL
“randomized sequential”

P-RLOCAL
“randomized distributed”
P-SLOCAL
“deterministic sequential”

deterministic network decomposition
[R., Ghaffari STOC’20]
direct

P-LOCAL
“deterministic distributed”
Proof: for P-SLOCAL algorithm with locality $r$, construct network decomposition on $G'$ in $O(r \log^7 n)$ rounds with $C=O(\log n)$, $D=O(\log^3 n)$; iterate over color classes and simulate the sequential algorithm in $O(\log n \cdot r \log^3 n)$ rounds.
By the way: Network decomposition is a complete problem for this reduction.
Corollary: There is an efficient deterministic algorithm for $\Delta+1$ coloring, maximal independent set, strong diameter network decomposition.
P-SLOCAL
“deterministic sequential”

deterministic network decomposition
[R., Ghaffari STOC’20]

P-LOCAL
“deterministic distributed”

P-RLOCAL
“randomized distributed”

P-RSLOCAL
“randomized sequential”

randomized network decomposition
[Linial, Saks SODA ’91]
conditional expectation*  
[Ghaffari, Harris, Kuhn FOCS’18]

**P-RLOCAL**
“randomized sequential”

direct

**P-RSLOCAL**
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**P-LOCAL**
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deterministic network decomposition
[R., Ghaffari STOC’20]

**P-SLOCAL**
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deterministic network

* for problems locally checkable in poly(log n) rounds
Corollary: There is an efficient deterministic algorithm for Lovász local lemma,...
We see a clean first-order theory of the LOCAL model.
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   b. Δ+1 coloring, MIS, Lovász local lemma
$\Delta+1$ coloring

- **LOCAL**, deterministic
- **LOCAL**, randomized
- **MPC**, randomized
Δ+1 coloring

**LOCAL**, deterministic
poly(log n)

**LOCAL**, randomized

**MPC**, randomized
Δ+1 coloring

- **LOCAL**, deterministic, \( \text{poly}(\log n) \)
- **LOCAL**, randomized, \( \text{poly}(\log \log n) \)
- **MPC**, randomized

- Shattering [Chang, Li, Pettie STOC’18]
- Network decomposition [R Ghaffari STOC’20]
Δ+1 coloring

amplification of success probability
[Chang, Kopelowitz, and Pettie FOCS’16]

LOCAL, deterministic
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+ network decomposition [R Ghaffari STOC’20]

graph exponentiation
[Chang, Fischer, Ghaffari, Uitto, Zheng PODC’19]
Δ+1 coloring

amplification of success probability
[Chang, Kopelowitz, and Pettie FOCS’16]

conditioned on hardness of connectivity
in MPC [Ghaffari, Kuhn, Uitto FOCS’19]

LOCAL, deterministic
poly(log n)

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shattering [Chang, Li, Pettie STOC’18]

network decomposition [R Ghaffari STOC’20]

graph exponentiation
[Chang, Fischer, Ghaffari, Uitto, Zheng PODC’19]
### Maximal independent set (MIS)

<table>
<thead>
<tr>
<th></th>
<th>Upper bound</th>
<th>Lower bound</th>
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</thead>
<tbody>
<tr>
<td><strong>LOCAL, deterministic</strong></td>
<td>$O(\log^7 n)$ [R. Ghaffari STOC’20]</td>
<td>$\Omega(\log n / \log \log n)$ [Balliu et al. FOCS’19]</td>
</tr>
<tr>
<td><strong>LOCAL, randomized</strong></td>
<td>$O(\log \Delta) + \text{poly}(\log \log n)$ [Ghaffari SODA’16]</td>
<td>$\Omega(\log \Delta / \log \log \Delta)$ [Kuhn et al. J.ACM’16]</td>
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\[o(\Delta) + o(\log \log n / \log \log \log n)\] is impossible [Balliu et al. FOCS’19]
Lovász local lemma

Each node corresponds to “bad” event independent on all but neighbouring events.

If probability of each event is small enough, can we instantiate them so that no bad event occurs?
### Lovász local lemma \((p = \Delta^{-10})\)

<table>
<thead>
<tr>
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<tr>
<td>randomized</td>
<td>[Moser, Tardos J.ACM’10]</td>
<td>[Brandt et al., SODA’16]</td>
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<td><strong>O</strong>((\Delta^2)) + poly**(log log (n))**</td>
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<td>[Fischer, Ghaffari DISC’17]</td>
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Lovász local lemma

**Theorem** [Chang, Pettie FOCS’17]:

In graphs of degree $O(1)$, problems checkable with locality $O(1)$ have randomized complexity of either $\Omega(\log n)$ or $O(T_{LLL})$.

Here, $T_{LLL}$ is the randomized complexity of Lovász local lemma on constant degree graphs.
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   c. **CONGEST** model and open problems
CONGEST model

Recall: In one round, only $O(\log n)$ bits of information can be sent through an edge.
CONGEST model

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In general, we cannot collect the whole topology of a cluster.
CONGEST model

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In general, we cannot collect the whole topology of a cluster

**Theorem:** [Censor-Hillel, Parter, Schwartzman DISC'17; R. Ghaffari STOC'20]

There is poly$(\log n)$-round CONGEST algorithm for MIS.

**Theorem:** [Bamberger, Kuhn, Maus PODC'20; R. Ghaffari STOC'20]

There is poly$(\log n)$-round CONGEST algorithm for $\Delta+1$ coloring.
Open problems

Find deterministic algorithm for MIS, Δ+1 coloring, … in the LOCAL model faster than state-of-the-art algorithm for network decomposition.

Find a combinatorial deterministic poly(log n)-round algorithm for MIS, Δ+1 coloring, … in the CONGEST model.
Summary

Network decomposition
Summary

Network decomposition

Distributed ball carving
Summary

Network decomposition

Distributed ball carving

Big picture of the LOCAL model

P-SLOCAL ↔ P-RSLOCAL

P-LOCAL → P-RLOCAL
Summary

Network decomposition

Big picture of the LOCAL model

Det. LOCAL ↔ Rand. LOCAL ↔ MPC

Δ+1 coloring and connections across models