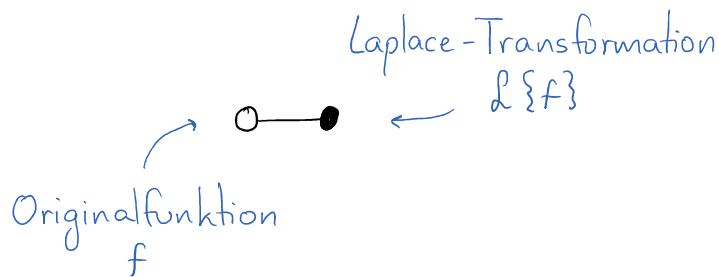


Theorie

1. Laplace-Transformation

$$\mathcal{L}\{f\}(s) := \int_0^{\infty} f(t) e^{-st} dt$$

→ Darstellung (Doetsch-Symbole $\circ \bullet$)



Beispiel:

$$\begin{aligned} f(t) &\circ \bullet F(s) \\ \frac{d}{dt} f(t) &\circ \bullet sF(s) - f(0) \\ t^n e^{-at} &\circ \bullet \frac{n!}{(s+a)^{n+1}} \end{aligned}$$

→ Rücktransformation

Beispiel: Löse $\ddot{y}(t) - y(t) = e^t$, $\dot{y}(0) = y(0) = 0$

$$\left[t^n e^{-at} \right] \circ \bullet \frac{n!}{(s+a)^{n+1}}$$

$$\begin{aligned} \rightarrow \mathcal{L}\left\{\frac{d^2}{dt^2} y(t)\right\}(s) &= s \mathcal{L}\left\{\frac{d}{dt} y(t)\right\}(s) - \left.\frac{d}{dt} y(t)\right|_{t=0} = s(s \mathcal{L}\{y(t)\}(s) - y(0)) - \dot{y}(0) \\ &= s^2 Y(s) - s y(0) - \dot{y}(0) = s^2 Y(s) \end{aligned}$$

$$\rightarrow \mathcal{L}\{y(t)\}(s) = Y(s)$$

$$\begin{aligned} \rightarrow \mathcal{L}\{e^t\}(s) = \frac{1}{s-1} &\Rightarrow s^2 Y(s) - Y(s) = \frac{1}{s-1} \Rightarrow Y(s) = \frac{1}{(s^2-1)(s-1)} \\ &= \frac{1}{(s+1)(s-1)^2} \end{aligned}$$

→ Rücktransformation: $y(t) = \mathcal{L}^{-1}\{Y(s)\}(t)$

$$Y(s) = \frac{1}{(s+1)(s-1)^2} = \frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{s+1} = \frac{-1/4}{s-1} + \frac{1/2}{(s-1)^2} + \frac{1/4}{s+1}$$

$$\begin{aligned} y(t) &= \mathcal{L}^{-1}\left\{-\frac{1}{4} \frac{1}{s-1}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{2} \frac{1}{(s-1)^2}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{4} \frac{1}{s+1}\right\} = -\frac{1}{4} e^t + \frac{1}{2} t e^t + \frac{1}{4} e^{-t} \\ &\quad \begin{matrix} n=0 \\ a=-1 \end{matrix} \quad \begin{matrix} n=1 \\ a=-1 \end{matrix} \quad \begin{matrix} n=0 \\ a=1 \end{matrix} \\ &= \frac{1}{2} t e^t - \frac{1}{2} \sinh(t) \end{aligned}$$

Beispiel: $Y(s) = \frac{3}{(s+1)^4}$

$$Y(s) = \frac{3}{(s+1)^4} = \frac{3}{3!} \frac{3!}{(s+1)^4} \Rightarrow \mathcal{L}^{-1}\left\{\frac{1}{2} \frac{3!}{(s+1)^4}\right\} = \frac{1}{2} t^3 e^{-t}$$

$$\begin{aligned} n &= 3 \\ a &= 1 \end{aligned}$$

$$\Rightarrow y(t) = \frac{1}{2} t^3 e^{-t}$$