Theorie

1. Parsevalsche Identität

$$\boxed{\frac{1}{2\pi}\int_{-\pi}^{\pi}[f(x)]^2\,dx = \sum_{k=-\infty}^{\infty}|c_k|^2 = \frac{a_0^2}{4}+\frac{1}{2}\sum_{k=1}^{\infty}\left(|a_k|^2+|b_k|^2\right)}$$

2. Dirichlet-Kern

Fourierkoeffizienten

→ $S_N f(t) = \sum_{k=-N}^{N} c_k \, e^{\frac{2\pi i k t}{T}}$ wobei $c_k = \hat{f}(k) = \frac{1}{T}\int_{-T/2}^{T/2} f(x)\,e^{-\frac{2\pi i k x}{T}}\,dx$

→ Jetzt für 1-periodische Funktionen $(T=1)$:

$$S_N f(t) = \sum_{k=-N}^{N}\int_{-1/2}^{1/2} f(x)\,e^{-2\pi i k x}\,dx \cdot e^{2\pi i k t} = \int_{-1/2}^{1/2} f(x)\underbrace{\sum_{k=-N}^{N} e^{2\pi i k (t-x)}}_{=:\,D_N(t-x)}\,dx$$

$$\boxed{D_N(x) = \sum_{k=-N}^{N} e^{2\pi i k x} = \frac{\sin\left(2\pi\left(N+\frac{1}{2}\right)x\right)}{\sin\left(\frac{2\pi x}{2}\right)}}$$