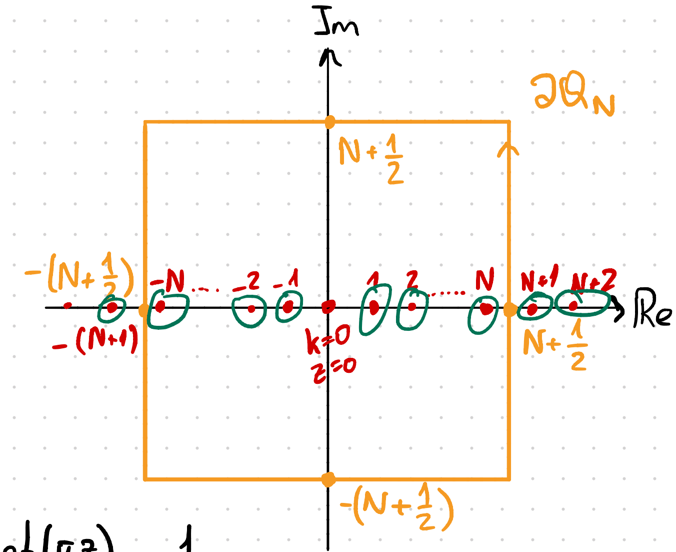
S9 A1

$$f(z) = \pi \cot(\pi z) = \pi \frac{\cos(\pi z)}{\sin(\pi z)}$$

$$\lim_{N\to\infty} \int_{\partial Q_N} \frac{f(z)}{g(z)}\, dz = 0, \text{ falls } g(z) \gg z$$

$$\lim_{N\to\infty} \int_{\partial Q_N} \frac{f(z)}{z^4}\, dz = 0 \quad \left[\sum_{k=1}^{\infty} \frac{1}{k^4} = ? \right]$$

$$= 2\pi i \sum \mathrm{Res}_{\partial Q_N}$$



Sing.

$$z = k \ [\text{Sing. von } f(z)]: \ \mathrm{Res}(f|k) = \lim_{z\to k} (z-k) \frac{\pi\cot(\pi z)}{z^4} = \frac{1}{k^4}$$

$$z = 0: \ \mathrm{Res}\left(\frac{f(z)}{z^4} \Big| 0\right) = -\frac{\pi^4}{45}$$

$$0 = 2\pi i \left(-\frac{\pi^4}{45} + \sum_{k=-\infty}^{-1} \frac{1}{k^4} + \sum_{k=1}^{\infty} \frac{1}{k^4} \right) \Rightarrow \left(-\frac{\pi^4}{45} + 2\sum_{k=1}^{\infty} \frac{1}{k^4} \right) = 0 \Rightarrow \sum_{k=1}^{\infty} \frac{1}{k^4} = \frac{\pi^4}{2\cdot 45} = \boxed{\frac{\pi^4}{90}}$$

$$\frac{\pi\cot(\pi z)}{z^4} = \frac{1}{z^5} - \frac{\pi^2}{3} z^{-3} \underbrace{- \frac{\pi^4}{45} z^{-1}}_{c_{-1}} - \frac{2\pi^6}{945} z + \cdots$$

$$\sum \frac{1}{k^2+a^2} \to \lim_{N\to\infty} \int_{\partial Q_N} \frac{f(z)}{z^2+a^2}\, dz = 0 = 2\pi i \sum \mathrm{Res}$$

$\to$ $z = \pm ia$ Pole 1. Ordnung

$$\mathrm{Res}\left(\frac{f(z)}{z^2+a^2}\,\middle|\, ia\right) = \lim_{z\to ia} (z-ia)\,\frac{\pi \cot(\pi z)}{z^2+a^2} = \dots =$$

$$z^2+a^2 = (z-\underline{ia})(z+\underline{ia})$$

$$\pi\cot(\pi z) \to z = k$$

komplex

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{2\pi i n t}{T}}}$$

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} \underline{f(x)}\, \underline{e^{-\frac{2\pi i n x}{T}}}\, dx$$

f $\left.\begin{matrix}\text{gerade} \\ \text{ungerade}\end{matrix}\right\} \Rightarrow c_n =$ keine Aussage

$e^{-x} \neq e^{x} \Rightarrow$ nicht g

$e^{-x} \neq -e^{x} \Rightarrow$ nicht u

reell

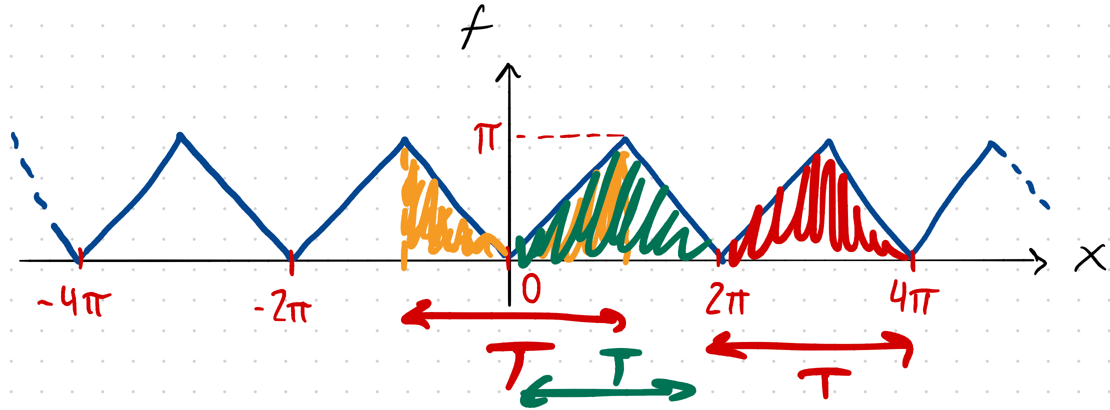
$$\boxed{f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n t}{T}\right) + b_n \sin\left(\frac{2\pi n t}{T}\right)}$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \cos\left(\frac{2\pi n x}{T}\right) dx \quad (n \geq 0)$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \sin\left(\frac{2\pi n x}{T}\right) dx \quad (n \geq 1)$$

f gerade $\Rightarrow b_n = 0, \; a_n = 2\int_0^{T/2} \cdots$

f ungerade $\Rightarrow a_n = 0, \; b_n = 2\int_0^{T/2} \cdots$



$$c_n, a_n, b_n = \cdots \int_{-T/2+\alpha}^{T/2+\alpha} f(x)\, \mathrm{trig}\left(\frac{2\pi}{T} n x\right) dx \qquad \alpha \in \mathbb{R}$$

T-periodisch

$$\underset{c_n}{\exp},\ \underset{a_n}{\cos},\ \underset{b_n}{\sin}$$

$$= \int_0^T \cdots = \int_{3T}^{4T} \cdots = \int_{\frac{3}{2}T}^{T+\frac{3}{2}T} \cdots ; \int_a^b \cdots$$

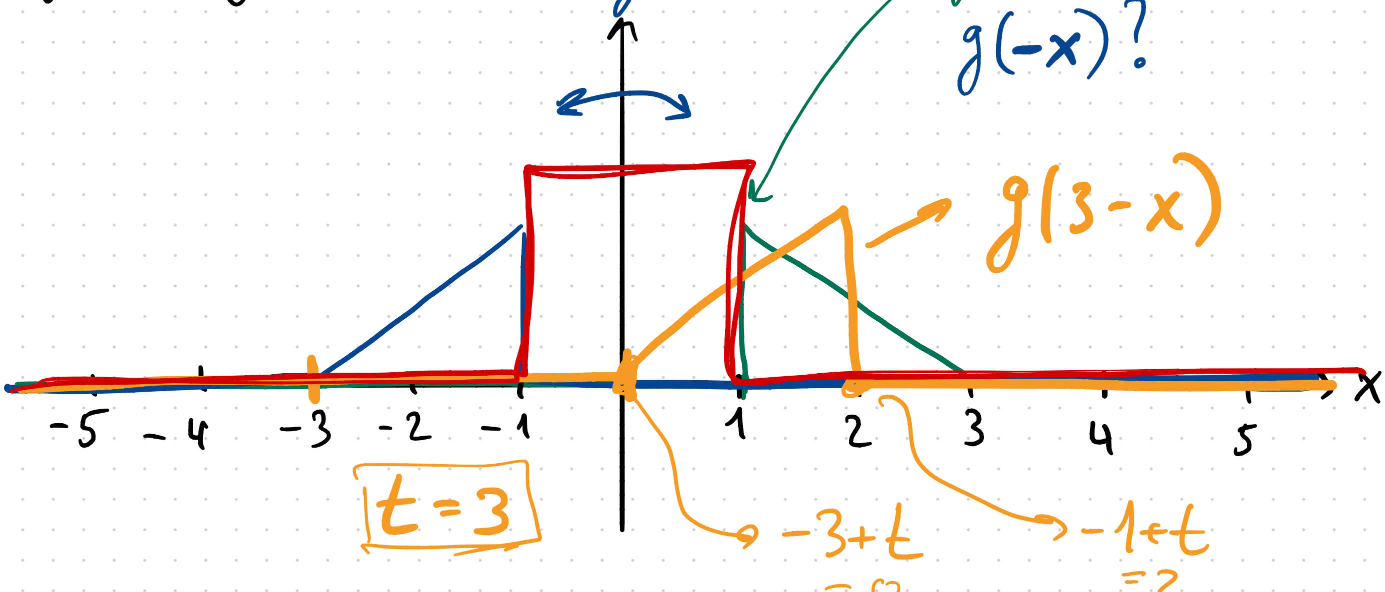
$$(b-a) = T$$

Faltung

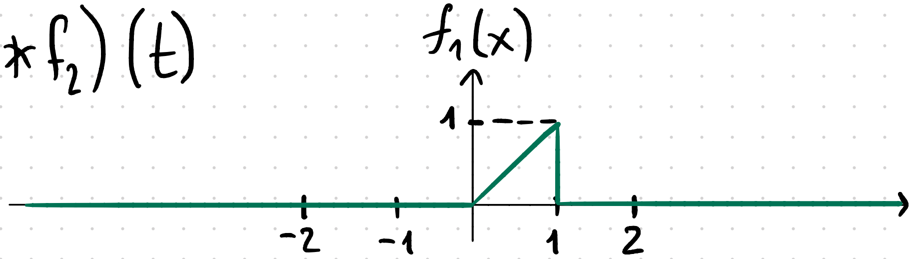
→ Funktional

$$\boxed{(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau}$$

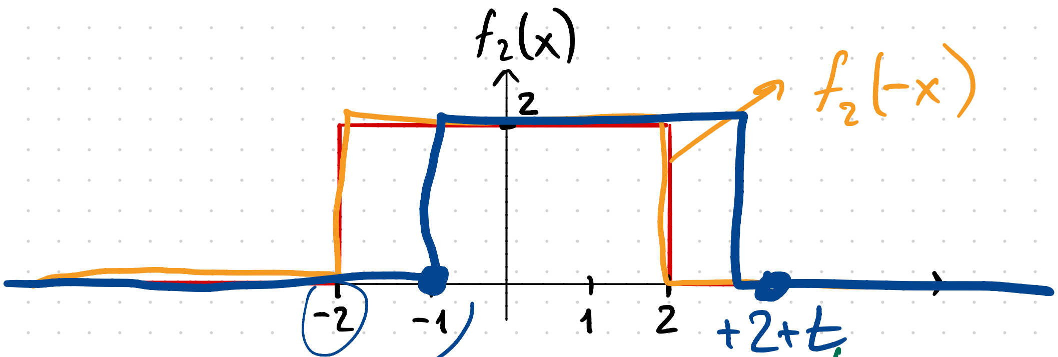
→ $(f * g)(t) = (g * f)(t)$



$(f_1 * f_2)(t)$



$$f_1(x) = \begin{cases} x, & 0 < x < 1 \\ 0, & \text{sonst} \end{cases}$$



$$f_2(x) = \begin{cases} 2, & |x| < 2 \\ 0, & \text{sonst} \end{cases}$$

$-2+t = -1$

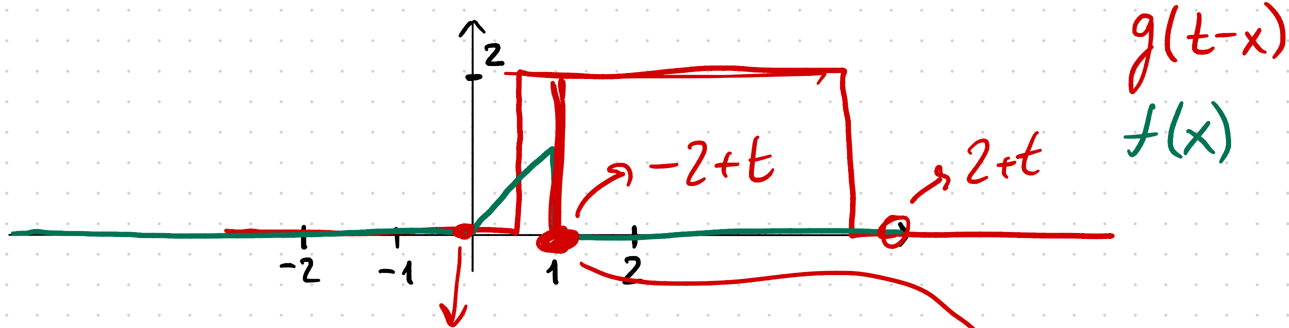
$\boxed{t = 1}$

f_1

f_2

$$(f*g)(t) = \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau$$

$$(f_1 * f_2)(t) = \begin{cases} 0, & t < -2 \\ (t+2)^2, & -2 \leq t < -1 \\ 1, & -1 \leq t < 2 \\ 1-(t-2)^2, & 2 \leq t < 3 \\ 0, & t \geqslant 3 \end{cases}$$



$2+t<0 \Rightarrow t<-2$

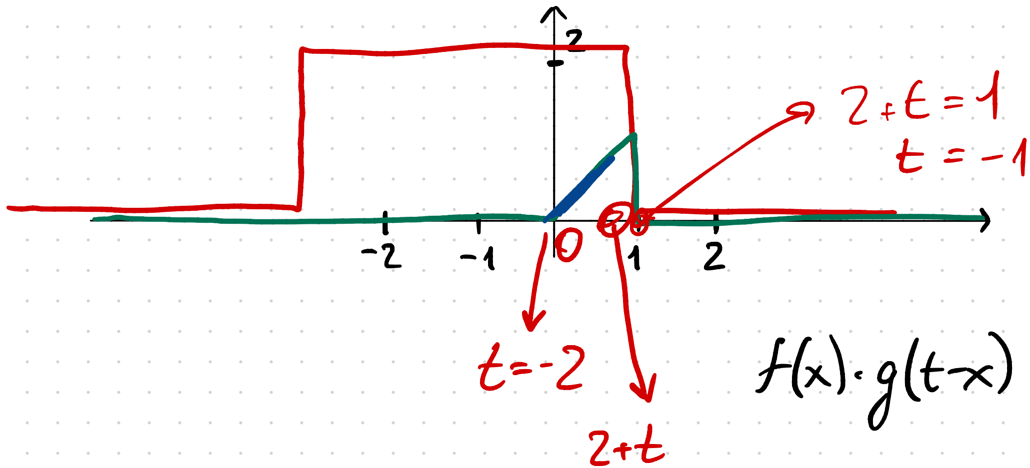
$-2+t > 1$

$t > 3$

$$(f_1 * f_2)(t<-2,\ t>3) = \int_{-\infty}^{\infty} f(x)\,g(t-x)\,dx = \int_{-\infty}^{\infty} 0 = 0$$

$g(t-x)$

$f(x)$

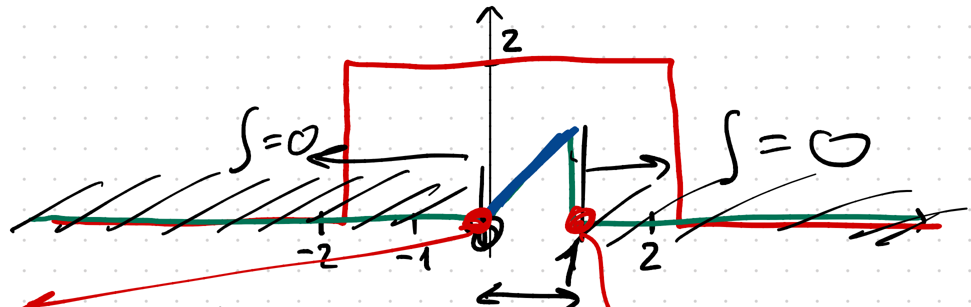


$$f(x)\cdot g(t-x) = 2\cdot x$$

$$(f_1 * f_2)(-2<t<-1) = \int_0^{2+t} 2x\,dx = \underline{(t+2)^2}$$

$g(t-x)$

$f(x)$



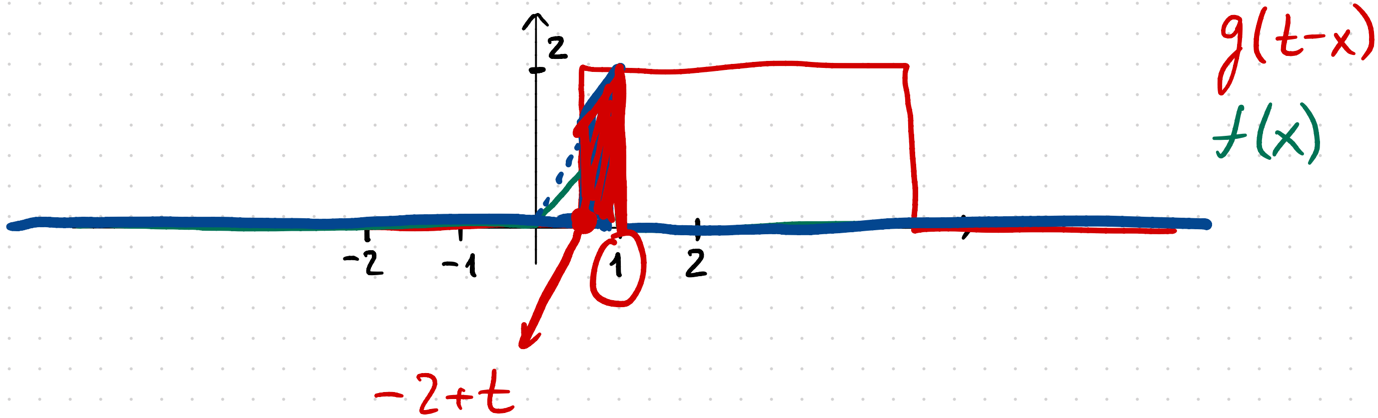
$-2+t=0$

$$\int_0^1 2x\,dx = 1$$

$+2+t=1$

$t=-1$

$$(f_1 * f_2)(-1<t<2) = \int_0^1 2x\,dx = \underline{\underline{1}}$$



$$(f_1 * f_2)(\underline{2<t<3}) = \int_{-2+t}^{1} 2x\,dx = 1-(t-2)^2$$

$$\int_{\alpha}^{\alpha+L} f(t)\,dt = \int_{\alpha}^{L} f(t)\,dt + \underbrace{\int_{\boxed{L}}^{\boxed{\alpha+L}} f(t)\,dt}$$

$\boxed{t = s+L}$ $\quad s = t-L$

$\boxed{\frac{ds}{dt} = 1} \Rightarrow dt = ds$

$$\int_{0}^{\alpha} f(s+L)\,ds$$

$t = s+L \rightarrow t \in [L, \alpha+L]$

bei L ist $s = 0$ $\quad [L = s+L \Rightarrow s=0]$

bei $\alpha+L$ ist $s = \alpha$ $\quad [\alpha+L = s+L \Rightarrow s=\alpha]$

$$\int_{\alpha}^{\alpha+L} f(t)\,dt = \int_{\alpha}^{L} f(t)\,dt + \int_{0}^{\alpha} f(t)\,dt = \int_{0}^{L} f(t)\,dt$$