1. Fourierreihe → Periodische Funktionen [T=2π]

→ Satz von Parseval

$$\boxed{\frac{1}{2\pi}\int_{-\pi}^{\pi}|f(x)|^2\,dx = \sum_{k=-\infty}^{\infty}|c_k|^2 = \frac{a_0^2}{4}+\frac{1}{2}\sum_{k=1}^{\infty}\left(|a_k|^2+|b_k|^2\right)}$$

2. Fouriertransformation → Alle [integrierbare] Funktionen

→ Satz von Plancherel

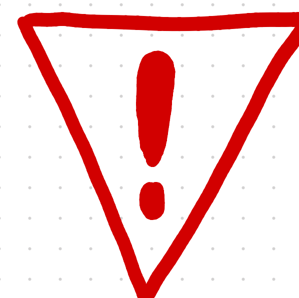
$$\boxed{\int_{-\infty}^{\infty}|f(t)|^2\,dt = \int_{-\infty}^{\infty}|\hat{f}(\omega)|^2\,d\omega}$$

Fouriertransformation

$$\mathcal{F}\{f\}(\omega) = \hat{f}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\,e^{-i\omega t}\,dt$$

→ Inverse FT: $\mathcal{F}^{-1}\{\hat{f}\}(t) = f(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \hat{f}(\omega)\,e^{i\omega t}\,d\omega$

$$\Rightarrow \underbrace{\mathcal{F}^{-1}\{\underbrace{\mathcal{F}\{f\}(\omega)}_{\text{FT}}\}(t)}_{\text{Inv FT}} = f(t)$$



$$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} \, dt$$

$F(t)$	$\widehat{F}(\omega)$	Notes	(0)
$f(t)$	$\int_{-\infty}^{\infty} f(t)e^{-i\omega t}\,dt$	Definition.	(1)
$\frac{1}{2\pi}\int_{-\infty}^{\infty} \widehat{f}(\omega)e^{i\omega t}\,d\omega$	$\widehat{f}(\omega)$	Inversion formula.	(2)
$\widehat{f}(-t)$	$2\pi f(\omega)$	Duality property.	(3)
$e^{-at}u(t)$	$\frac{1}{a+i\omega}$	a constant, $\Re e(a)>0$	(4)
$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$	a constant, $\Re e(a)>0$	(5)
$\beta(t)=\begin{cases}1, & \text{if } \|t\|<1,\\ 0, & \text{if } \|t\|>1\end{cases}$	$2\,\mathrm{sinc}(\omega)=2\frac{\sin(\omega)}{\omega}$	Boxcar in time.	(6)
$\frac{1}{\pi}\,\mathrm{sinc}(t)$	$\beta(\omega)$	Boxcar in frequency.	(7)
$f'(t)$	$i\omega\widehat{f}(\omega)$	Derivative in time.	(8)
$f''(t)$	$(i\omega)^2\widehat{f}(\omega)$	Higher derivatives similar.	(9)
$t\,f(t)$	$i\frac{d}{d\omega}\widehat{f}(\omega)$	Derivative in frequency.	(10)
$t^2 f(t)$	$i^2\frac{d^2}{d\omega^2}\widehat{f}(\omega)$	Higher derivatives similar.	(11)
$e^{i\omega_0 t}f(t)$	$\widehat{f}(\omega-\omega_0)$	Modulation property.	(12)
$f\left(\frac{t-t_0}{k}\right)$	$ke^{-i\omega t_0}\widehat{f}(k\omega)$	Time shift and squeeze.	(13)
$(f*g)(t)$	$\widehat{f}(\omega)\widehat{g}(\omega)$	Convolution in time.	(14)

$\frac{1}{\sqrt{2\pi}}$

$$\widehat{\left(\frac{d}{dt}f(t)\right)}(\omega) = i\omega\hat{f}(\omega)$$

$$\widehat{(f*g)}(\omega) = \hat{f}(\omega)\cdot\hat{g}(\omega)$$

Beispiel: Finde $\hat{f}(\omega)$ in Abhängigkeit von $\hat{g}(\omega)$

$$f(t) := \int_0^\infty g(t-s)\,ds$$

Methode 1:

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$$f(t) := \int_0^\infty g(t-s)\,ds$$

Methode 2:

Reelle Integrale mit exp, sin, cos

$$\int_{|z|=1} f(z)\,dz \xrightarrow[t\in[0,1]]{\gamma(t):=e^{2\pi i t}} \int_0^1 f(\underbrace{e^{2\pi i t}}_{\gamma(t)})\cdot \underbrace{2\pi i\, e^{2\pi i t}}_{\gamma'(t)}\,dt$$

$$f(z) = \frac{z}{1+2z}$$

Beispiel

$$f(e^{it}) = \frac{e^{it}}{1+2e^{it}}$$

Reelle Integrale mit exp, sin, cos

$$\int_{|z|=1} f(z)\,dz \xrightarrow[t\in[0,1]]{\gamma(t):=e^{2\pi i t}} \int_0^1 f(\underbrace{e^{2\pi i t}}_{\gamma(t)})\cdot \underbrace{2\pi i e^{2\pi i t}}_{\gamma'(t)}\,dt$$

$$\boxed{\int_0^T f\left(e^{\frac{2\pi}{T}it}\right) dt \xrightarrow{z:=e^{\frac{2\pi}{T}it}} \int_{|z|=1} f(z)\cdot\frac{1}{2\pi i z}\,dz}$$

$$\left[\frac{dz}{dt} = \frac{2\pi}{T} i e^{\frac{2\pi}{T}it} = \frac{2\pi}{T} i z \quad \Rightarrow dt = \frac{1}{2\pi i z}\right]$$

1. Alle period. Fkt. haben gleiche Periode
2. Grenzen $\Rightarrow$ <u>komplette</u> Umdrehungen (Vielfaches der Periode)

Beispiel: Finde $\int_0^{4\pi} \frac{e^{it}}{1+2e^{it}} \, dt$