Beispiel: Finde $\hat{f}(\omega)$ in Abhängigkeit von $\hat{g}(\omega)$

$$f(t) := \int_0^\infty g(t-s)\,ds$$

$$\sqrt{2\pi}\,\hat{f}(\omega) = \cdots = \hat{g}(\omega) \underbrace{\int_0^\infty e^{-i\omega s}\,ds}$$

$$\frac{1}{-i\omega} e^{-i\omega s}\Big|_0^\infty = \frac{1}{-i\omega}\left(\lim_{s\to\infty} e^{-i\omega s} - 1\right)$$

$F(t)$	$\widehat{F}(\omega)$	Notes	(0)
$f(t)$	$\int_{-\infty}^{\infty} f(t)e^{-i\omega t}\,dt$	Definition.	(1)
$\frac{1}{2\pi}\int_{-\infty}^{\infty}\widehat{f}(\omega)e^{i\omega t}\,d\omega$	$\widehat{f}(\omega)$	Inversion formula.	(2)
$\widehat{f}(-t)$	$2\pi f(\omega)$	Duality property.	(3)
$e^{-at}u(t)$	$\frac{1}{a+i\omega}$	a constant, $\Re(a)>0$	(4)
$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$	a constant, $\Re(a)>0$	(5)
$\beta(t)=\begin{cases}1, & \text{if } \lvert t\rvert<1,\\ 0, & \text{if } \lvert t\rvert>1\end{cases}$	$2\,\mathrm{sinc}(\omega)=2\frac{\sin(\omega)}{\omega}$	Boxcar in time.	(6)
$\frac{1}{\pi}\,\mathrm{sinc}(t)$	$\beta(\omega)$	Boxcar in frequency.	(7)
$f'(t)$	$i\omega\widehat{f}(\omega)$	Derivative in time.	(8)
$f''(t)$	$(i\omega)^2\widehat{f}(\omega)$	Higher derivatives similar.	(9)
$tf(t)$	$i\frac{d}{d\omega}\widehat{f}(\omega)$	Derivative in frequency.	(10)
$t^2f(t)$	$i^2\frac{d^2}{d\omega^2}\widehat{f}(\omega)$	Higher derivatives similar.	(11)
$e^{i\omega_0 t}f(t)$	$\widehat{f}(\omega-\omega_0)$	Modulation property.	(12)
$f\left(\frac{t-t_0}{k}\right)$	$ke^{-i\omega t_0}\widehat{f}(k\omega)$	Time shift and squeeze.	(13)
$(f*g)(t)$	$\widehat{f}(\omega)\widehat{g}(\omega)$	Convolution in time.	(14)

$\frac{1}{\sqrt{2\pi}}$

$$\widehat{\left(\frac{d}{dt}f(t)\right)}(\omega) = i\omega\hat{f}(\omega)$$

$$\widehat{(f*g)}(\omega) = \hat{f}(\omega)\cdot\hat{g}(\omega)$$

Laplacetransformation

$$\mathcal{L}\{f\}(s) = F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$

→ Existenz

1. f ist von exponentieller Ordnung

   $\exists\, c, s_0 > 0$ s.d. $|f(t)| \leq C e^{s_0 t}, t > 0$ $\Rightarrow$ $\mathcal{L}$ existiert für $\mathrm{Re}\{s_0\}$

2. Integrierbarkeit

   $\int_0^T |f(t)|\, dt < \infty, T > 0$ $\Rightarrow$ f stetig zwischen 0 und ∞

Beispiel:

→ $f(t) = e^{3t}$

→ $f(t) = e^{t^2}$

→ $f(t) = \frac{1}{t-3}$

→ $f(t) = \frac{1}{t+3}$

Laplacetransformation

$$\boxed{\mathcal{L}\{f\}(s) = F(s) = \int_0^\infty f(t)\, e^{-st}\, dt}$$

→ Eigenschaften

1. $f(t) \circ\!\!-\!\!\bullet\ F(s)$
2. $f(t-a) \circ\!\!-\!\!\bullet\ e^{-as} F(s)$
3. $\frac{d}{dt} f(t) \circ\!\!-\!\!\bullet\ sF(s) - f(0)$
4. $t^n e^{-at} \circ\!\!-\!\!\bullet\ \frac{n!}{(s+a)^{n+1}}$

⋮

	$f(t) = \mathfrak{L}^{-1}\{F(s)\}$	$F(s) = \mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$
3.	$t^n, \quad n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2 + a^2}$
9.	$t \sin(at)$	$\frac{2as}{(s^2 + a^2)^2}$
11.	$\sin(at) - at\cos(at)$	$\frac{2a^3}{(s^2 + a^2)^2}$
13.	$\cos(at) - at\sin(at)$	$\frac{s(s^2 - a^2)}{(s^2 + a^2)^2}$
15.	$\sin(at + b)$	$\frac{s\sin(b) + a\cos(b)}{s^2 + a^2}$

Beispiel: Gegeben ist $F(s) = \frac{1}{s(s-1)}$. Finde $f(t)$

$$\left[t^n e^{-at} \circ\!\!-\!\!\bullet \frac{n!}{(s+a)^{n+1}} \right]$$

Beispiel: Löse $\dot{f}(t) = e^t$, $f(0) = 0$

$\hookrightarrow \frac{d}{dt} f(t)$

$$\left[\begin{array}{lcl} \frac{d}{dt} f(t) & \circ\!\!-\!\!\bullet & sF(s) - f(0) \\ t^n e^{-at} & \circ\!\!-\!\!\bullet & \frac{n!}{(s+a)^{n+1}} \end{array}\right]$$

Beispiel: Was ist $\mathcal{L}\{\cdot\}$ von $\ddot{y}(t)$ in Abhängigkeit von $Y(s) = \mathcal{L}\{y\}$?

$\hookrightarrow \frac{d^2}{dt^2} y(t)$

$$\left[\frac{d}{dt} f(t) \;\circ\!\!-\!\!\bullet\; sF(s) - f(0) \right]$$