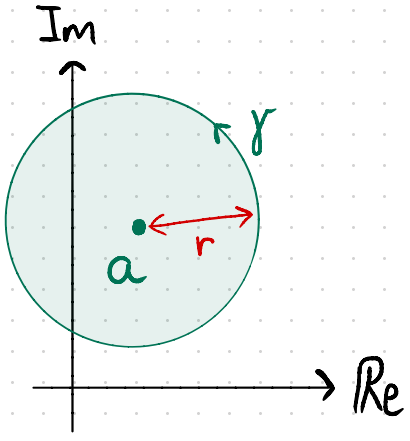
Mittelwertsatz

A5



→ Cauchy-Integralformel

$$f(a) = \frac{1}{2\pi i} \int_{\gamma} \frac{f(z)}{z-a}\, dz$$

$\gamma(t) = a + re^{2\pi i t}$

=

A5

Min-/Maxprinzip

→ Mittelwertsatz ⟹ $f(a) =$ „$\sum f(\text{Umgebung})$"

Beispiel: Noten der Basisprüfung

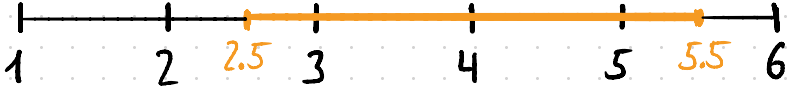
Min-/Maxprinzip

→ Mittelwertsatz ⟹ $f(a) =$ „$\sum f(\text{Umgebung})$"

Beispiel: Noten der Basisprüfung

⇝ Professor: „Note von Lukas = Mittelwert aller Noten"

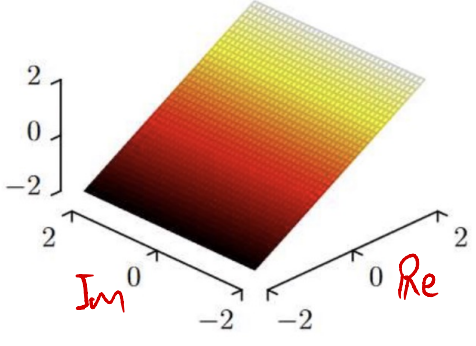
Student	Note
1.	2.5
2.	2.5
3.	3.0
4.	4.0
5.	4.5
6.	4.5
7.	5.0
8.	5.0
9.	5.0
10.	5.5



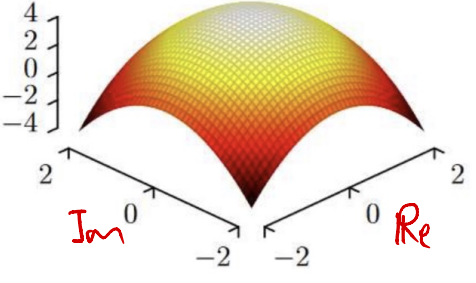
A5

Beispiel: Welche der folgenden Graphen stellen sicherlich ***nicht*** den Realteil einer ***holomorphen*** Funktion dar?

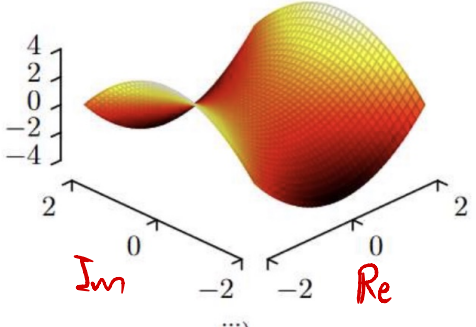
$$f: U \subset \mathbb{C} \to \mathbb{C}$$
$$\downarrow$$
$$U = [-2,2]^2$$



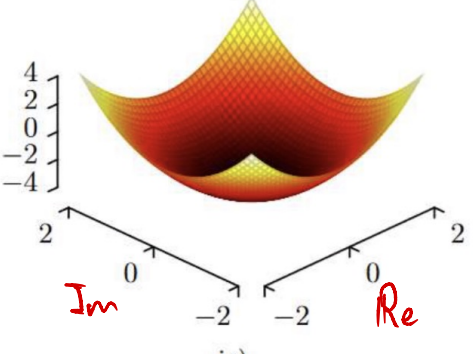
i)



ii)



iii)



iv)

nicht holomorph:

kann holomorph sein:

Laurententwicklung

A1-A4

→ $f: U \subset \mathbb{C} \to \mathbb{C}$ holomorph

→ Geometrische Reihe

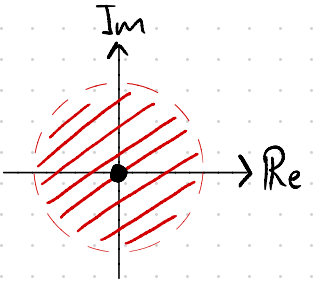
$$f(z) = \frac{1}{1-z} = \sum_{k=0}^{\infty} z^k, \quad |z| < 1$$

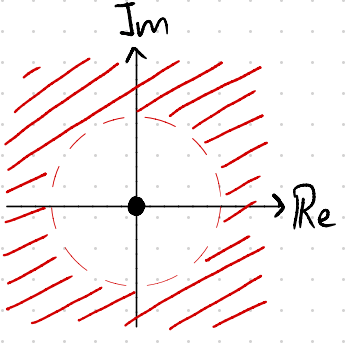
→ Partialbruchzerlegung [Theorie Woche 7]

$$\frac{1}{(z-a)(z-b)\cdots} = \frac{A}{z-a} + \frac{B}{z-b} + \cdots = \underbrace{\sum_{k=0}^{\infty} \cdots}_{\text{G.R.}} + \underbrace{\sum_{k=0}^{\infty} \cdots}_{\text{G.R.}} +$$

A1-A4

Beispiel: $f(z) = \frac{z}{3-2z}$





$$f(z) = \begin{cases} S1, & |z| < \\ S2, & |z| > \end{cases}$$

Trigonometrische Funktionen

A1-A4

→ Taylorentwicklung mit Entwicklungspunkt $z_0 = 0$ $\rightsquigarrow$ $\sum_{k=-\infty}^{\infty} c_n z^k$ $\quad c_{k<0} = 0$

→ Konvergenzradius $\Rightarrow \rho \to \infty \Rightarrow$ ganz $\mathbb{C}$

$$\exp(z) = \sum_{k=0}^{\infty} \frac{z^k}{k!}$$

$$\sin(z) = \sum_{k=0}^{\infty} (-1)^k \frac{z^{2k+1}}{(2k+1)!}$$

$$\ln(1-z) = \sum_{k=1}^{\infty} -\frac{z^k}{k}$$

$$\cos(z) = \sum_{k=0}^{\infty} (-1)^k \frac{z^{2k}}{(2k)!}$$

$$\ln(1+z) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{z^k}{k}$$

Beispiel: $f(z) = \frac{e^{z^2} \cdot z}{3 - 2z}$