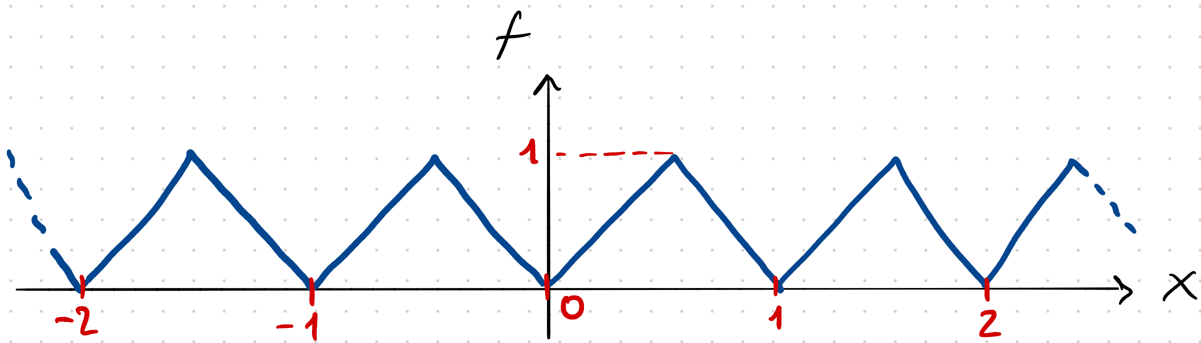
$$\int_{-\infty}^{\infty} \frac{1}{x^6+1}\,dx = 2\pi i \sum_{\operatorname{Im}>0} \operatorname{Res}(f|z_k)$$

Fourierreihen

→ f Periodisch $\Rightarrow$ $\exists\, T > 0$ so dass $f(x+kT) = f(x)$, $k \in \mathbb{Z}$
(kleinste T = Fundamentalperiode)



→ Darstellung von periodische Funktionen durch trigonometrische Funktionen
$\exp(i\cdot)$, $\cos$, $\sin$

komplex

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{2\pi i n t}{T}}$$

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} f(x)\, e^{-\frac{2\pi i n x}{T}} dx$$

$$a_n = c_n + c_{-n}$$

$$b_n = i(c_n - c_{-n})$$

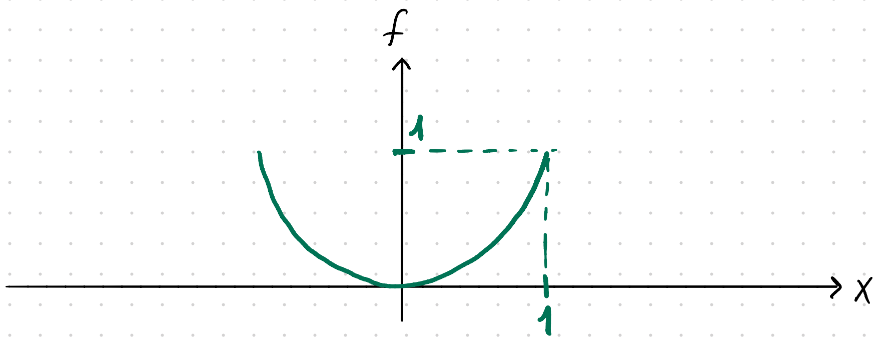
reell

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n t}{T}\right) + b_n \sin\left(\frac{2\pi n t}{T}\right)$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \cos\left(\frac{2\pi n x}{T}\right) dx \quad (n \geq 0)$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \sin\left(\frac{2\pi n x}{T}\right) dx \quad (n \geq 1)$$

<u>Beispiel:</u> $\tilde{f}(x) = x^2$ für $x \in [-1, 1] \rightsquigarrow$ Periode $T = 2$

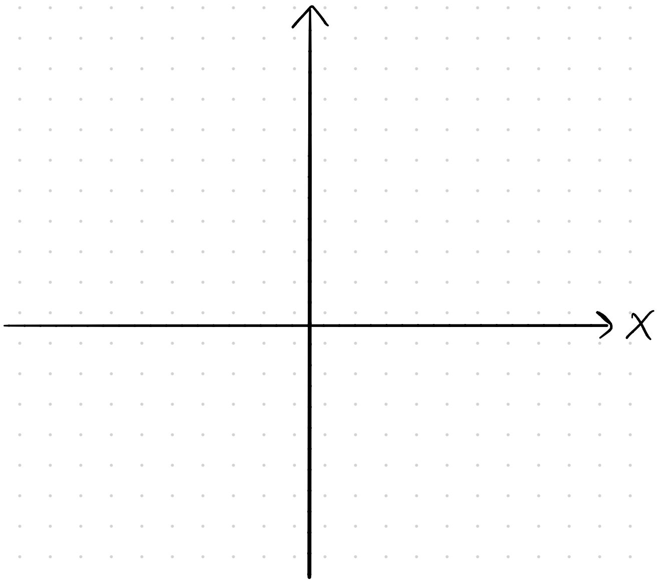


$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \cos\left(\frac{2\pi n}{T} t\right) dt$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \sin\left(\frac{2\pi n}{T} t\right) dt$$

$f(x)$ gerade $\Rightarrow f(-x) = f(x)$ Bsp.: $\cos(x)$

$f(x)$ ungerade $\Rightarrow f(-x) = -f(x)$ Bsp.: $\sin(x)$



$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(x)\cos\left(\frac{2\pi n}{T}t\right)dt$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(x)\sin\left(\frac{2\pi n}{T}t\right)dt$$

<u>f gerade</u>:

$a_n =$

<u>f ungerade</u>:

$a_n =$

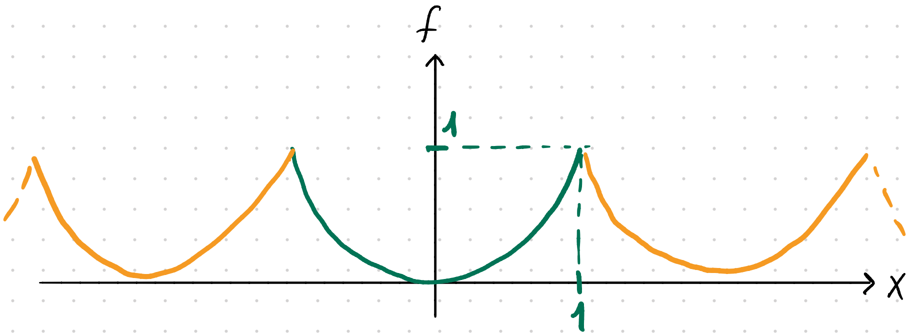
<u>f gerade</u>:

$b_n =$

<u>f ungerade</u>:

$b_n =$

Beispiel: $\tilde{f}(x) = x^2$ für $x \in [-1,1] \rightsquigarrow$ Periode $T = 2$



$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \cos\left(\frac{2\pi n}{T} t\right) dt$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \sin\left(\frac{2\pi n}{T} t\right) dt$$

$a_n =$

$b_n =$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{\frac{2\pi i n t}{T}}, \qquad f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi n t}{T}\right) + b_n \sin\left(\frac{2\pi n t}{T}\right)$$

Beispiel: $f(t) = \sin^2(t)$