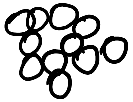
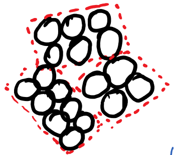
① Crystal Structures

i. Amorphus



No order (caos)

ii. Polycristaline

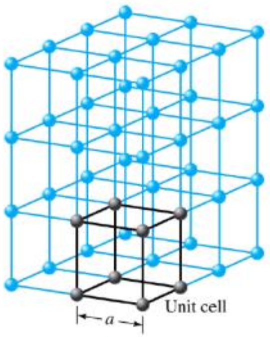


more or less order

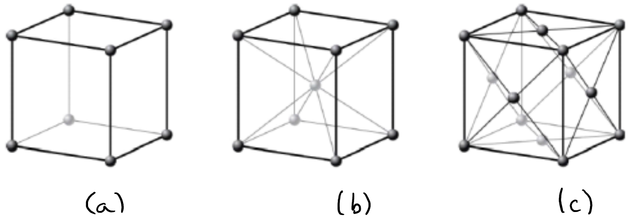
iii. Single-crystal

ordered cells

→ **Coordination number**: Number of nearest neighbours any atom has in a given crystal lattice. By definition a crystal lattice is periodic in 3D.

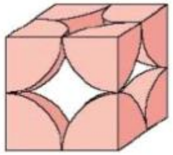


↳ lattice constant



(a) simple cubic (sc) CN = 4
(b) body-centered (bcc) CN = 8
(c) face-centered cubic (fcc) CN = 12

→ **Packing Density**: Ratio between the volume occupied by the atoms and the volume of one entire unit cell. The volume of the atoms is the volume determined by the sphere with radius equal to half the distance to the nearest neighbor multiplied by the relative number of atoms inside the unit cell.

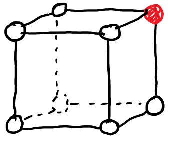


$$\text{Packing density} = \frac{\text{Volume occupied by atoms in one unit cell}}{\text{Volume of one unit cell}}$$

- The relative number of atoms inside a unit cell is, for each individual atom, its contribution to one unit cell (factor with which it is shared in the crystal)

sc → 1 bcc → 2 fcc → 4 diamond → 8

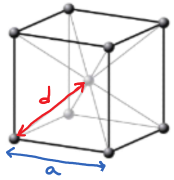
Example: Atom in red is shared between 8 cells in the crystal.



⇒ Its contribution = $1/8$
Since there are 8 equal cells in this unit cell
⇒ relative number of atoms $= 8 \cdot \frac{1}{8} = 1$

$$\rightsquigarrow \text{Packing density} = \frac{(\text{smallest atom volume})\cdot(\text{relative number of atoms})}{\text{unit cell volume} = a^3}$$

Example: Packing density of bcc structure



d = distance to nearest neighbor $= \frac{\sqrt{3}}{2}a$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{r}{2}\right)^3 = \frac{4}{3}\frac{\pi d^3}{8} = \frac{\pi d^3}{6} = \frac{\pi\cdot 3\sqrt{3}a^3}{6\cdot 8} = \frac{\sqrt{3}\pi a^3}{16}$$

$$\#\text{atoms} = 8\cdot\frac{1}{8} + 1 = 2$$

$$\Rightarrow \text{Packing density} = \frac{\frac{\sqrt{3}\pi a^3}{16}\cdot 2}{a^3} = \frac{\sqrt{3}}{8}\pi$$

simple cubic Po

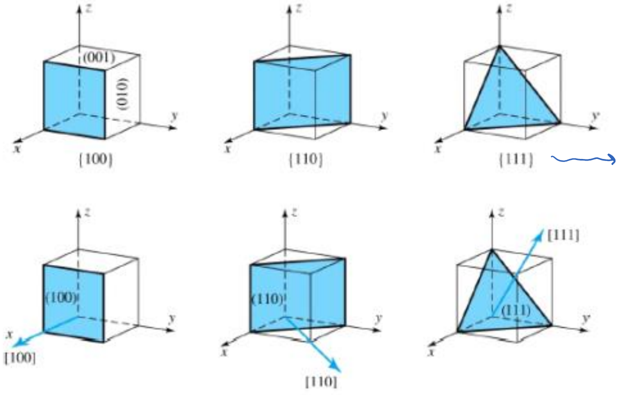
body-centered cubic Li, Na, K, Cr, Fe, Nb, Mo

face-centered cubic Al, Ar, Ni, Cu, Kr, Pd, Ag, Xe, Ta, W, Pt, Au

diamond Si

sc → $\pi/6$
bcc → $\frac{\sqrt{3}}{8}\pi$
fcc → $\frac{\sqrt{2}}{6}\pi$
diamond → $\frac{\sqrt{3}}{16}\pi$

→ **Miller Index**: The crystal has different periodicities with different directions, ie., it does not look the same in all directions. Different planes and directions generally have different properties

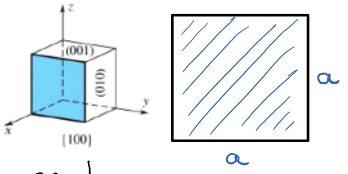


⟶ Miller Index

→ Atomic density per unit area

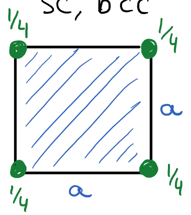
$$\text{atomic density per unit area} = \frac{\text{relative number of atoms on the plane}}{\text{area of the face in the cell}} \quad [cm^{-2}]$$

[100]



Area a^2

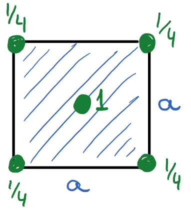
sc, bcc



$\#\text{atoms} = 4\cdot\frac{1}{4} = 1$

$Pd = \boxed{\frac{1}{a^2}}$

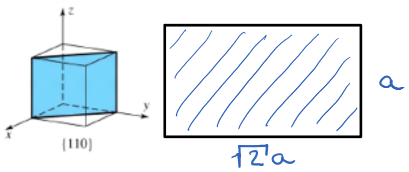
fcc, diamond



$\#\text{atoms} = 4\cdot\frac{1}{4} + 1 = 2$

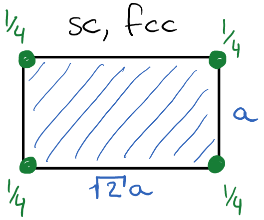
$Pd = \boxed{\frac{2}{a^2}}$

[110]



Area $\sqrt{2}a^2$

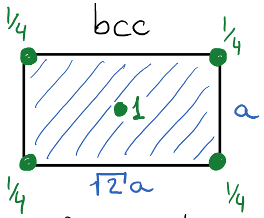
sc, fcc



$\#\text{atoms} = 4\cdot\frac{1}{4} = 1$

$Pd = \boxed{\frac{1}{\sqrt{2}a^2}}$

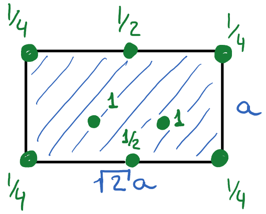
bcc



$\#\text{atoms} = 4\cdot\frac{1}{4} + 1 = 2$

$Pd = \frac{2}{\sqrt{2}a^2} = \boxed{\frac{\sqrt{2}}{a^2}}$

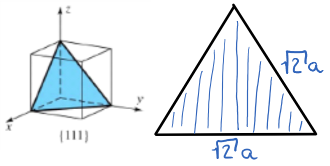
Diamond



$\#\text{atoms} = 4\cdot\frac{1}{4} + 2\cdot\frac{1}{2} + 2 = 4$

$Pd = \frac{4}{\sqrt{2}a^2} = \boxed{\frac{2\sqrt{2}}{a^2}}$

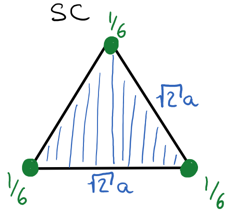
[111]



Area

$\frac{\sqrt{2}}{2}a\cdot\sqrt{\frac{3}{2}}a = \frac{\sqrt{3}}{2}a^2$

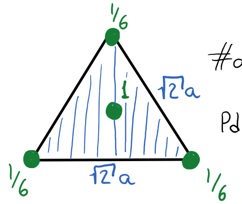
sc



$\#\text{atoms} = 3\cdot\frac{1}{6} = \frac{1}{2}$

$Pd = \frac{1/2}{\frac{\sqrt{3}}{2}a^2} = \boxed{\frac{1}{\sqrt{3}a^2}}$

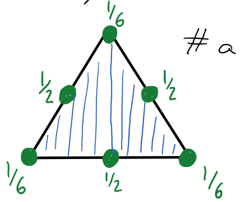
bcc



$\#\text{atoms} = 3\cdot\frac{1}{6} + 1 = \frac{3}{2}$

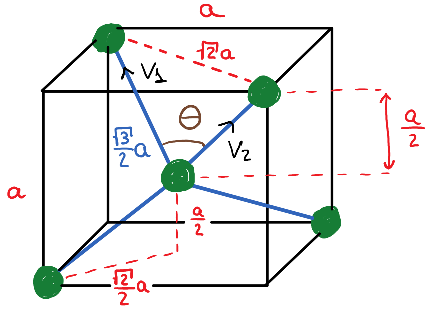
$Pd = \frac{3/2}{\frac{\sqrt{3}}{2}a^2} = \boxed{\frac{\sqrt{3}}{a^2}}$

fcc, diamond



$\#\text{atoms} = 3\cdot\frac{1}{6} + 3\cdot\frac{1}{2} = 2$

$Pd = \frac{2}{\frac{\sqrt{3}}{2}a^2} = \boxed{\frac{4}{\sqrt{3}a^2}}$



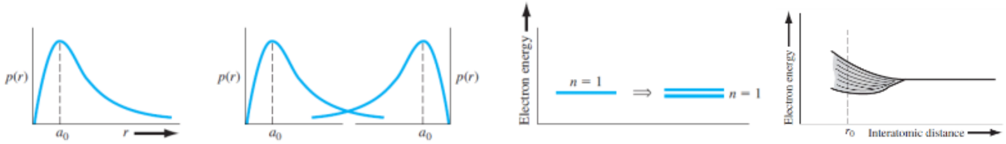
$V_1 = \begin{pmatrix}-1\\-1\\1\end{pmatrix} \quad |V_1| = |V_2| = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$V_2 = \begin{pmatrix}1\\1\\1\end{pmatrix} \quad V_1\cdot V_2 = |V_1||V_2|\cos(\theta)$

$\theta = \arccos\left(\frac{V_1\cdot V_2}{|V_1||V_2|}\right) = \arccos\left(-\frac{1}{3}\right) \approx 109{,}47°$

② Quantum Theory of Solids

1. Energy Bands



Splitting of energy state ↵

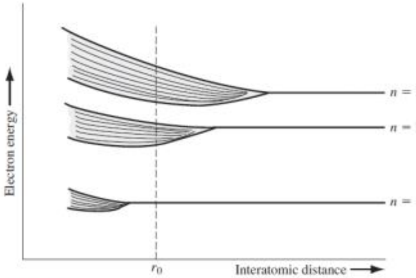
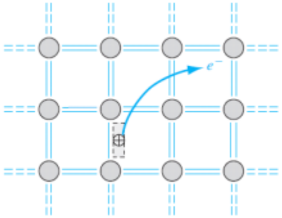


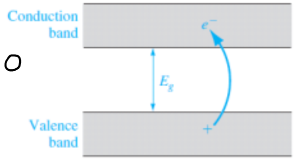
Figure 3.3 | Schematic showing the splitting of three energy states into allowed bands of energies.

When the atoms are close enough even the electrons on the $n = 2, 1$ shell start to interact ⇒ splitting of energy states

2. Electrical conduction



$T > 0$ ⇒



e^- moves from valence band to the next band (conduction band) → because of thermal excitation

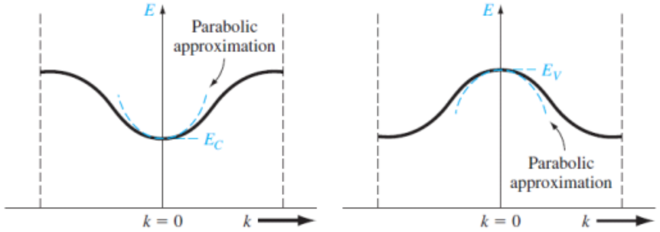
- The semiconductor is neutrally charged: # e^- in conduction band is equal to # holes in valence band.

→ **Effective Mass:** The movement of an electron in a lattice and in free space is different

$$F_{total} = F_{ext} + F_{int} = m \cdot a \xrightarrow{approx.} F_{ext} \approx m^* \cdot a$$

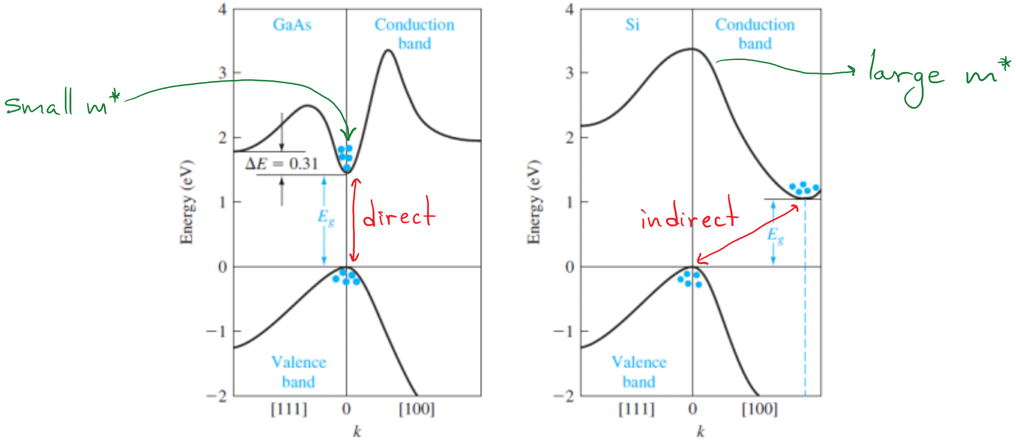
effective mass ↵

m^* takes into account the real particle's mass pluss any other internal force (m^* is an „apparent" mass)



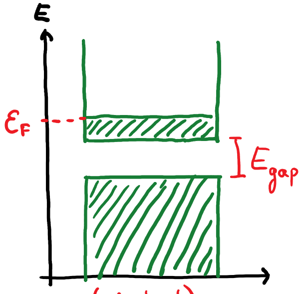
$$\boxed{\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}}$$

- small curvature ‿ ⇒ large m^*
- large curvature V ⇒ small m^*

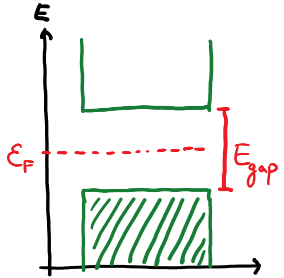


→ **Direct bandgap:** maximum valence band energy has the same k-value as the minimum conduction band energy

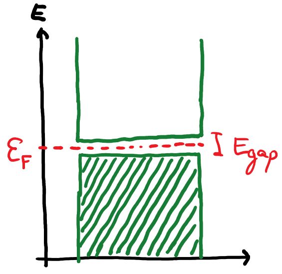
- Direct → good for lasers and other optical devices



(Metal)



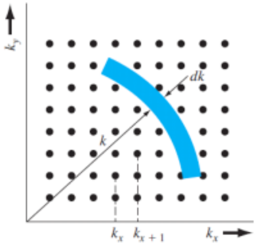
(Isolator)

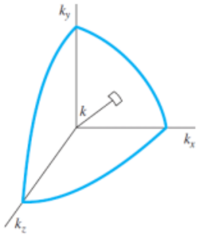


(Semiconductor)

3. Density of states

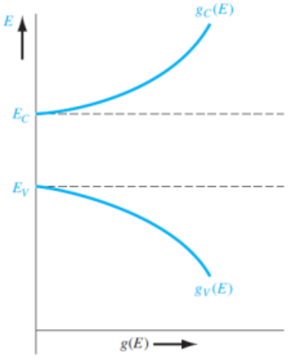
→ Interested in the possible numbers of states for a given energy level





$$\boxed{g(E) = \frac{4\pi(2m)^{3/2}}{h^3}\sqrt{E}}$$

„number of quantum states between energy E and E+dE per unit volume of the crystal"



$$\boxed{g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3}\sqrt{E-E_c}}$$

$$E \approx E_c + \frac{\hbar^2k^2}{2m_n^*}$$

$$\boxed{g_v(E) = \frac{4\pi(2m_p^*)^{3/2}}{h^3}\sqrt{E_v-E}}$$

$$E \approx E_c - \frac{\hbar^2k^2}{2m_p^*}$$

4. Fermi-Dirac Probability function

→ We want to find the probability the a specific energy state is occupied by an electron/hole

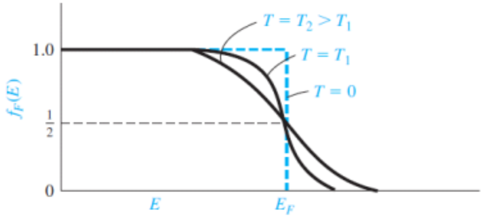
$$\boxed{\frac{N(E)}{g(E)} = f_F(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{kT}\right)}}$$

$N(E)$ = number of particles per unit volume per unit energy

$g(E)$ = number of states per unit volume per unit energy

E_F = Fermi energy level → where $f_F = 1/2$ $(f(E_F) = 1/2)$

Probability for holes $\Rightarrow$ $1 - f_F(E)$



for T=0 are all lowest states on the valence band occupied.

→ Boltzmann-Approximation: Approximation for the Fermi probability function for energies far away from E_F

$$E - E_F \gg kT \Rightarrow \boxed{f_F(E) \approx \exp\left(\frac{E_F-E}{kT}\right)}$$

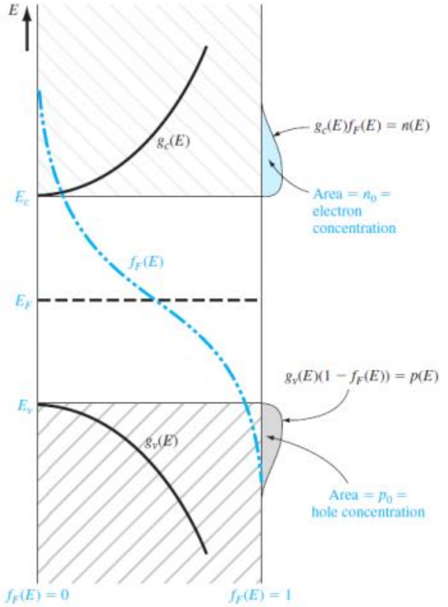
③ Semiconductor in Equilibrium

→ **Intrinsic semiconductor:** pure semiconductor with no impurities.

In an intrinsic semiconductor the number of electrons in the conduction band is equal to the number of holes in the valence band.

$$\boxed{n_o = p_o = n_i}$$

→ intrinsic carrier concentration



For the intrinsic case

i. E_F is near midgap

ii. $n_o = p_o = n_i \quad \forall T$

iii. $n_o = \int_{E_c}^{\infty} g_c(E) \cdot f_F(E)\, dE$

$$p_o = \int_{-\infty}^{E_v} g_v(E)(1-f_F(E))\, dE$$

1. The n_o and p_o equations

→ Using the Boltzmann approximation:

i. electron concentration $n_o = \int_{E_c}^{\infty} g_c(E) f_F(E)\,dE$

$$\boxed{n_o = N_c e^{-\left(\frac{E_c-E_F}{kT}\right)}} \quad \boxed{N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}}$$

ii. hole concentration $p_o = \int_{-\infty}^{E_v} g_c(E)[1-f_F(E)]\,dE$

$$\boxed{p_o = N_v e^{-\left(\frac{E_F-E_v}{kT}\right)}} \quad \boxed{N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}}$$

iii. Intrinsic concentration

$$n_o = n_i = p_o \Rightarrow n_i^2 = n_o \cdot p_o = N_c N_v e^{-\left(\frac{E_c-E_F}{kT}\right)} = N_c N_v e^{\frac{-E_g}{kT}}$$

$$\boxed{n_i^2 = N_c N_v e^{\frac{-E_G}{kT}}} \quad \text{where} \quad E_G = E_c - E_v$$

iv. Position of Fermi energy

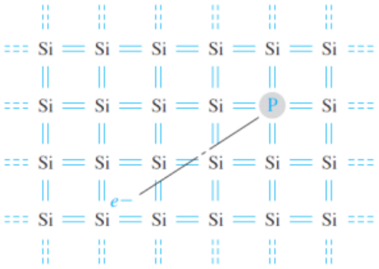
$$n_o = p_o \Rightarrow N_c e^{-\left(\frac{E_c-E_F}{kT}\right)} = N_v e^{-\left(\frac{E_F-E_v}{kT}\right)}, \quad E_{mid} = \frac{1}{2}(E_c+E_v)$$

$$\boxed{E_{Fi} - E_{mid} = \frac{1}{2}kT\ln\left(\frac{N_v}{N_c}\right) = \frac{3}{4}kT\ln\left(\frac{m_p^*}{m_n^*}\right)}$$

2. Dopant Atoms and energy levels

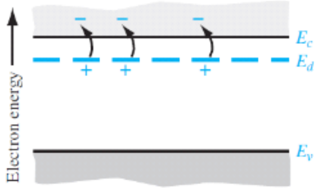
→ Add small small, controlled amounts of specific dopants
⇒ extrinsic material (has impurities)

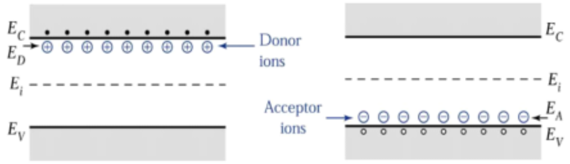
The energy required to elevate the donor electron into the conduction band is considerably less than that for electrons involved in the covalent bonding

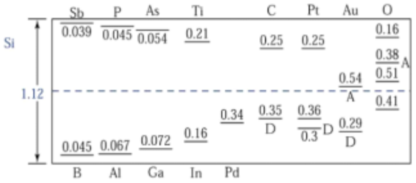


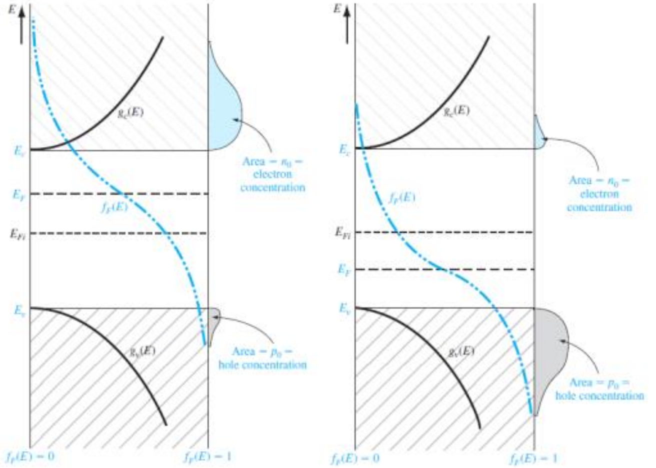
Donor (P) "gives" e⁻ to conduction →

n-type semiconductor









$$\boxed{n_o = n_i e^{\frac{E_F-E_{Fi}}{kT}}}$$

$$\boxed{p_o = n_i e^{-\frac{(E_F-E_{Fi})}{kT}}}$$

$$\boxed{E_F - E_c = kT\ln\left(\frac{n_o}{N_c}\right)} \quad \boxed{E_v - E_F = kT\ln\left(\frac{p_o}{N_v}\right)}$$

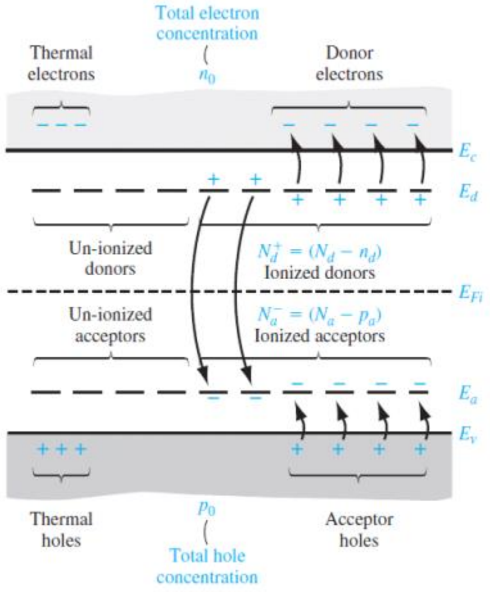
$$\boxed{E_F - E_{Fi} = kT\ln\left(\frac{n_o}{n_i}\right)}$$

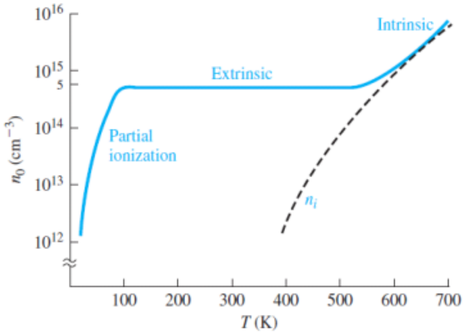
→ Mass action law

$$\boxed{n_0 p_0 = n_i^2} \rightarrow \text{independent of dobing levels}$$

→ Compensated semiconductor: contains both donors and acceptors

→ Electroneutrality:





$$\boxed{\begin{aligned} n_0 + N_A^- &= p_0 + N_D^+ \\ n_0 + (N_A - p_A) &= p_0 + (N_D - n_A) \end{aligned}}$$

type	condition	electrons	holes
n-type	$N_D \gg N_A, n_i$	$n_{n0} \approx N_D$	$p_{n0} \approx \frac{n_i^2}{n_0}$
p-type	$N_A \gg N_D, n_i$	$n_{p0} \approx \frac{n_i^2}{p_0}$	$p_{p0} \approx N_A$

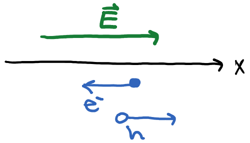
$$\boxed{n_0 = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2}} \qquad \boxed{p_0 = \frac{(N_a - N_d)}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2}}$$

④ Carrier Transport Phenomena

1. Drift Currents

→ μ-Mobility: Tells us how well a particle can move on an E-Field

$$\boxed{|V_E| = \mu E} \quad \text{and} \quad \boxed{J_{drf} = \rho V_E} \rightarrow \rho = \text{charge density}: (\pm) e \frac{\#\text{particles}}{\text{Volume}}$$



electrons: $V_E = -\mu_n E$ $\quad \rho = -en$

holes: $V_E = \mu_p E$ $\quad \rho = ep$

$$\boxed{J_{drf,e} = (-\mu_n E)(-en) = e\mu_n n E} \qquad \boxed{J_{drf,e} = (-\mu_n E)(-en) = e\mu_n n E}$$

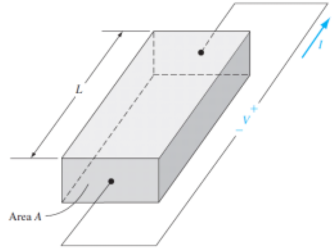
$$\boxed{J_{drf} = J_{drf,e} + J_{drf,p} = e(\mu_n n + \mu_p p)E}$$

	μ_n (cm²/V-s)	μ_p (cm²/V-s)
Silicon	1350	480
Gallium arsenide	8500	400
Germanium	3900	1900

electrons are in general faster than holes!

→ Conductivity: Current $J \propto E \longrightarrow J = \sigma E$, $[\sigma] = (\Omega \text{cm})^{-1}$

→ Resistivity: Reciprocal of conductivity: $J = \frac{1}{\rho} E$, $[\rho] = \Omega \text{cm}$



$$\boxed{\rho = \frac{1}{\sigma} = \frac{1}{e(\mu_n n + \mu_p p)}}$$

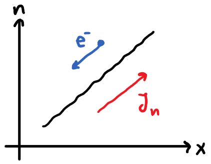
Resistance R: $\boxed{R = \rho \frac{L}{A} = \frac{1}{\sigma} \frac{L}{A}}$

$[R] = \Omega$

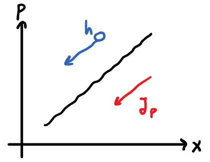
2. Diffusion Currents

→ Flow from a region of high concentration to a region of low concentration.

$$F_n = -V_{th} \ell \frac{dn}{dx} \quad \text{and} \quad J = -eF_n \Rightarrow J = eV_{th} \ell \frac{dn}{dx} = eD_n \frac{dn}{dx}$$



$$\boxed{J_{diff,n} = eD_n \frac{dn}{dx}}$$



$$\boxed{J_{diff,p} = -eD_p \frac{dp}{dx}}$$

$$\boxed{J_{diff} = eD_n \frac{dn}{dx} - eD_p \frac{dp}{dx}}$$

3. Total Current Density

$$\boxed{J_{tot} = J_{drf} + J_{diff} = en\mu_n E_x + ep\mu_p E_x + eD_n\frac{dn}{dx} - eD_p\frac{dp}{dx}}$$

→ Einstein's Relation

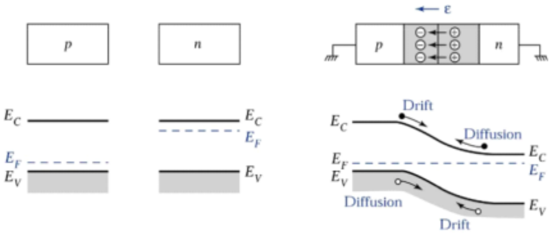
No connection ⇒ no current ⇒ $J_n = en\mu_n E + eD_n\frac{dn}{dx} \overset{!}{=} 0$

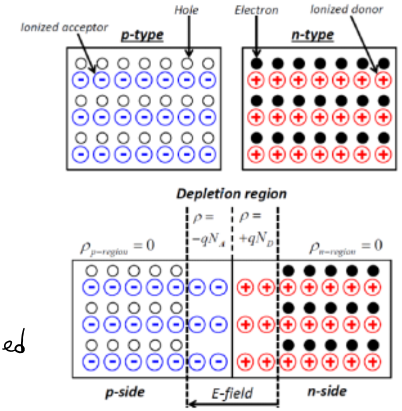
⇒ $\frac{D_n}{\mu_n} = \frac{kT}{e}$ same for hole concentration ↵

$$\boxed{\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}}$$

⚠ Electrons move in opposite direction as E-Field, holes move in the same direction as the E-Field.

⑤ The pn Junction



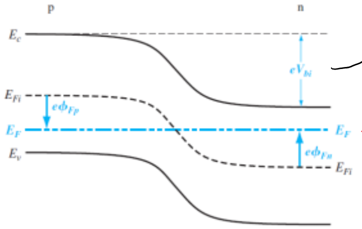


→ The electrons of the donors move to the p side, leaving a positive charged atom ⇒ ε-Field

→ E-Field → from positive to negative ⇒ from n to p.

1. Built-in Voltage

→ Since the Fermi-level has to be constant (same probabilites for electrons), the conduction and valence band energies must bend



→ electrons see a potential barrier when trying to go to the p-side ⇒ built-in potential barrier (V_{bi})

→ E_F constant

$$\boxed{E = \frac{1}{q}\frac{\partial E_c}{\partial x} = \frac{1}{q}\frac{\partial E_v}{\partial x} = \frac{1}{q}\frac{\partial E_i}{\partial x}}$$

→ Comparing the energies on the p and n side:

$$(E_c - E_F) = (E_c - E_F) + eV_{bi} \Rightarrow -\frac{(E_c-E_F)}{kT} = -\frac{(E_c-E_F)}{kT} - \frac{qV_{bi}}{kT} \Rightarrow \underbrace{N_c e^{-\frac{(E_c-E_F)}{kT}}}_{n_{po}} = \underbrace{N_c e^{-\frac{(E_c-E_F)}{kT}}}_{n_{no}} e^{-\frac{qV_{bi}}{kT}}$$

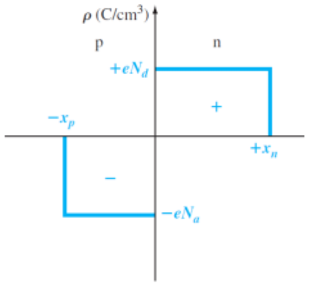
$$\Rightarrow \boxed{n_{po} = n_{no}\, e^{-\frac{qV_{bi}}{kT}}} \quad \boxed{p_{no} = p_{po}\, e^{-\frac{qV_{bi}}{kT}}}$$

- Consider $n_{no} \approx N_D$ and $p_{po} \approx N_A \Rightarrow n_{po} = \frac{n_i^2}{N_A}$

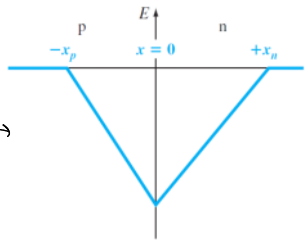
$$\Rightarrow \frac{n_i^2}{N_A} = N_D\, e^{-\frac{qV_{bi}}{kT}} \xrightarrow{\text{solve for } V_{bi}} \boxed{V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right) = V_t \ln\left(\frac{N_A N_D}{n_i^2}\right)}$$

2. Electric Field

→ Poisson's equation: $\boxed{\frac{d^2\phi}{dx} = \frac{-\rho(x)}{\varepsilon_s} = \frac{dE(x)}{dx}}$



$$\xrightarrow{\int \frac{\rho(x)}{\varepsilon_s}dx}$$



↗ we consider to have E = 0 on the boundaries ($x = -x_p$ and $x = x_n$)

$$\left.\begin{aligned} E(-x_p \le x \le 0) &= \int \frac{-eN_a}{\varepsilon_s}dx = \frac{-eN_a}{\varepsilon_s}x + C \\ E(0 \le x \le x_n) &= \int \frac{eN_d}{\varepsilon_s}dx = \frac{eN_d}{\varepsilon_s}x + C \end{aligned}\right\} \text{initial conditions} \quad \begin{aligned} E(-x_p) &= 0 \\ E(x_n) &= 0 \end{aligned}$$

$$E(-x_p \le x \le 0) = \frac{-eN_a}{\varepsilon_s}(x + x_p)$$

$$E(0 \le x \le x_n) = \frac{-eN_d}{\varepsilon_s}(x_n - x)$$

→ $E(x=0)$ continuous ⇒ $\boxed{N_a x_p = N_d x_n}$

$\phi(x) = -\int E(x)\,dx$ → The initial condition is not that important. We want the potential difference rather than the absolute value

Set $\phi(x=-x_p) = 0 \Rightarrow \phi(-x_p \le x \le 0) = \frac{eN_a}{2\varepsilon_s}(x + x_p)^2$

for $0 \le x \le x_n$ we use $\phi(x=0) = \longrightarrow$ at $x = 0$

$$\phi(-x_p \le x \le 0) = \frac{eN_a}{2\varepsilon_s}(x + x_p)^2 \quad \text{and} \quad \phi(0 \le x \le x_n) = \frac{eN_d}{\varepsilon_s}\left(x_n x - \frac{x^2}{2}\right) + \frac{eN_a}{2\varepsilon_s}x_p^2$$

→ We have $V_{bi} = \phi(x=x_n) - \phi(x=-x_p)$

$\Rightarrow$ $V_{bi} = \frac{e}{2\varepsilon_s}(N_d x_n^2 + N_a x_p^2)$ → use $x_p N_a = x_n N_d$

$$\boxed{x_n = \left[\frac{2\varepsilon_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}} \qquad \boxed{x_p = \left[\frac{2\varepsilon_s V_{bi}}{e}\left(\frac{N_d}{N_a}\right)\left(\frac{1}{N_a+N_d}\right)\right]^{1/2}}$$

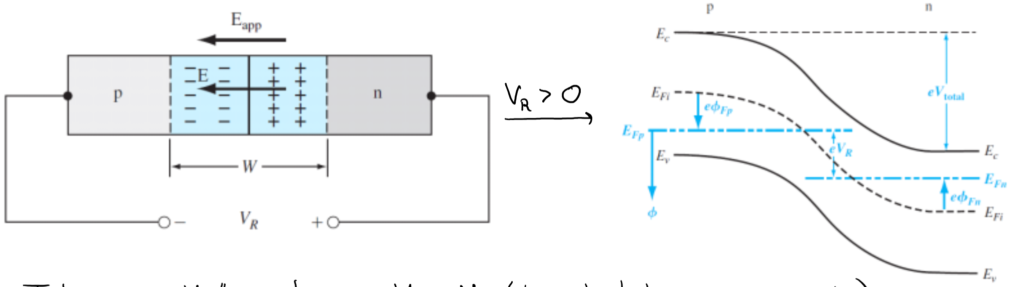
„depletion width" → $\boxed{W = x_n + x_p = \left[\frac{2\varepsilon_s V_{bi}}{e}\left(\frac{N_a+N_d}{N_a N_d}\right)\right]^{1/2}}$

→ E_{max}: Maximum electric field in the depletion region (at $x=0$)

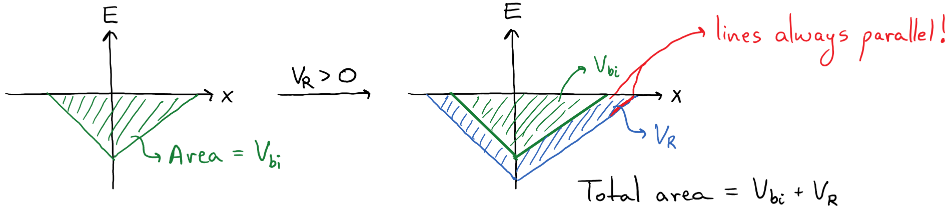
$$\boxed{E_{max} = \frac{-eN_d x_n}{\varepsilon_s} = \frac{-eN_a x_p}{\varepsilon_s} = -\left[\frac{2e}{\varepsilon_s}\left(\frac{N_a N_d}{N_a+N_d}\right)\right]^{1/2} = \frac{-2V_{bi}}{W}}$$

3. Bias

→ Reverse bias: substitute V_{bi} with $V_{bi} + V_R$
Forward bias: substitute V_{bi} with $V_{bi} - V_F$



- Take „new V_{bi}" as being $V_{bi} + V_R$ (to calculate x_n, x_p, W etc)

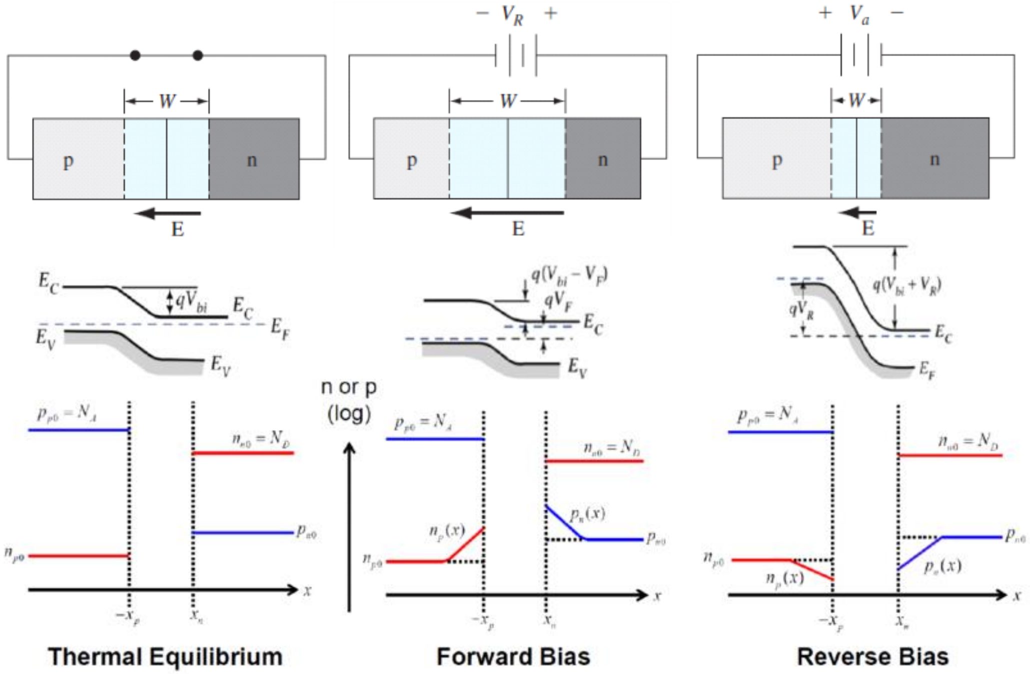


$$n_p = n_{n0} e^{\frac{-q(V_{bi}-V_F)}{kT}} = n_{p0} e^{\frac{qV_F}{kT}} \longrightarrow \boxed{n_p = n_{p0} e^{\frac{qV_F}{kT}}} \quad \boxed{p_n = p_{n0} e^{\frac{qV_F}{kT}}}$$

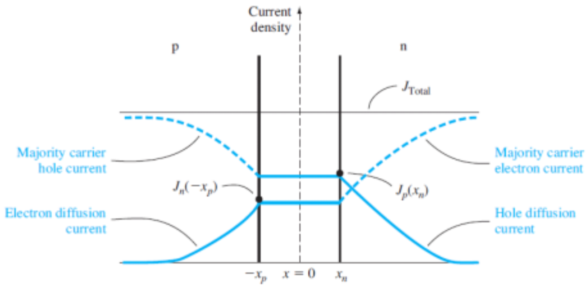
4. Schockley - Boundary Condition

$$\boxed{n_p(-x_p) = n_{p0} e^{\frac{-qV_R}{kT}}} \quad \boxed{p_n(x_n) = p_{n0} e^{\frac{-qV_R}{kT}}} \quad \boxed{p_{n0} = p_{p0} e^{\frac{-qV_{bi}}{kT}}}$$

$$\boxed{n_p(-x_p) = n_{p0} e^{\frac{qV_F}{kT}}} \quad \boxed{p_n(x_n) = p_{n0} e^{\frac{qV_F}{kT}}} \quad \boxed{n_{p0} = n_{n0} e^{\frac{-qV_{bi}}{kT}}}$$



⑥ The Diode



There is no electric field outside the depletion region ⇒ no drift currents, only diffusion

$$\boxed{J_{tot} = J_{n,diff}(-x_p) + J_{p,diff}(x_n)}$$

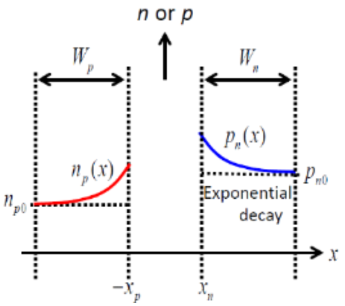
determined by minority carriers

$p_n(x) = p_{n0} + (p_n(x_n) - p_{n0}) e^{-\frac{x-x_n}{L_p}}$ and $J_p(x_n) = -qD_p \frac{\partial}{\partial x}(x)\Big|_{x=x_n}$

$$\Rightarrow \boxed{J = \left(\frac{qD_n n_{po}}{L_n} + \frac{qD_p p_{no}}{L_p}\right)\left(e^{\frac{qV_F}{kT}} - 1\right)}$$

$$\boxed{J = J_s\left(e^{\frac{qV_F}{kT}} - 1\right) \approx J_s e^{\frac{qV_F}{kT}}} \rightarrow \text{Approximation}$$

→ Short diode: if W_p, W_n are smaller than L_p, L_n values of $n_p(x_n)$, n_{p_o} (at beginning of the diode) and $p_n(x_n)$, p_{no} (at the end of the diode) have to be maintained

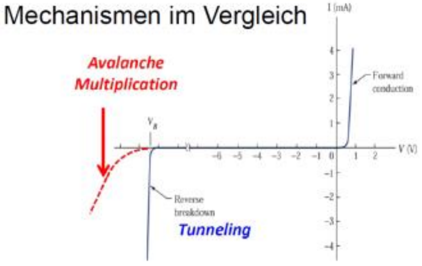


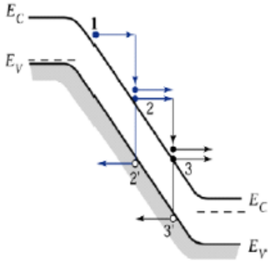
$$\Rightarrow \boxed{J = \left(\frac{qD_n n_{po}}{L_n} + \frac{qD_p p_{no}}{L_p}\right)\left(e^{\frac{qV_F}{kT}} - 1\right)}$$

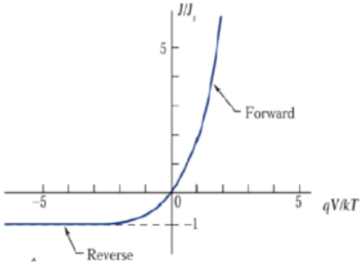
→ Breakdown

i. Tunneling: Valence and conduction band are very close from each other ⇒ tunneling of electrons

ii. Avalanche Multiplication: Electrons have enough energy to ionize other electrons before they recombine

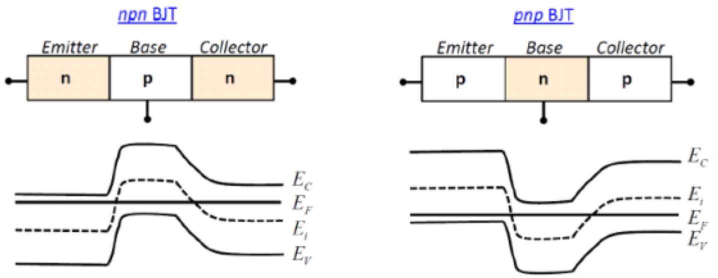


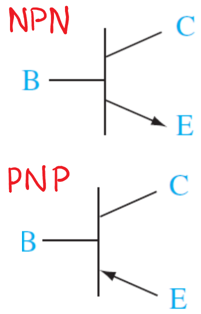


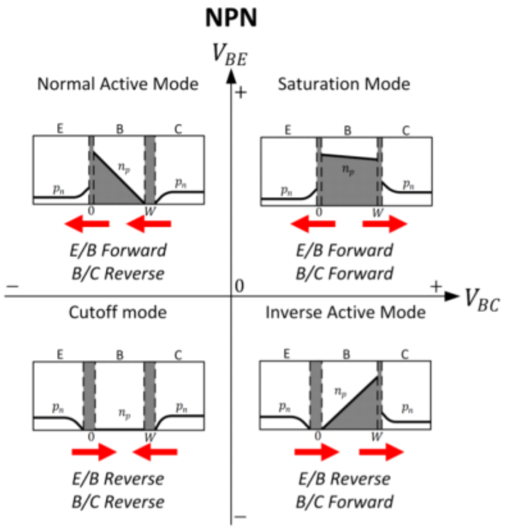


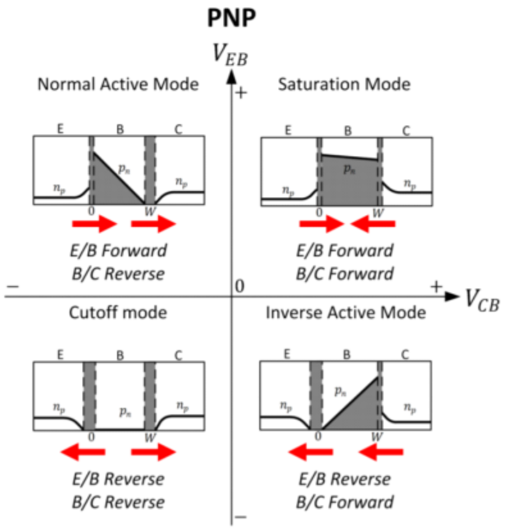
There is a maximum current for reverse bias → J_s

⑦ Bipolar Junction Transistor (BJT)





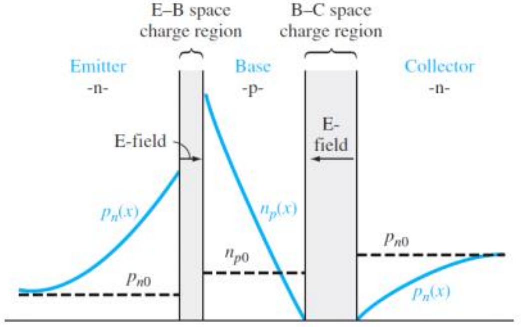




→ Normal active mode:

i. Minority carrier injection (forward bias) at E/B
⇒ electrons (minority carriers) injected into base

ii. Minority carrier extraction (reverse bias) at B/C
⇒ electrons (minority carriers) extracted from base to collector.

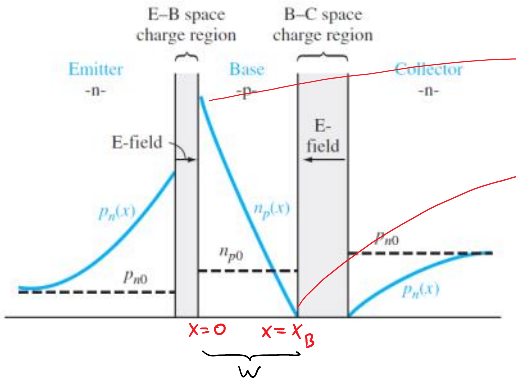


→ We want the electrons from the emitter to go to the collector (through the base) without recombining

⇒ Small base width

→ Ideal minority carrier concentration: We want a high extraction of minority carriers at the B/C junction. V_{CB} is therefore reversed biased, bringing the minority carriers close to zero.

⇒ Since it is close to zero, the current pratically does not depend on V_{CB}



$$n_B(0) = n_{B0}\, e^{\frac{qV_{BE}}{kT}}$$

$$n_B(x_B) \approx 0$$

$$J_c = eD_n \frac{dn}{dx} \cong eD_n \frac{\overset{n_B(0)}{n_{B0}\, e^{\frac{qV_{BE}}{kT}}} - \overset{n_B(x_B)}{0}}{0 - x_B}$$

$$\Rightarrow \boxed{J_c = -\frac{qD_{nB}}{W} n_{B0}\left(e^{\frac{qV_{BE}}{kT}} - e^{\frac{qV_{BC}}{kT}}\right) \approx -\frac{qD_{nB}}{W} n_{B0}\left(e^{\frac{qV_{BE}}{kT}} - 1\right)}$$

for small values of V_{BC}, where $e^{\frac{qV_{BC}}{kT}} \ll 1 \Rightarrow \boxed{I_c = I_{sc}\, e^{\frac{qV_{BE}}{kT}}}$

$$\boxed{J_B = -\frac{qD_{pE}}{L_{pE}} p_{E0}\left(e^{\frac{qV_{BE}}{kT}} - 1\right)} \quad \text{and} \quad I_E = I_C + I_B$$

⚠ Multiply J with area to get current I → $\boxed{I = J \cdot A}$

1. Current Gain

→ $\beta := \frac{I_c}{I_B}$ for $V_{BC} = 0$

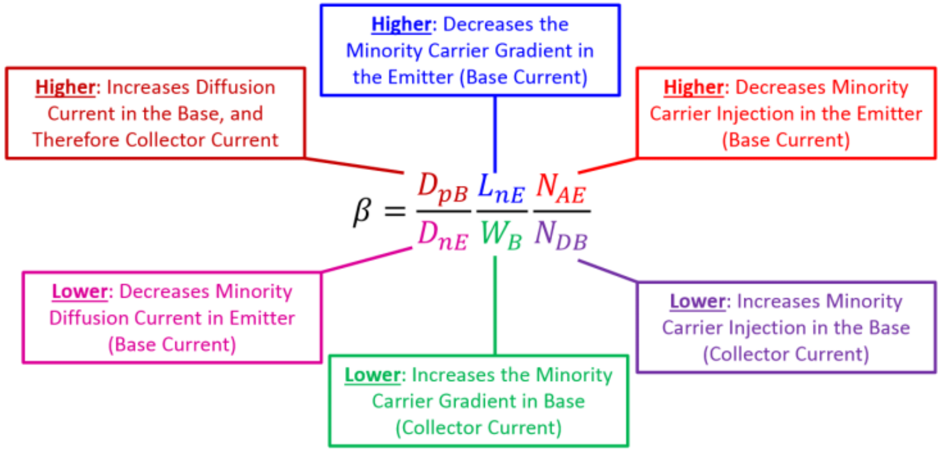
$$\Rightarrow \beta = \frac{I_c}{I_B} = \frac{-A_c \frac{qD_{nB}}{W} \overset{\frac{n_i^2}{N_{AB}}}{n_{B0}} \cancel{\left(e^{\frac{qV_{BE}}{kT}} - 1\right)}}{-A_E \frac{qD_{pE}}{L_{pE}} \underset{\frac{n_i^2}{N_{DE}}}{p_{E0}} \cancel{\left(e^{\frac{qV_{BE}}{kT}} - 1\right)}} = \frac{A_c}{A_E} \cdot \frac{D_{nB}}{D_{pE}} \cdot \frac{L_{pE}}{W} \cdot \frac{N_{DE}}{N_{AB}}$$

Normal:

$$\beta = \frac{I_{pC}}{I_{nE}} = \frac{I_{pE}}{I_{nE}} \quad \text{(pnp)}$$
$$= \frac{I_{nC}}{I_{pE}} = \frac{I_{nE}}{I_{pE}} \quad \text{(npn)}$$
$$\beta = \frac{D_{pB}}{D_{nE}} \frac{L_{nE}}{W_B} \frac{N_{AE}}{N_{DB}} \quad \text{(pnp)}$$
$$= \frac{D_{nB}}{D_{pE}} \frac{L_{pE}}{W_B} \frac{N_{DE}}{N_{AB}} \quad \text{(npn)}$$

Reverse:

$$\beta_{\text{reverse}} = \frac{D_{pB}}{D_{nC}} \frac{L_{nC}}{W_B} \frac{N_{DC}}{N_{AB}} \cdot \frac{A_E}{A_C} \quad \text{(pnp)}$$
$$= \frac{D_{nB}}{D_{pC}} \frac{L_{pC}}{W_B} \frac{N_{DC}}{N_{AB}} \cdot \frac{A_E}{A_C} \quad \text{(npn)}$$



Higher: Decreases the Minority Carrier Gradient in the Emitter (Base Current)

Higher: Increases Diffusion Current in the Base, and Therefore Collector Current

Higher: Decreases Minority Carrier Injection in the Emitter (Base Current)

$$\beta = \frac{D_{pB}}{D_{nE}} \frac{L_{nE}}{W_B} \frac{N_{AE}}{N_{DB}}$$

Lower: Decreases Minority Diffusion Current in Emitter (Base Current)

Lower: Increases Minority Carrier Injection in the Base (Collector Current)

Lower: Increases the Minority Carrier Gradient in Base (Collector Current)

→ Common Emitter Current Gain

$$\boxed{\beta = \frac{I_c}{I_B} = \frac{\alpha}{1-\alpha} \approx \frac{1}{\frac{N_B D_E}{N_E D_B} \frac{x_B}{x_E} + \frac{1}{2}\left(\frac{x_B}{L_B}\right)^2 + \frac{J_{ro}}{J_{so}} \exp\left(\frac{-eV_{BE}}{2kT}\right)}}$$

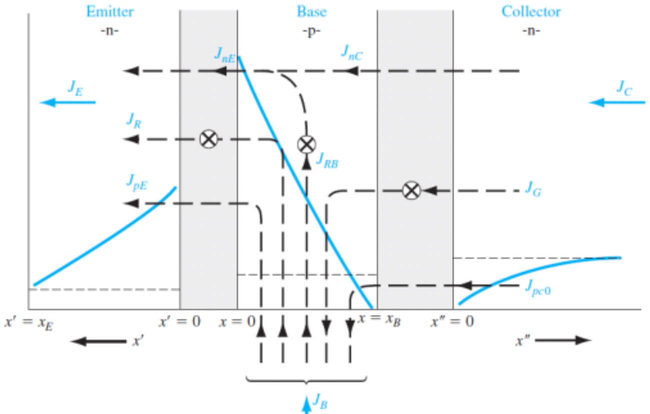
→ Common Base Current Gain

$$\boxed{\alpha = \frac{I_c}{I_E} = \frac{\beta}{1+\beta} = \gamma \alpha_T \delta \approx \frac{1}{1 + \frac{N_B D_E}{N_E D_B} \frac{x_B}{x_E} + \frac{1}{2}\left(\frac{x_B}{L_B}\right)^2 + \frac{J_{ro}}{J_{so}} \exp\left(\frac{-eV_{BE}}{2kT}\right)}}$$

→ Emitter Injection Efficiency (npn)

$$\boxed{\gamma = \frac{I_{nE}}{I_{nE} + I_{pE}} \approx \frac{1}{1 + \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}}}$$

→ Base Transport Factor (npn) $\boxed{\alpha_T = \frac{I_{nc}}{I_{nE}} \approx \frac{1}{\cosh\left(x_B/L_B\right)} \approx \frac{1}{1 + \frac{1}{2}\left(\frac{x_B}{L_B}\right)^2}}$



J_{nE}: Due to the diffusion of minority carrier electrons in the base at $x = 0$.

J_{nC}: Due to the diffusion of minority carrier electrons in the base at $x = x_B$.

J_{RB}: The difference between J_{nE} and J_{nC}, which is due to the recombination of excess minority carrier electrons with majority carrier holes in the base. The J_{RB} current is the flow of holes into the base to replace the holes lost by recombination.

J_{pE}: Due to the diffusion of minority carrier holes in the emitter at $x' = 0$.

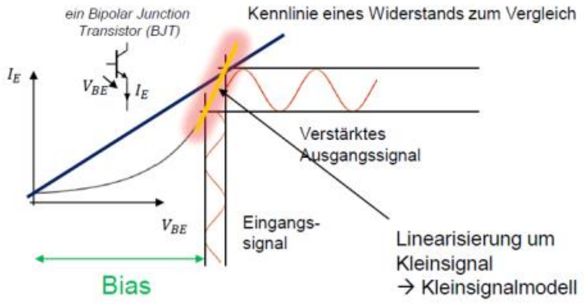
J_R: Due to the recombination of carriers in the forward-biased B–E junction.

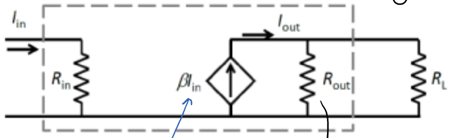
J_{pc0}: Due to the diffusion of minority carrier holes in the collector at $x'' = 0$.

J_G: Due to the generation of carriers in the reverse-biased B–C junction.

2. Small Signal Equivalent

→ AC-Signals ⇒ linearization of the output of small AC signals





$I_c = \beta I_B$

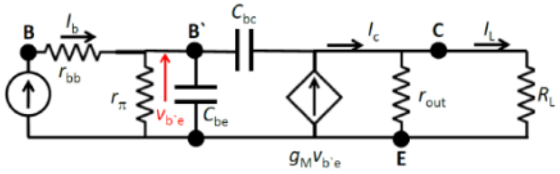
Early effect (ideally $R_{out} \to \infty$)

$$\boxed{\beta I_B = g_m V_{BE}} \rightarrow g_m = \frac{dI_c}{dV_{BE}}$$

$$\Rightarrow \boxed{g_m = \frac{dI_c}{dV_{BE}} = \frac{I_c}{V_t}} \rightarrow V_t = \frac{kT}{q} \qquad \boxed{r_\pi = \frac{1}{g_m}}$$

→ Power Gain ($R_{out} \to \infty$) ⇒ $\boxed{G = \frac{P_{out}}{P_{in}} = \frac{I_{out}^2 R_L}{I_{in}^2 R_{in}} = \beta^2 \frac{R_L}{R_{in}}}$

→ Frequency Response:



$$Z_\pi = r_\pi \parallel \underbrace{(C_{be} + C_{bc})}_{C_\pi}$$

$$I_c = g_m \cdot V_{be} \, , \quad I_b = \frac{V_{be}}{Z_\pi}$$

→ Common Emitter Current Gain: $\boxed{\beta(\omega) = g_m Z_\pi = \frac{r_\pi g_m}{1 + j\omega r_\pi C_\pi}}$

↓ Transistor is a low-pass filter!

→ Cut-off frequency f_{T_0}:

$$\boxed{f_{T_0} = \beta_0 \frac{1}{2\pi r_\pi C_\pi} = \frac{1}{2\pi} \frac{g_m}{C_\pi}} \rightarrow \tau_0 = \frac{C_\pi}{g_m}$$

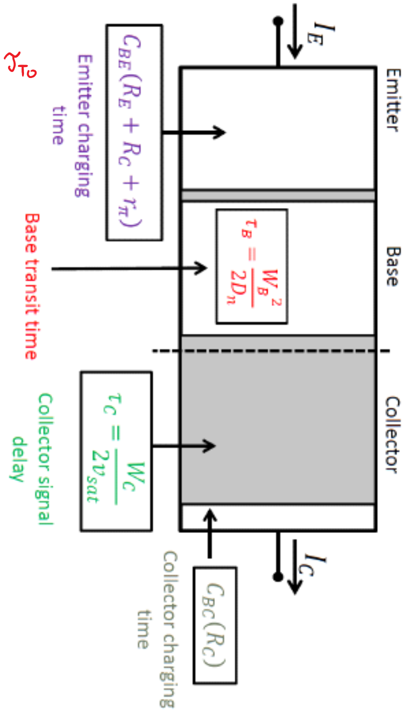
i. Fundamental transistor Time $\boxed{\frac{C_\pi}{g_m}}$ τ_{T_0}

ii. Base Transit Time $\boxed{\frac{W_B^2}{2D_n}}$ τ_B

iii. Collector Signal Delay $\boxed{\frac{W_c}{2V_{sat}}}$ τ_c

iv. Emitter Charging Time $\boxed{C_{BE}(R_E + R_c + r_\pi)}$

v. Collector Charging Time $\boxed{C_{BC}(R_c)}$



$$f_T = \frac{1}{2\pi \tau_T} \, , \quad \tau_T = \tau_{T_0} + \tau_B + \cdots$$

$$\boxed{f_T = \alpha_0 f_{\alpha\tau} = \frac{1}{2\pi \tau_T}}$$

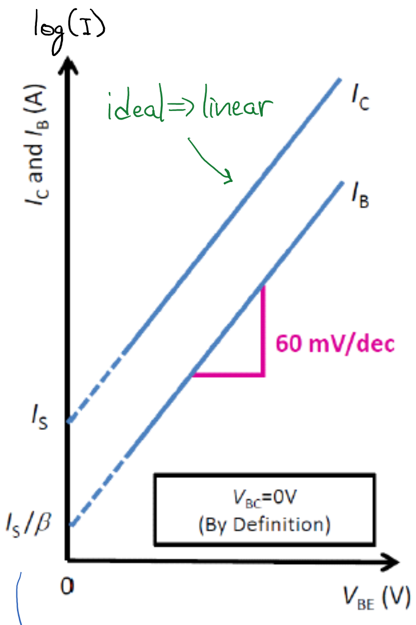
α_0, β_0 = Current gain (without the influence of frequency → capacitors)

$$\boxed{\tau_B = \frac{Q_B}{J_c}} \qquad \boxed{f_\alpha = \frac{f_T}{\alpha_0}} \qquad \boxed{f_\beta = f_\alpha (1 - \alpha_0)}$$

→ Gummel-plot: semi-logarithmic plot V_{BE} vs I_C, I_B

$$I_C = A_E \frac{qD_{pB}}{W_B}\frac{n_i^2}{N_{DB}}\left(e^{qV_{BE}/kT}-1\right) = I_s\left(e^{qV_{BE}/kT}-1\right) \approx I_s e^{qV_{BE}/kT}$$

$$I_B = A_E \frac{qD_{nE}}{L_nE}\frac{n_i^2}{N_{AE}}\left(e^{qV_{BE}/kT}-1\right) = \frac{I_s}{\beta}\left(e^{qV_{BE}/kT}-1\right) \approx \frac{I_s}{\beta} e^{qV_{BE}/kT}$$



actually $\log_{10}(I_s/\beta)$

$$\log_{10}(I_c) = \frac{\log(I_c)}{\log(10)} = \frac{\log(I_s) + \log\left(e^{qV_{BE}/kT}\right)}{\log(10)}$$

$$\Rightarrow \frac{d}{dV_{BE}}\log(I_c) = \frac{q}{kT\log(10)} V_{BE}$$

$$\frac{\Delta y}{\Delta x} = \frac{q}{kT\log(10)} \longrightarrow \text{dec} \Rightarrow \Delta y = 1 \quad (\log_{10}(10) = 1)$$

$$\Rightarrow \Delta x = \frac{kT\log(10)}{q} \approx 60\,mV$$

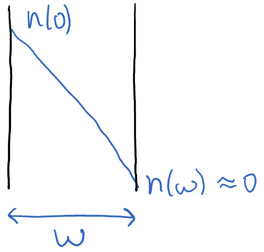
Quality factor η → $I_c = I_s e^{qV_{BE}/\eta kT}$

(I guess)

$$\boxed{\text{new slope would be } \frac{\eta kT\log(10)}{q}\ mV/dec}$$

Questions

- Why is $J_B = \frac{W_B^2}{2D_n}$?



$$J_c = qD_n\frac{dn}{dx} = qD_n\frac{n(w)-n(0)}{w}$$

$$\approx -qD_n\frac{n(0)}{w} \quad \text{(negative sign egal)}$$

$$Q_c = q\frac{1}{2}w\left(n(0)-n(w)\right) \approx q\frac{w\,n(0)}{2}$$

$$J_B := \frac{Q_c}{J_c} = \frac{\cancel{q}\frac{w\,\cancel{n(0)}}{2}}{\frac{\cancel{q}D_n\cancel{n(0)}}{w}} = \boxed{\frac{w^2}{2D_n}}$$

- Why is $N_E > N_B > N_C$?

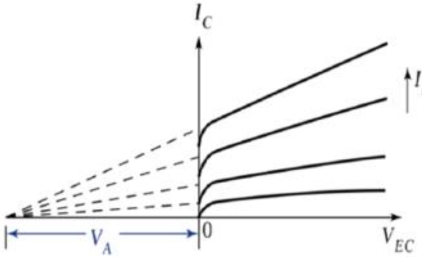
$N_E > N_B \Rightarrow$ big β, $N_B > N_C \Rightarrow$ reduction of Early-Effect

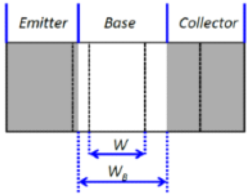
3. Early Effect

→ The increase of V_{CE} increases the depletion region (B/C), which decreases the base width

smaller base width W ⇒ higher $\frac{dn}{dx}$ gradient ⇒ higher I_c

we expect higher currents for increasing V_{CE} ↵





$$\boxed{\frac{\partial I_c}{\partial V_{EC}} = \frac{I_c}{V_A + V_{EC}} =: \frac{1}{R_{out}}}$$

or r_o ↵

$$\boxed{V_A + V_{CE} = \frac{q N_{DB} W_B}{C_{BC}}}$$

↑ Base doping ⇒ ↑ V_A

4. Base Drift Field

→ A non constant doping level in the base region creates an E-Field. Now we don't have only diffusion, but also drift (which can help or not the current flow).

$$\text{total } J \rightarrow J_n = qD_n\frac{dn}{dx} + qn\mu_n E \longrightarrow \frac{dn}{dx} + \frac{nE}{kT/q} - \frac{J_n}{qD_n} = 0$$

$$\boxed{n(x) = -\frac{J_n W_B}{qD_n} \cdot \frac{1-\exp(-\eta(1-x/W_B))}{\eta}} \rightarrow E \equiv \frac{kT/q}{x_0}, \quad \eta = \frac{W_B}{x_0}$$

$$\boxed{J_{n,diff} = J_n \exp\left[-\eta\left(1-\frac{x}{W_B}\right)\right]} \qquad \boxed{J_{n,drift} = J_n\left\{1-\exp\left[-\eta\left(1-\frac{x}{W_B}\right)\right]\right\}}$$

$$\boxed{\tau_B = \frac{W_B^2}{D_n}\left(\frac{\eta - 1 + e^{-\eta}}{\eta^2}\right) = \frac{W_B^2}{D_n}\left(\frac{\eta-1}{\eta^2}\right), \quad \eta > 3}$$

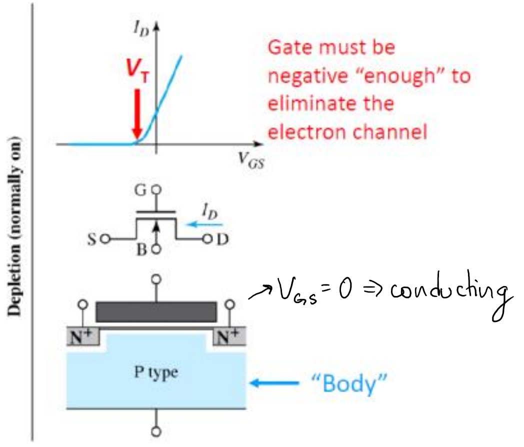
⑧ Metal-oxide-semiconductor field-effect transistor (MOSFET)

→ Surface-controlled majority carrier devices (BJTs are minority carrier devices)

→ Two types of MOSFETs:

i. Channel is present in equilibrium:
Normally-ON, Depletion-mode device

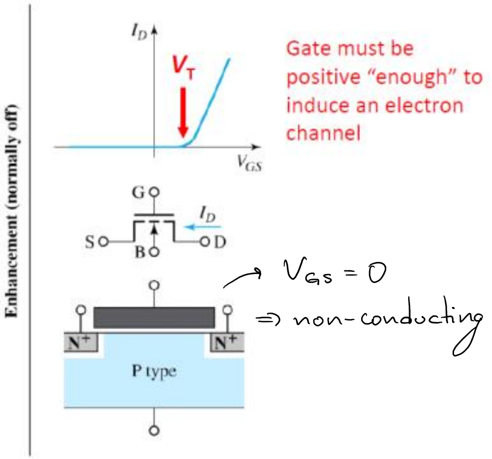
ii. no channel present in equilibrium:
Normally-OFF, Enhancement-mode device



Gate must be negative "enough" to eliminate the electron channel

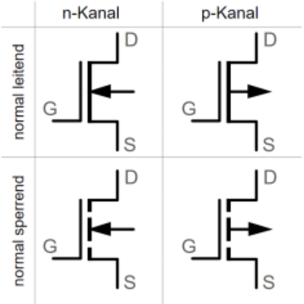
↗ $V_{GS} = 0$ ⇒ conducting

← "Body"



Gate must be positive "enough" to induce an electron channel

↗ $V_{GS} = 0$
⇒ non-conducting



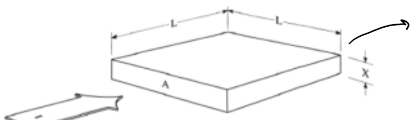
depletion ←

Enhancement ←

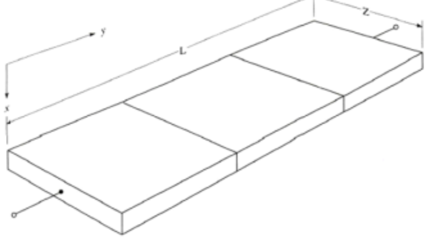
1. Sheet resistance

→ $R = \rho\frac{L}{A} = \rho\frac{L}{X \cdot L} = \rho\frac{1}{X} =: R_s$ ← Sheet resistance $\Omega/\square$

↑ per sheet



$\rightarrow R_s = \rho \frac{1}{X}$



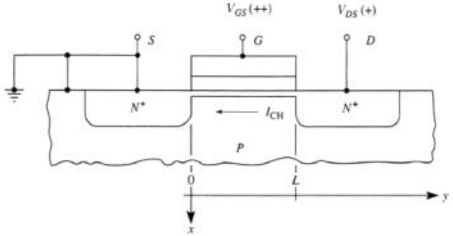
$\rightarrow R_{tot} = 3 \cdot R_s$

→ Charge in the channel per unit area → $$[n] = \left[\frac{1}{V}\right] = \left[\frac{1}{A \cdot X}\right]$$ (charge density ↓)

$$Q_n = -qnX = -q \cdot n \cdot \frac{\rho}{R_s} \longrightarrow \rho = \frac{1}{q \cdot n \cdot \mu_n}$$

↳ $[Q_n] = C/A$

$$\Rightarrow n \cdot x = \frac{1}{A} \text{ („per unit area")}$$



$$\Rightarrow Q_n = -q \cdot n \cdot \frac{1}{q \cdot n \cdot \mu_n \cdot R_s} = -\frac{1}{\mu_n R_s}$$

→ Now assuming we have a capacitance between gate and the oxide

$$Q_n = -C_{ox}(V_{GS} - V_T) \longrightarrow R_s = \frac{1}{\mu_n \cdot C_{ox}(V_{GS} - V_T)}$$

Assuming we can change V_{DS}, the resistance will change along the y-axis. $V(y)$ will vary from 0 at the source to V_{DS} at the drain

$$\boxed{R_s(y) = \frac{1}{\mu_n C_{ox}(V_{GS} - V_T - V(y))}}$$

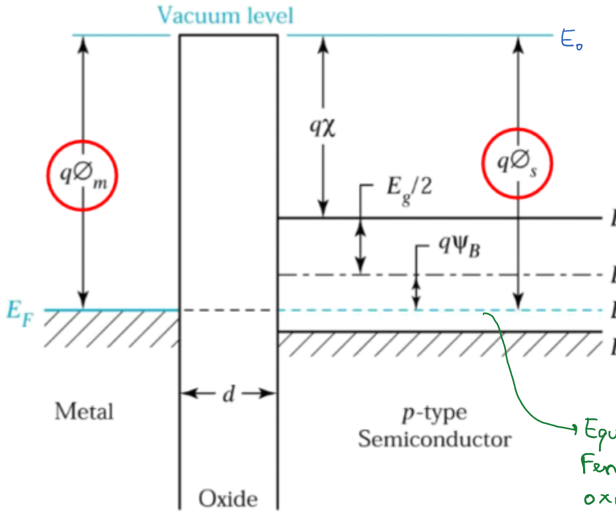
$$dR = \frac{dy}{Z} R_s(y) = \frac{dy}{Z \cdot \mu_n C_{ox}(V_{GS} - V_T - V(y))}$$ where $\frac{dy}{Z}$ = # squares (sheets)

↳ „Resistance of channel element of length dy and width z at position y"

→ Current $I_{CH} = \frac{dV}{dR}$

$$I_{CH} = \frac{Z \mu_n C_{ox}(V_{GS} - V_T - V(y))\, dV}{dy} \Longleftrightarrow I_{CH} \int_0^L dy = Z \mu_n C_{ox} \int_0^{V_{DS}} (V_{GS} - V_T - V(y))\, dV$$

$$\Rightarrow \boxed{I_{CH} = \frac{\mu_n C_{ox}}{2} \frac{Z}{L} \left[2(V_{GS} - V_T) V_{DS} - V_{DS}^2\right] = I_D}$$



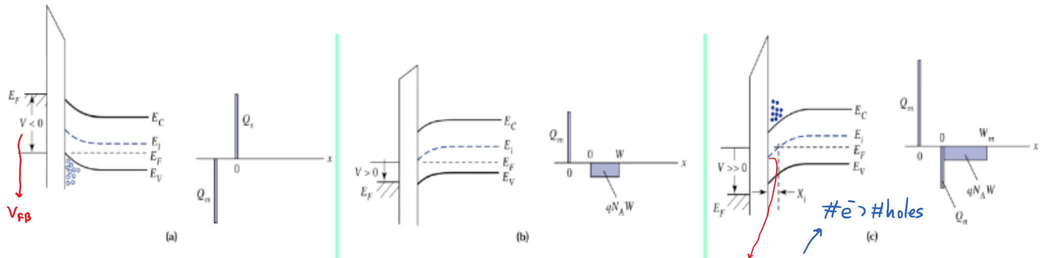
E_F below E_i (p-type)

Equilibrium ⇒ Flat (constant) Fermi-level (through metal, oxide and semiconductor)

- Vacuum level E_0: Energy outside the material [eV]
- Work function $\varnothing$: Energy difference between Fermi-level and vacuum-level [V]
- Electron affinity χ: Energy difference between conduction band and vacuum-level [V]
- Bulk potential Ψ_B: Energy difference between Fermi-level and intrinsic Fermi-level [V]
- Surface potential Ψ_s: Energy difference between E_i of Bulk and oxide [V]

$$\begin{tikzcd} \text{Voltage [V]} \arrow[r, bend left, "\times q"] & \text{Energy [eV]} \arrow[l, bend left, "\times 1/q"] \end{tikzcd}$$

$$\boxed{\Psi_B = \frac{kT}{q} \ln\left(\frac{N_A}{n_i}\right)}$$



E_F below E_i ⇒ „transformation of a p-type into a n-type semiconductor!"

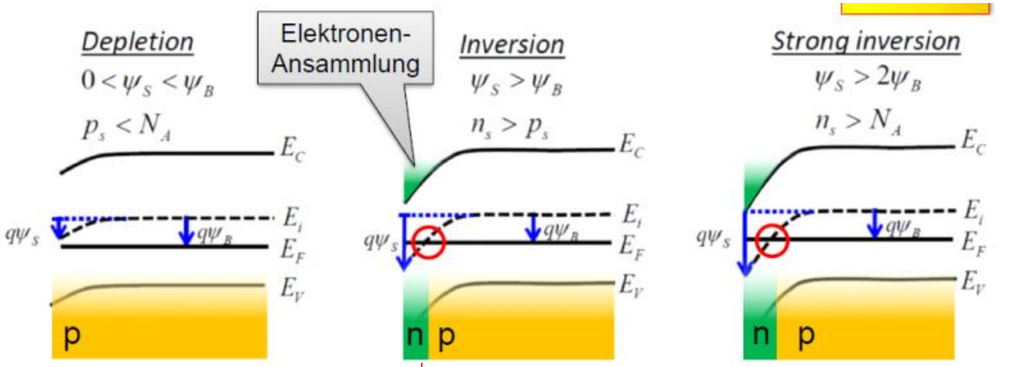
Accumulation (p-Substrate)

Depletion (Negative Space Charge)

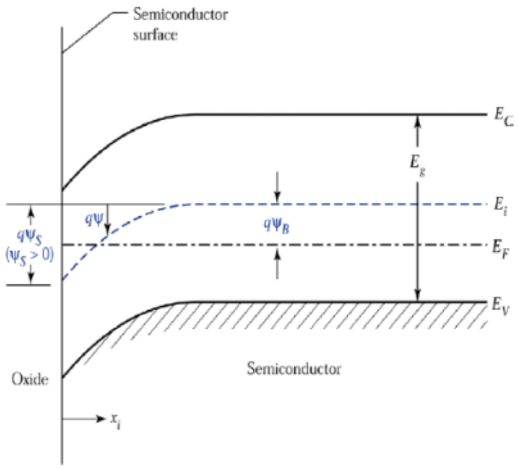
Inversion (Electron Channel in p-Substrate)

V_{FB} = Energy difference between Fermi-levels (oxide and semiconductor)

$V_{FB} = \varnothing_m - \varnothing_s$



„transformation into n-type" (more e⁻ than holes)



$$\boxed{p_p = n_i e^{\frac{E_i - E_F}{kT}}} \quad \boxed{n_p = n_i e^{\frac{E_F - E_i}{kT}}}$$

$$\boxed{p_p = n_i e^{\frac{\Psi_B - \Psi}{kT}}} \quad \boxed{n_p = n_i e^{\frac{\Psi - \Psi_B}{kT}}}$$

$$\boxed{\begin{aligned} p_p(x) &= n_i \exp\left(\frac{q(\Psi_B - \Psi(x))}{kT}\right) \\ n_p(x) &= n_i \exp\left(\frac{q(\Psi(x) - \Psi_B)}{kT}\right) \end{aligned}}$$

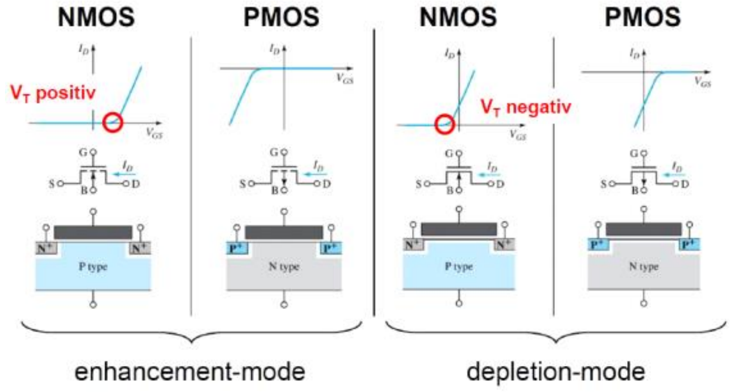
→ n(0) and p(0) are called surface carrier densities

→ „Inversion" ⇒ $\Psi_s = \Psi_B \rightsquigarrow \boxed{\Psi_{s,inv} = \frac{kT}{q} \ln\left(\frac{N_A}{n_i}\right)}$

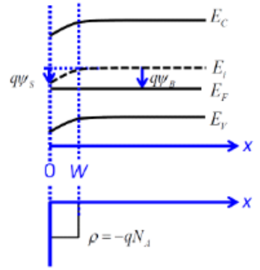
→ Strong inversion ⇒ $\Psi_s = 2\Psi_B \rightsquigarrow \boxed{\Psi_{s,inv} = \frac{2kT}{q} \ln\left(\frac{N_A}{n_i}\right)}$

↳ when $n_s = N_A$

→ We define the real inversion where $n_s = N_A \Rightarrow \Psi_s = 2\Psi_B$



V_T also defines where current „starts" flowing



2. Depletion width

→ Charge density $\rho = q(p - n + N_D - N_A) \approx -qN_A$

→ E-Field: $E(x) = \frac{qN_A}{\varepsilon_r \varepsilon_0}(W - x)$

→ Electric potential: $\Psi(x) = \frac{qN_A}{2\varepsilon_r \varepsilon_0}(W - x)^2$ for $0 \leq x \leq W$

$$\boxed{E(0) = E_s = \frac{qN_A}{\varepsilon_s} W} \longrightarrow E_s \qquad \boxed{E_s \varepsilon_s = E_{ox} \varepsilon_{ox}}$$

$$\boxed{\Psi_s = \frac{qN_A}{2\varepsilon_r \varepsilon_0} W^2}$$

→ Potential on the surface (x = 0)

→ $\varepsilon_s = \varepsilon_r \varepsilon_0$

$$\boxed{W = \sqrt{\frac{2\varepsilon_r \varepsilon_0 \Psi_s(V_B)}{qN_A}}}$$

⇝ $\Psi_s = E_{i,Bulk} - E_i(0)$

⇝ $\Psi_B = E_{i,Bulk} - E_F$

$$\varepsilon_s = \varepsilon_r \varepsilon_0$$

$$W_{max} = \sqrt{\frac{2\varepsilon_r \varepsilon_0 \Psi_{s,inv}}{qN_A}} \Rightarrow \boxed{W_{max} = 2\sqrt{\frac{\varepsilon_r \varepsilon_0 \Psi_B}{qN_A}} = 2\sqrt{\frac{\varepsilon_r \varepsilon_0 kT \ln(N_A/n_i)}{q^2 N_A}}}$$

↳ $\Psi_{s,inv} = 2\Psi_B$

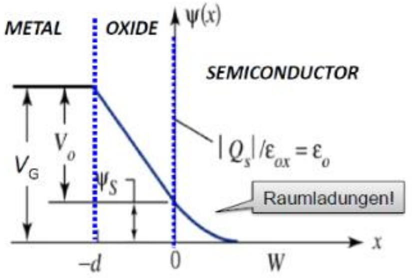
3. Gate Potential

→ $V_G = V_{ox} + \Psi_s$ (ideal)

→ No free particles inside oxide ⇒ constant E-field equal to E-field at the border (of the semiconductor → E(0))

$$\boxed{V_{ox} = d \cdot E_{ox} = d \cdot E(0) = d \cdot \frac{qN_A}{\varepsilon_{ox}} W}$$

$$\boxed{V_{ox} = \frac{\sqrt{2q\varepsilon_s N_A \Psi_s}}{C_{ox}}} \rightarrow C_{ox} = \frac{\varepsilon_{ox}}{d_{ox}} \quad [F \cdot cm^{-2}]$$

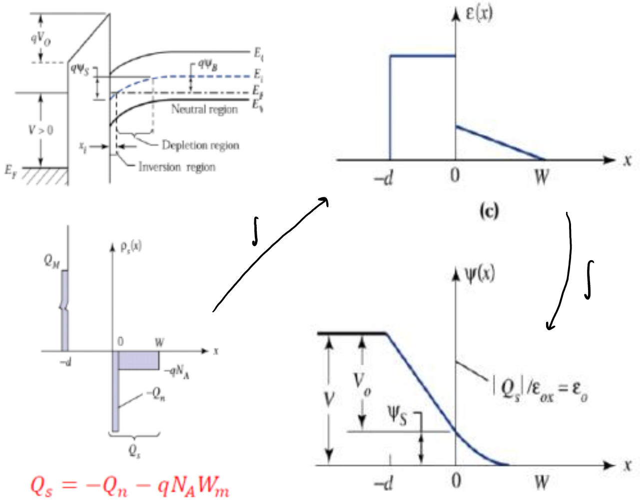


$$\boxed{V_G = V_{ox} + \Psi_s = \frac{\sqrt{2q\varepsilon_s N_A \Psi_s}}{C_{ox}} + \Psi_s + V_{FB}}$$

$$\boxed{V_T = \frac{\sqrt{4q\varepsilon_s N_A \Psi_B}}{C_{ox}} + 2\Psi_B} \rightarrow V_T = \text{threshold voltage}$$

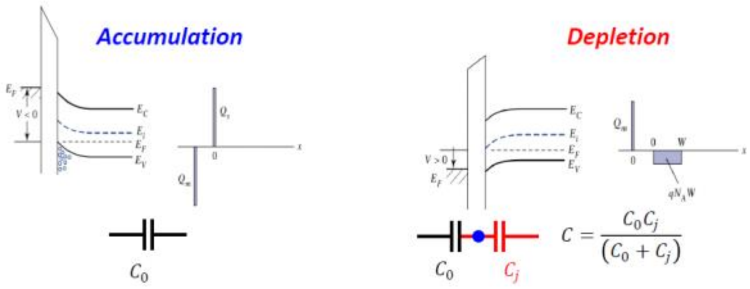
$$V_{FB} = \varnothing_{ms} - \varnothing_s = \varnothing_{ms} - \frac{Q_F}{C_{ox}}$$ → Free charge in the oxide

↳ Flat band voltage takes into consideration non-idealities ($\varnothing_{ms} \neq 0$ and free particles in the oxide)



$Q_s = -Q_n - qN_A W_m$

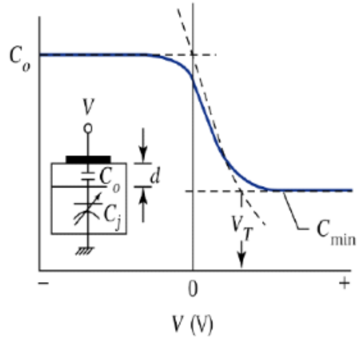
4. MOS Capacitance



$C = \frac{C_0 C_j}{(C_0 + C_j)}$

$$\boxed{\frac{C}{C_{ox}} = \frac{1}{\sqrt{1 + \frac{2\varepsilon_{ox}^2 V_G}{qN_A\varepsilon_s d^2}}}}$$

↓ Normalized to Oxide (ε_{ox}/d)



$$\boxed{C_{min} = \frac{\varepsilon_{ox}}{d + \frac{\varepsilon_{ox}}{\varepsilon_s} W_{max}}}$$

This only for p-type

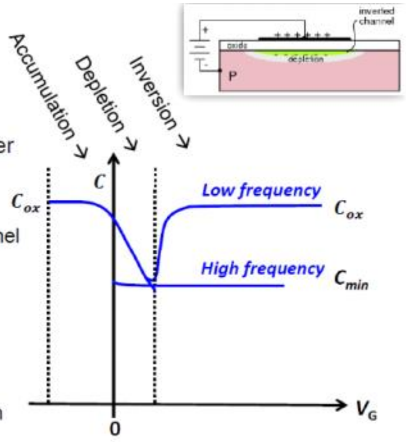
$$\boxed{C_{j,min} = \frac{\varepsilon_s}{W_{max}}}$$

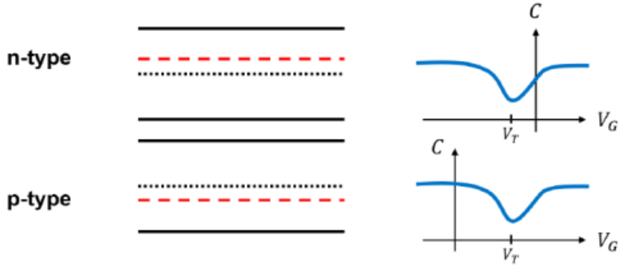
$C_0 = C_{ox}$

Betrachtete Gatespannung: $v_G(t) = \underbrace{V_G}_{\text{DC}} + \underbrace{v_0 \sin\omega t}_{\text{AC}}$

- **Accumulation**: Unmengen an Ladungsträgern im Channel, keine depletion region → nur Oxidkapazität
- **Depletion**: Eine Raumladungszone ohne freie Ladungsträger bildet sich ($C_j > 0$). Dadurch sinkt die Kapazität ab
- **Inversion**: Die depletion region wird nicht mehr grösser

  Bei ändernder Spannung muss sich die Elektronendichte im Channel ändern. Dies geschieht durch Generation und Diffusion. Beide Vorgänge haben eine fixe Geschwindigkeit.
  - **Tiefe Frequenzen**: Es können genügend Elektronen erzeugt werden, um der Gate-Spannung zu folgen. Daher hat die Kapazität der depletion region keinen Einfluss mehr
  - **Hohe Frequenzen**: Elektronenkonzentration im Channel kann sich kaum ändern, daher wirkt die Kapazität der depletion region noch.

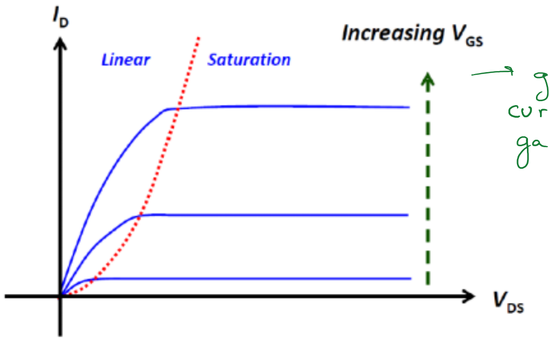




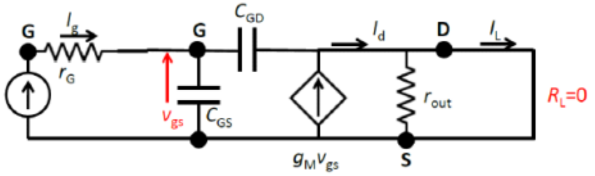
5. Transconductance

→ In saturation region:

$$\boxed{g_m = \frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox}\frac{Z}{L}(V_{GS}-V_T)}$$



→ g_m tells us how fast the saturation-current changes with a change on the gate voltage (V_{GS})

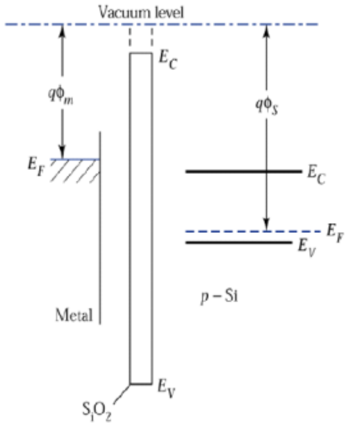


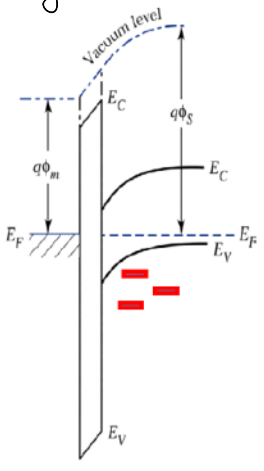
$$\boxed{A(\omega) = \frac{I_D}{I_G} = \frac{g_m}{j\omega(C_{GS}+C_{GD})}}$$

→ Cut-off frequency ($A(\omega)=1$):

$$\boxed{f_T = \frac{3\mu_n(V_{GS}-V_T)}{4\pi L^2}} \qquad \boxed{f_T = \frac{g_m}{2\pi(C_{GS}+C_{GD})}}$$

→ BJTs are better if we want high values of $A(\omega)$





$$\boxed{\begin{aligned} p_p(x) &= n_i \exp\left(\frac{q(\Psi_B-\Psi(x))}{kT}\right) \\ n_p(x) &= n_i \exp\left(\frac{q(\Psi(x)-\Psi_B)}{kT}\right)\end{aligned}}$$

→ Set $\Psi(x=0) = \Psi_s$ to get concentrations at the oxide-semiconductor border

$$\boxed{\Psi_B = \frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right)}$$

→ for a p-type semiconductor

→ Energy difference between E_i and E_F (doping ≠0)

$$\boxed{E_s = \frac{qN_A}{\varepsilon_s}W}$$ → E-field at $x=0$

$$\boxed{E_{ox} = \frac{qN_A}{\varepsilon_{ox}}W}$$

$$E_{ox} = \frac{\varepsilon_s}{\varepsilon_{ox}}E_s$$

Conversion s ↔ ox

$$\boxed{E_s\varepsilon_s = E_{ox}\varepsilon_{ox}}$$

$$\boxed{\Psi_s = \frac{qN_A}{2\varepsilon_s}W^2}$$ → Potential at $x=0$

E-field inside oxide is constant (can be calculated from $E(0)$)

$$\Rightarrow \boxed{V_{ox} = d\cdot E_{ox} = d\,\frac{qN_A}{\varepsilon_{ox}}W = \frac{\sqrt{2\varepsilon_s q N_A \Psi_s}}{C_{ox}}} \xrightarrow{\text{using}} \boxed{C_{ox} = \frac{\varepsilon_{ox}}{d}}$$

$$\boxed{W = \sqrt{\frac{2\varepsilon_s\Psi_s}{qN_A}}} \qquad \boxed{W_f = \sqrt{\frac{2\varepsilon_s\Psi_B}{qN_A}}} \qquad \boxed{W_{max} = 2\sqrt{\frac{\varepsilon_s\Psi_B}{qN_A}}}$$

flat-band (no band-bending) ⇒ intrinsic semiconductor ($\Psi_s = \Psi_B$)

inversion ($\Psi_s = 2\Psi_B$)

$$\boxed{V_G = V_{ox} + \Psi_s}$$

→ $\Psi_s = \Psi_B \Rightarrow$ flat-band

→ $\Psi_s = 2\Psi_B \Rightarrow$ inversion ($n_s = N_A$)

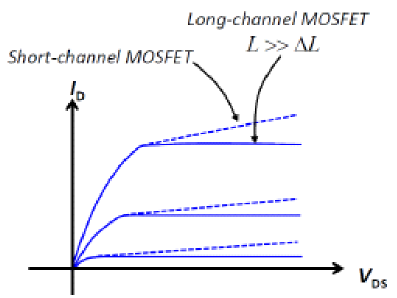
- V_T = threshold gate voltage $(\Psi_s = 2\Psi_B) = V_{ox}(\Psi_s = 2\Psi_B) + 2\Psi_B$

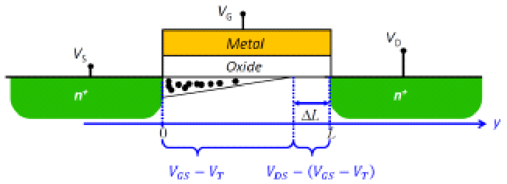
4.4 Nichtidealitäten im MOSFET
Channel Length Modulation (CLM)

- In der Herleitung des Saturation Currents sind wir davon ausgegangen, dass der pinch-off-Bereich ΔL sehr klein ist verglichen mit der gesamten Channel-Länge
- Bei kurzem Channel stimmt dies nicht mehr, und da $I_D \propto \frac{Z}{L}$ steigt der Strom, wenn der **Channel kürzer wird**, also

$$I_D = \mu_n C_{ox} \frac{Z}{L - \Delta L} \frac{(V_{GS} - V_T)^2}{2}$$

- Dies ist das Analogon zum Early-Effekt in einem BJT und führt zu einem Ausgangswiderstand





4.4 Nichtidealitäten im MOSFET
Short Channel Effects: Threshold Voltage Shift

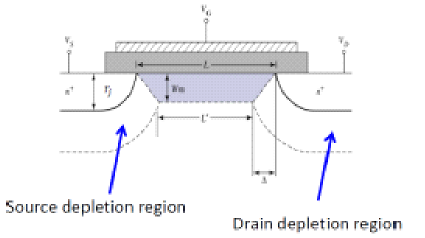
PRÜFUNGSTIPP!
Vorallem Konzept wichtig…

Charge Sharing:

- Zwischen dem Bulk und den Kontakten für Source und Drain bilden sich jeweils pn-junctions aus
- Wenn der Channel sehr kurz ist, «fressen» diese **pn-junction depletion regions** einen signifikanten Teil des Channels weg, wodurch er noch kürzer wird
- Dadurch sinkt die benötigte Gate-Spannung um Inversion zu erreichen, also die Threshold-Spannung V_T

$$\Delta V_T = -\frac{q N_A W_m r_j}{C_0 L}\left(\sqrt{1 + \frac{2W_m}{r_j}} - 1\right)$$

Ein **kürzerer Channel** hat viele Vorteile: Man erhöht die Transconductance g_m sowie die Geschwindigkeit und reduziert die Baugrösse. Es treten allerdings mehr Nichtidealitäten, so genannte Short Channel Effects, auf.



4.4 Nichtidealitäten im MOSFET
Short Channel Effects: Threshold Voltage Shift

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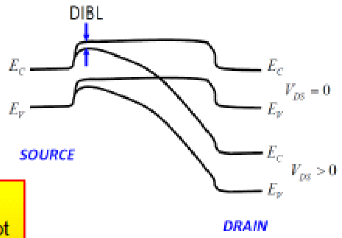
Drain-induced barrier lowering (DIBL):

- Bei grosser Drain/Source-Spannung dehnt sich die Drain-Bulk pn-junction depletion region sehr weit aus und beginnt, mit derjenigen der Source zu interargieren
- Das heisst, der Channel wird praktisch ganz von den depletion regions um die Source und die Drain aufgefressen
- Die Energiehürde für ein Elektron, um über den Channel zu springen, sinkt plötzlich
- Daher sinkt die Threshold-Spannung ab.

Ein **kürzerer Channel** hat viele Vorteile: Man erhöht die Transconductance g_m sowie die Geschwindigkeit und reduziert die Baugrösse. Es treten allerdings mehr Nichtidealitäten, so genannte Short Channel Effects, auf.

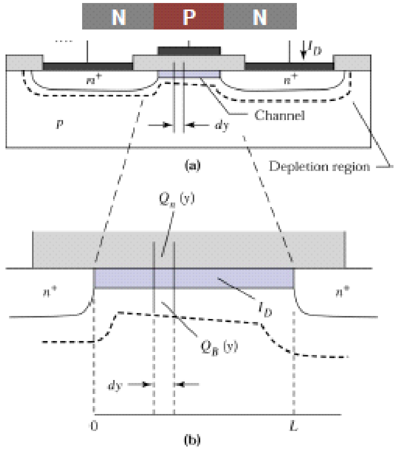


PRÜFUNGSTIPP!
Vorallem Konzept wichtig…

4.4 Nichtidealitäten im MOSFET
Subthreshold Swing

- Kurz bevor wir den Channel mit inversion erzeugen, haben wir im Grunde zwischen Drain und Source eine NPN (oder PNP) Struktur wie in einem BJT. Durch leichte Variation der Gate-Spannung verhält sich der MOSFET in diesem Bereich ähnlich wie ein BJT (**subthreshold régime**)
- Der Subthreshold Swing gibt an, wie effizient der MOSFET in diesem Bereich ist, i.e. wie effizient er ein- und ausgeschaltet werden kann

$$S = \frac{1}{\frac{\partial \log_{10} I_D}{\partial V_G}} \approx \frac{\Delta V_G}{\log_{10} I_D|_{V_G = V_T} - \log_{10} I_D|_{V_G = 0}}$$



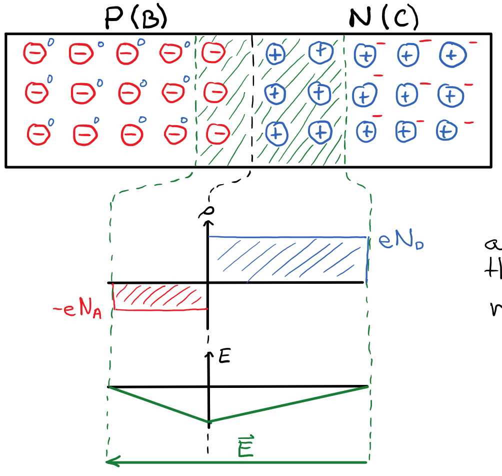
4.4 Nichtidealitäten im MOSFET
Gate Oxide Scaling

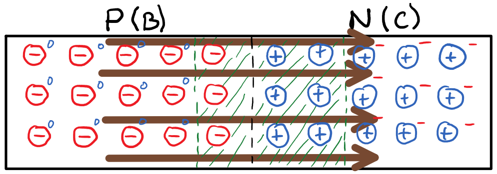
- Wir wollen den MOSFET möglichst mit der Gate-Spannung kontrollieren
- Reduzieren wir die **Dicke des Oxids** (SiO_2), so erhöht sich die Oxid-Kapazität C_{ox}. Dies verbessert die Steuerbarkeit des MOSFET.
- Sehr dünne Oxidschichten können allerdings von Elektronen durchtunnelt werden
- Um **Tunneling** zu verhindern, muss dann ein Oxid mit höherem κ (Leitfähigkeit) verwendet werden
- Die **equivalent oxide thickness** wird verwendet, um verschiedene Oxide zu vergleichen

$$EOT = d \cdot \frac{\varepsilon_{SiO_2}}{\varepsilon_{\mathrm{high}\kappa}}$$

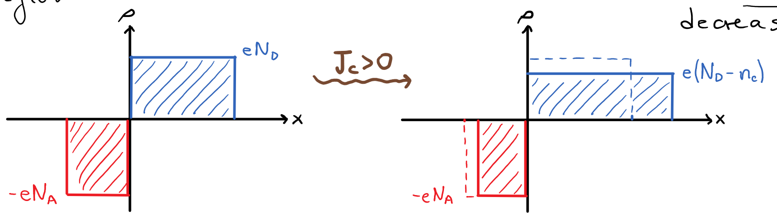
4. Kirk Effect

→ Ae gambada, aqui lhes explicarei os conceitos fundamentais por trás do Kirk Effect (é 13 memo porra)





- electrons go through the junction and, since ($N_{DC} < N_{AB}$), kind of neutralize the positive charge in the B/C depletion region



The drift current in the depletion region is given by

$$J_c = q n_c \mu_n E = q n_c v_c \Rightarrow n_c = \frac{J_c}{q v_c}$$

where n_c is the moving electron density (the extra electron density moving through the depletion region because of the current J_c)

The effective electron space charge will be decreased

→ Effects of $n_c > 0$:

1. Lower effective space charge → $e(N_{DC} - n_c)$
2. Higher depletion width ⇒ lower E-field

→ On $N_{DC} = n_c$ we have a total neutralization of the positive charge on Collector side ⇒ no depletion region

→ On $n_c > N_{DC}$ we have an inversion, since we now have a negative space charge (E-field inversion)
↳ ⇒ effective base width increases

→ On inversion (where $N_D = N_A = 0$ on the border):

$$V_{CB} = \frac{q}{2\varepsilon_s}(n_c - N_{DC})\overbrace{(W_c - W_K)}^{=:W_{Csc}}{}^2$$

$$V_{cd} = \frac{q N_{DC}}{2\varepsilon_s} W_c^2$$

↗ Voltage to make the depletion region occupie the whole Collector

$$\boxed{W_K = W_c - W_{Csc} = \sqrt{\frac{2\varepsilon_s V_{cd}}{q N_{DC}}} - \sqrt{\frac{2\varepsilon_s V_{CB}}{q(n_c - N_{DC})}} = W_c\left[1 - \sqrt{\frac{V_{CB}/V_{cd}}{(n_c/N_{DC}) - 1}}\right]}$$

$$\boxed{n_{cK} = N_{DC}\left(1 + \frac{V_{CB}}{V_{cd}}\right)}$$

→ threshold n_c where we start having a base shift ($n_{cK} \Rightarrow W_K = 0$)

$$\boxed{\begin{aligned} J_K &:= q V_{sat} n_{cK} = q V_{sat} N_{DC}\left(1 + \frac{V_{CB}}{V_{cd}}\right) \\ J_K &= q V_{sat} N_{DC}\left(1 + \frac{2\varepsilon_s V_{CB}}{q N_{DC} W_c^2}\right) \end{aligned}}$$

- New transit time for $J_c > J_K$ is $\boxed{\tau_B = \frac{(W_B + W_K)^2}{2 D_n}}$

new base width $W_B' = W_B + W_K$