1. Using the same technique as we used to prove the Cauchy–Davenport Theorem and the Erdős–Heilbronn Conjecture, prove the following: if $A, B \subseteq \mathbb{Z}_p$, then

$$|\{a + b : a \in A, b \in B, ab \neq 1\}| \geq \min\{p, |A| + |B| - 3\}$$

   Additionally, prove that this bound is best possible for any choice of $|A|$ and $|B|$.

2. In class, we saw that there are two fundamentally different ways to cover all but one of the vertices of the hypercube $\{0, 1\}^n$ using exactly n hyperplanes: one method covers based on the number of coordinates equal to 1 in each vertex, and another method covers using hyperplanes orthogonal to the axes.

   Similarly, find two fundamentally different ways to cut all the edges of the hypercube using exactly n hyperplanes. Can you prove that n is the best possible?

3. Generalize the Erdős–Heilbronn Conjecture we proved in class, as follows. For any set $A \subseteq \mathbb{Z}_p$, and any $s \in \mathbb{N}$, let $s \odot A$ denote the set of all sums of s *distinct* elements of A. Prove that

$$|s \odot A| \geq \min\{p, s|A| - s^2 + 1\}$$