1. (a) Prove that every 2-coloring of $E(K_N)$ contains a monochromatic N-vertex tree.
   (b) Prove that for every $q \geqslant 2$, there exists some $\delta > 0$ such that the following holds. In any q-coloring of $E(K_N)$, one of the color classes contains *all* of the trees on δN vertices.

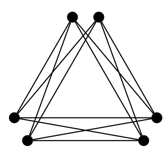
2. Prove that the q-color Ramsey number $r(2, 3, \ldots, q, q+1)$ satisfies the bounds

$$2^{cq^2} \leqslant r(2, 3, \ldots, q, q+1) \leqslant q^{Cq^2}$$

   for some absolute constants $c, C > 0$.

3. Prove that if N is sufficiently large, then the following holds. Among any N points in the plane, there are three of them that determine an angle greater than $179°$.

4. Let G be a graph. The *s-blowup* of G, denoted $G[s]$, is the graph obtained by replacing each vertex of G by s vertices, and replacing each edge of G by a complete bipartite graph $K_{s,s}$. For example, here is a picture of $K_3[2]$:



   (a) Prove that for every $s \geqslant 2$, there exists some $N = N(s)$ such that $K_6[N]$ is Ramsey for $K_3[s]$.
   (b) Prove that $N(s) > 2^s$ for all $s \geqslant 4$.

5. Let $k \geqslant 3$ and $N \geqslant 3k$ be integers. Recall that k-AP is short for k-term arithmetic progression.
   (a) Prove that there are at least $N^2/(6k)$ distinct k-APs in $[\![N]\!]$.
   (b) Let $A \subseteq [\![N]\!]$. Prove that A intersects at most $\binom{k}{2}|A|^2$ k-APs in more than one point.
   (c) Prove the *canonical van der Waerden theorem*, which states the following. For every $k \geqslant 3$, there exists some N such that the following holds. In any coloring of $[\![N]\!]$ with an arbitrary number of colors, there is a monochromatic or rainbow k-AP.

6. The *Ramsey game* is played between two players, called Builder and Painter. The game starts with an infinite set of vertices and no edges. At every turn, Builder selects a pair of vertices that are not joined by an edge, and builds an edge between them. Painter then immediately has to assign that edge a color, red or blue. The game ends when a monochromatic K_k is produced. Builder's goal is to end the game as soon as possible, while Painter's goal is to continue for as long as possible. The *online Ramsey number*, denoted $\widetilde{r}(k)$, is the minimum number of edges built during the game if both players play optimally.
   (a) Prove that $\widetilde{r}(k) \geqslant \frac{1}{2} \cdot 2^{k/2}$.
   (b) Prove that $\widetilde{r}(k) \leqslant 4^k$.